International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 **Issue:** III **Month of publication:** March 2023

DOI: https://doi.org/10.22214/ijraset.2023.49478

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue III Mar 2023- Available at www.ijraset.com

Solutions of the Transcendental Equations

$$p^2 - \sqrt[3]{p^5 + q^5 - (pq)^3} + \sqrt[3]{r^2 + s^2} = k^3(n^2 + 1)$$

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Abstract: ***We make an effort and elucidate the integral solutions of the transcendental equation*** $p^2 - \sqrt[3]{p^5 + q^5 - (pq)^3} + \sqrt[3]{r^2 + s^2} = k^3(n^2 + 1)$ ***under multiple patterns with certain numerical examples.***

Keywords: Transcendental Equation, Integral solution, Diophantine equation.

I. INTRODUCTION

A transcendental equation is one with the transcendental functions of the variables that need to be resolved. These equations are solved easily until the variables are roughly known. Numerous equations in which the variables appear to provide an argument for only elementary solutions are used to solve transcendental functions.

We frequently label a function as transcendental when an analytical function cannot be solved by a polynomial equation. It cannot be formulated in terms of a finite sequence of addition, multiplication, and root extraction operations in pure mathematics.

The well-known transcendental functions include the logarithmic, exponential, trigonometric, hyperbolic, and inverse of all of the aforementioned. Some unexpected transcendental functions are included together with specific functions of analysis like elliptic zeta and gamma.

[1-2] has been recommended for fundamental notions and concepts in number theory. For fundamental theories and concepts regarding number theory, [3-5] has been analyzed. For Transcendental equation-related ideas and problems and various methods of solving Diophantine type equations [6-14] were observed.

II. TECHNIQUE FOR ANALYSIS

The equation to be solved is,

$$p^2 - \sqrt[3]{p^5 + q^5 - (pq)^3} + \sqrt[3]{r^2 + s^2} = k^3(n^2 + 1) \qquad (1)$$

The following linear transformation, $p = (u - v)^3$, $q = (v - u)^3$, $r = u^3 - 3uv^2$, $s = 3u^2v - v^3$ leads to $p^2 - \sqrt[3]{p^5 + q^5 - (pq)^3} + \sqrt[3]{r^2 + s^2} = u^2 + v^2$. Hence, $p^2 - \sqrt[3]{p^5 + q^5 - (pq)^3} + \sqrt[3]{r^2 + s^2} = u^2 + v^2$ reduces to,

$$u^2 + v^2 = k^3(n^2 + 1) \qquad (2)$$

Now we find various patterns of solutions of (1) using (2).

A. *Pattern 1*

Let $k = y^2 + z^2$, for $y, z \geq 0$

$$u^2 + v^2 = (y^2 + z^2)^3(n^2 + 1) \qquad (3)$$

Using factorization and equating real and imaginary parts we get,

$$u = n(y^3 - 3yz^2) - (3y^2z - z^3)$$

$$v = (y^3 - 3yz^2) + n(3y^2z - z^3)$$

Therefore, $u = nf(y,z) - g(y,z)$ and $v = f(y,z) + ng(y,z)$

where, $f(y,z) = y^3 - 3yz^2$ and $g(y,z) = 3y^2z - z^3$

Hence, for all $n \geq 0$ the non-zero integral solutions are found to be,

$$p = (n-1)f(y,z) - (n+1)g(y,z)$$
$$q = (n+1)g(y,z) - (n-1)f(y,z)$$
$$r = (nf(y,z) - g(y,z))^3 - 3(nf(y,z) - g(y,z))(f(y,z) + ng(y,z))^2$$
$$s = 3(nf(y,z) - g(y,z))^2 (f(y,z) + ng(y,z)) - (f(y,z) + ng(y,z))^3$$

Numerical examples satisfying the solution are listed below.

TABLE 1

n	p	q	r	s	k	LHS=RHS
1	-64	64	-64	0	2	16
2	-125	125	5500	-14625	5	625
0	-85184	85184	7696	30672	10	1000
3	1259712	-1259712	-511758	354294	9	7290

B. Pattern 2

Equation (3) can be rewritten as,

$$u^2 + v^2 = 1 \cdot (y^2 + z^2)^3 (n^2 + 1)$$

Taking , $1 = \left(\dfrac{24^2 + 10^2}{26^2} \right)$

$$u^2 + v^2 = \left(\frac{24^2 + 10^2}{26^2} \right)(y^2 + z^2)^3 (n^2 + 1)$$

Using factorization and equating real and imaginary parts we get,

$$u = \frac{n(24y^3 - 30y^2z - 72yz^2 + 10z^3) - (10y^3 + 72y^2z - 30yz^2 - 24z^3)}{26}$$
$$v = \frac{n(10y^3 + 72y^2z - 30yz^2 - 24z^3) + (24y^3 - 30y^2z - 72yz^2 + 10z^3)}{26}$$

Take $y = 26A$ and $z = 26B$ we get,

$$u = n(16224C^3 - 20280C^2D - 48672CD^2 + 6760D^3) - (6760C^3 + 48672C^2D - 20280CD^2 - 16224D^3)$$
$$v = n(6760C^3 + 48672C^2D - 20280CD^2 - 16224D^3) + (16224C^3 - 20280C^2D - 48672CD^2 + 6760D^3)$$

Therefore, $u = nf(C,D) - g(C,D)$ and $v = ng(C,D) + f(C,D)$

where, $f(C,D) = 16224C^3 - 20280C^2D - 48672CD^2 + 6760D^3$ and

$$g(C,D) = 6760C^3 + 48672C^2D - 20280CD^2 - 16224D^3$$

Hence, for all $n \geq 0$ the non-zero integral solutions are found to be,

$$p = (n-1)f(C,D) - (n+1)g(C,D)$$
$$q = (n+1)g(C,D) - (n-1)f(C,D)$$
$$r = (nf(C,D) - g(C,D))^3 - 3(nf(C,D) - g(C,D))(ng(C,D) + f(C,D))^2$$
$$s = 3(nf(C,D) - g(C,D))^2 (ng(C,D) + f(C,D)) - (ng(C,D) + f(C,D))^3$$

Numerical examples satisfying the solution are listed below.

TABLE 2

n	p	q	r	s	k	LHS=RHS
1	34163613499392	-34163613499392	5965781466112	-14150813867008	676	617831552
2	-1313364071305730	1313364071305730	-1008323104966530000	641018953024699000	6084	1125998003520
0	-3379862333173250	3379862333173250	-6064392324463620	-4560736135141890	3380	38614472000
3	-4711889860001790	4711889860001790	-3802441815457790	796834651897856	1352	24713262080

C. Pattern 3

Taking (3) as in pattern 2 and replacing '1' as $\left(\frac{48^2+20^2}{52^2}\right)$, we get $u^2+v^2=\left(\frac{48^2+20^2}{52^2}\right)(y^2+z^2)^3(n^2+1)$

Using factorization and equating real and imaginary parts we get,

$$u=\frac{n(48y^3-60y^2z-144yz^2+20z^3)-(20y^3+144y^2z-60yz^2-48z^3)}{52}$$

$$v=\frac{n(20y^3+144y^2z-60yz^2-48z^3)+(48y^3-60y^2z-144yz^2+20z^3)}{52}$$

Take $y=52A$ and $z=52B$ we get,

$$u=n(129792C^3-162240C^2D-389376CD^2+54080D^3)-(54080C^3+389376C^2D-162240CD^2-129792D^3)$$

$$v=n(54080C^3+389376C^2D-162240CD^2-129792D^3)+(129792C^3-162240C^2D-389376CD^2+54080D^3)$$

Therefore, $u=nf(C,D)-g(C,D)$ and $v=ng(C,D)+f(C,D)$

where, $f(C,D)=129792C^3-162240C^2D-389376CD^2+54080D^3$ and

$g(C,D)=54080C^3+389376C^2D-162240CD^2-129792D^3$

Hence, for all $n\geq 0$ the non-zero integral solutions are found to be,

$$p=(n-1)f(C,D)-(n+1)g(C,D)$$

$$q=(n+1)g(C,D)-(n-1)f(C,D)$$

$$r=(nf(C,D)-g(C,D))^3-3(nf(C,D)-g(C,D))(ng(C,D)+f(C,D))^2$$

$$s=3(nf(C,D)-g(C,D))^2(ng(C,D)+f(C,D))-(ng(C,D)+f(C,D))^3$$

Numerical examples satisfying the solution are listed below.

TABLE 3

n	p	q	r	s	k	LHS=RHS
1	17491770111688700	-17491770111688700	3054480110649340	-7245216699908100	2704	39541219328
2	-17491770111688700	17491770111688700	-13429144542414600000	8537279501179030000	10816	6326595092480
3	-316389224617869000000	316389224617869000000	122760934844727000000	4823079768752130000	13520	24713262080000
0	10122552147968000	-10122552147968000	57961733599264800	24435840885194800	5408	158164877312

D. Pattern 4

Taking (3) as in pattern2 and replacing 1 as $\left(\dfrac{40^2+30^2}{50^2}\right)$

$$u^2+v^2=\left(\frac{40^2+30^2}{50^2}\right)\left(y^2+z^2\right)^3\left(n^2+1\right)$$

Using factorization and equating real and imaginary parts we get,

$$u=\frac{n\left(40y^3-90y^2z-120yz^2+30z^3\right)-\left(30y^3+120y^2z-90yz^2-40z^3\right)}{50}$$

$$v=\frac{n\left(30y^3+120y^2z-90yz^2-40z^3\right)+\left(40y^3-90y^2z-120yz^2+30z^3\right)}{50}$$

Take $y=50A$ and $z=50B$ we get,

$$u=n\left(100000C^3-225000C^2D-300000CD^2+75000D^3\right)\text{-}\left(75000C^3+300000C^2D-225000CD^2-100000D^3\right)$$

$$v=n\left(75000C^3+300000C^2D-225000CD^2-100000D^3\right)+\left(100000C^3-225000C^2D-300000CD^2+75000D^3\right)$$

Therefore, $u=nf(C,D)-g(C,D)$ and $v=ng(C,D)+f(C,D)$

where, $f(C,D)=100000C^3-225000C^2D-300000CD^2+75000D^3$

and $g(C,D)=75000C^3+300000C^2D-225000CD^2-100000D^3$

Hence, for all $n\geq 0$ the non-zero integral solutions are found to be,

$$p=(n-1)f(C,D)-(n+1)g(C,D)$$

$$q=(n+1)g(C,D)-(n-1)f(C,D)$$

$$r=(nf(C,D)-g(C,D))^3-3(nf(C,D)-g(C,D))(ng(C,D)+f(C,D))^2$$

$$s=3(nf(C,D)-g(C,D))^2(ng(C,D)+f(C,D))-(ng(C,D)+f(C,D))^3$$

Numerical examples satisfying the solution are listed below.

TABLE 4

n	p	q	r	s	k	LHS=RHS
0	-244140625000000000	244140625000000000	-488281250000000000	-26855468750000000 00	12500	1953125000000
2	-1000000000000000000	1000000000000000000	-11000000000000000000	-2000000000000000000	10000	5000000000000
3	166375000000000000	-166375000000000000	-15031250000000000	-59906250000000000	2500	156250000000
1	4218750000000000000000	-4218750000000000000000	-1718750000000000000000	312500000000000000000	25000	31250000000000

III. CONCLUSION

In this post, we have explained the essential solutions to the transcendental equation $p^2 - \sqrt[3]{p^5 + q^5 - (pq)^3} + \sqrt[3]{r^2 + s^2} = k^3(n^2 + 1)$ under complex patterns utilizing various numerical examples. Additionally, one could look for detailed answers to these similar types of equations.

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