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Solving MADM Problems based on Matrix Games with a New Defuzzification Method for Triangular Intuitionistic Fuzzy Numbers

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Abstract: The paper discusses about a new defuzzification method called the Comprehensive Li-Samuel ranking for Triangular Intuitionistic Fuzzy Numbers (TrIFNs) which is improved from the Li-Samuel ranking method associated with membership function and Li-Samuel ranking method associated with non-membership function. The Comprehensive Li-Samuel ranking for TrIFNs is utilised to derive the weights for the Multiple Attribute Decision Making (MADM) problems under TrIFN. The MADM problem is solved based on two-person zero-sum game which is converted into a pair of LPP by using Comprehensive Li-Samuel ranking. The process of transforming decision matrix into a matrix game and deriving decision maker's and attribute weights from the matrix game are discussed. Numerical illustration is given to justify the viability and effectiveness of the proposed method and comparisons are made with existing ranking methods. The comparison study reveals that the proposed method provides the best maximum optimal solution for the decision problem than the existing methods.
Keywords: Multiple Attribute Decision Making, Triangular Intuitionistic Fuzzy Numbers, Defuzzification method, Ranking method, Matrix game.

I. INTRODUCTION

Multiple Attribute Decision Making (MADM) problems are more prevalent in real-life situations. The problem of MADM is to find the desirable solution from a limited number of possible alternatives tested for multiple attributes, both standardized and limited. To choose a desirable solution, the decision maker often provides his or her preferred information in the form of numerical values. However, in most cases, numerical values are insufficient to model real-life decision making problems. Indeed, human judgments including the details of preference information can be stated in the form of intuitionistic fuzzy knowledge. Therefore, the problem of MADM under intuitionistic fuzzy nature are an interesting study area for recent researchers. The details about the weights of the attribute can sometimes be known, partially known or be completely unknown. MADM problems are assumed to have a predetermined, limited number of alternatives. MADM problem solving involves filtering and ranking and it can be viewed as another way to integrate information into decision matrix together with additional information from the decision maker to obtain final ranking or selecting from the set of alternatives. In addition to the information contained in the decision matrix, it requires additional information from the decision maker to reach at a final ranking / selection. In many cases decision makers have vague information about alternatives with respect to attributes. The Triangular Intuitionistic Fuzzy Set (TrIFS), which has its membership function and non-membership function is applied by many researchers in decision making theory. Robinson [14] and Robinson & Amirtharaj [15],[16],[17],[18] proposed correlation coefficient for various higher order IFS and applied them in MAGDM problems. Robinson & Amirtharaj [19] have given a MAGDM analysis for triangular and trapezoidal intuitionistic fuzzy sets. Robinson et al. [20] has proposed a decision support systems miner algorithm to solve MAGDM problems. A biological application of MAGDM was proposed in Robinson et al. [13]. A method for solving Triangular Intuitionistic Fuzzy Linear Programming Problem was introduced in Nirmalsingh & Robinson [11]. Li [5],[7], Li & Yang [8], Li & Wan [3] and Li et al. [9] introduced some linear programming approaches to multi attribute decision making with IFS. Li [6] presented the decision making process and game theory under IFS. Nan & Li [10] and An & Li [1] introduced linear programming approach to solve matrix games under IFS. Li [4] has given some arithmetic operations on triangular intuitionistic fuzzy sets and proposed a ratio ranking method to rank the triangular intuitionistic fuzzy numbers. Wang & Kerre [21] have introduced some properties for ordering the fuzzy values. Bhaumik et al. [2] has introduced a linear programming approach to solve triangular intuitionistic fuzzy matrix games using robust ranking method. Nirmalsingh et al.

[12] have proposed Li-Samuel ranking method associated with membership function and Li-Samuel ranking method associated with non-membership function for defuzzification of TrIFNs. Xu et al. [22] have converted the decision information of the MADM problem to two-person zero-sum game and used linear programming to solve the matrix game.

Definition 1 [12]: The Li-Samuel ranking method associated with membership function, $LS_{\mu}\left(\tilde{A}\right)$ for $\tilde{A}=\left(a_1,a_2,a_3;u_{\tilde{A}},v_{\tilde{A}}\right)$ is defined as:

$$LS_{\mu}\left(\tilde{A}\right)=\left(\frac{1}{\sqrt{3}}\right)\left[\frac{\left(a_1+2a_2+a_3\right)}{2}\right]u_{\tilde{A}} \tag{1}$$

Definition 2 [12]: The Li-Samuel ranking method associated with non-membership function, $LS_{\gamma}\left(\tilde{A}\right)$ for TrIFN $\tilde{A}=\left(a_1,a_2,a_3;u_{\tilde{A}},v_{\tilde{A}}\right)$ is defined as:

$$LS_{\gamma}\left(\tilde{A}\right)=\left(\frac{1}{\sqrt{3}}\right)\times\left[\frac{\left(a_1+2a_2+a_3\right)}{2}\right]\left(1-v_{\tilde{A}}\right) \tag{2}$$

The Li-Samuel ranking associated with membership function eq.(1) considers the membership, and not the non-membership degree and Li-Samuel ranking associated with non-membership function eq.(2) considers the non-membership, and not the membership degree. This urges the need to propose a new ranking method which considers all the parameters of the TrIFNs, given in the following section.

II. A NOVEL DEFUZZIFICATION METHOD FOR TRIFNS

The Li-Samuel ranking which is defined in definition 1 and 2 are for membership and non-membership functions separately. The new Comprehensive Li-Samuel ranking takes the membership and non-membership into account and it is defined as follows:

$$LS_{\lambda}\left(\tilde{A}\right)=\lambda LS_{\mu}\left(\tilde{A}\right)+\left(1-\lambda\right)LS_{\gamma}\left(\tilde{A}\right)$$

$$LS_{\lambda}\left(\tilde{A}\right)=\left(\frac{1}{\sqrt{3}}\right)\times\left[\frac{\left(a_1+2a_2+a_3\right)}{2}\right][\lambda u_{\tilde{A}}+\left(1-\lambda\right)\left(1-v_{\tilde{A}}\right)] \tag{3}$$

Where $\lambda\in\left[0,1\right]$ represents the decision maker's preference level and $\lambda\in[0,1/2)$ means that the decision maker's preference level is pessimistic; $\lambda\in(1/2,1]$ means that the decision maker's preference level is optimistic; $\lambda=1/2$ means that the decision maker's preference level maintains neutrality. Thus, the new Comprehensive Li-Samuel ranking reflects the decision maker's subjective preference level.

Definition 3: Let $\tilde{A}=\left(a_1,a_2,a_3;u_{\tilde{A}},v_{\tilde{A}}\right)$ and $\tilde{B}=\left(b_1,b_2,b_3;u_{\tilde{B}},v_{\tilde{B}}\right)$ be two TrIFNs. Based on the Comprehensive Li-Samuel ranking, for $\lambda\in\left[0,1\right]$, we have:

- If $LS_{\lambda}\left(\tilde{A}\right)<LS_{\lambda}\left(\tilde{B}\right)$, then $\tilde{A}<\tilde{B}$;
- If $LS_{\lambda}\left(\tilde{A}\right)>LS_{\lambda}\left(\tilde{B}\right)$, then $\tilde{A}>\tilde{B}$;
- If $LS_{\lambda}\left(\tilde{A}\right)=LS_{\lambda}\left(\tilde{B}\right)$, then $\tilde{A}=\tilde{B}$;

Theorem 1: $LS_{\lambda}\left(\tilde{A}\right)$ is a continuous non-increasing function of the parameter $\lambda\in\left[0,1\right]$.

Proof. $LS_\lambda\left(\tilde{A}\right)$ is a linear function of the variable $\lambda\in[0,1]$. Hence $LS_\lambda\left(\tilde{A}\right)$ is continuous on $\lambda\in[0,1]$. The Partial derivative of $LS_\lambda\left(\tilde{A}\right)$ with respect to $\lambda\in[0,1]$ can be calculated as follows: $\dfrac{\partial LS_\lambda\left(\tilde{A}\right)}{\partial\lambda}=LS_\mu\left(\tilde{A}\right)-LS_\gamma\left(\tilde{A}\right)$.It is noted that $LS_\mu\left(\tilde{A}\right)\le LS_\gamma\left(\tilde{A}\right)$. Hence, $\dfrac{\partial LS_\lambda\left(\tilde{A}\right)}{\partial\lambda}\le 0$. Therefore, $LS_\lambda\left(\tilde{A}\right)$ is a non-increasing function of the parameter $\lambda\in[0,1]$.

Theorem 2: Let $\tilde{A}$ and $\tilde{B}$ be two TrIFNs with $u_{\tilde{A}}=u_{\tilde{B}}$ and $v_{\tilde{A}}=v_{\tilde{B}}$. Then for any $\lambda\in[0,1]$,
$LS_\lambda\left(\tilde{A}+\tilde{B}\right)=LS_\lambda\left(\tilde{A}\right)+LS_\lambda\left(\tilde{B}\right)$ is true.

Proof. We know that $\tilde{A}+\tilde{B}=\left(a_1+b_1,a_2+b_2,a_3+b_3;\wedge\{u_{\tilde{A}},u_{\tilde{B}}\},\vee\{v_{\tilde{A}},v_{\tilde{B}}\}\right)$.

Since $u_{\tilde{A}}=u_{\tilde{B}}$ and $v_{\tilde{A}}=v_{\tilde{B}}$ then let the minimum of membership is $\wedge\{u_{\tilde{A}},u_{\tilde{B}}\}=U$ and maximum of non-membership is $\vee\{v_{\tilde{A}},v_{\tilde{B}}\}=V$.

Now, the Comprehensive Li-Samuel ranking of $\tilde{A}+\tilde{B}$ for $\lambda\in[0,1]$ is as follows:

$$\begin{aligned}LS_\lambda\left(\tilde{A}+\tilde{B}\right)&=\left(\frac{1}{\sqrt{3}}\right)\times\left[\frac{(a_1+b_1+2(a_2+b_2)+a_3+b_3)}{2}\right][\lambda U+(1-\lambda)(1-V)]\\&=\left(\frac{1}{\sqrt{3}}\right)\times\left[\frac{(a_1+2a_2+a_3)}{2}+\frac{(b_1+2b_2+b_3)}{2}\right][\lambda U+(1-\lambda)(1-V)]\\&=\left(\frac{1}{\sqrt{3}}\right)\times\left[\frac{(a_1+2a_2+a_3)}{2}\right][\lambda U+(1-\lambda)(1-V)]\\&\quad+\left(\frac{1}{\sqrt{3}}\right)\times\left[\frac{(b_1+2b_2+b_3)}{2}\right][\lambda U+(1-\lambda)(1-V)]\\&=LS_\lambda\left(\tilde{A}\right)+LS_\lambda\left(\tilde{B}\right).\end{aligned}$$

III. SOLVING TRIFN TWO-PERSON ZERO-SUM GAME WITH LINEAR PROGRAMMING TECHNIQUES

Let $E_i(i=1,2,\dots,m)$ and $F_i(i=1,2,\dots,m)$ be the sets of all pure strategies for player I and player II in a matrix game respectively, where $\tilde{M}$ be the payoff matrix with TrIFN entries. The TrIFN matrix game is denoted by $\tilde{\Gamma}=\left(I,II,S_I,S_{II},\tilde{M}\right)$ where S_I is the mixed strategies of the Player I and S_{II} is the mixed strategies of the Player II. Each element of $\tilde{M}=\left(\tilde{m}_{ij}\right)_{m\times n}$ where $\tilde{m}_{ij}=\left((m_1)_{ij},(m_2)_{ij},(m_3)_{ij};u_{\tilde{m}_{ij}},v_{\tilde{m}_{ij}}\right)$ informs us about the knowledge player I and player II indicate on their own payoffs provided that the player I and player II chooses pure strategy. The TrIFNs in the payoff matrix are assumed to be positive for simplicity.

The optimal strategies can be found using a pair of intuitionistic fuzzy linear programming problems which is given by;

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$$\max \tilde{X}$$
$$s.t\begin{cases}\sum_{i=1}^{m}\tilde{m}_{ij}w_i \geq \tilde{X},\ j=1,2,\dots,n \\ \sum_{i=1}^{m}w_i=1, w_i\geq 0, i=1,2,\dots,m\end{cases} \tag{4}$$

and

$$\min \tilde{Y}$$
$$s.t\begin{cases}\sum_{j=1}^{n}\tilde{m}_{ij}\omega_j \leq \tilde{Y}, i=1,2,\dots,m \\ \sum_{j=1}^{n}\omega_j=1, \omega_j\geq 0,\ j=1,2,\dots,n\end{cases} \tag{5}$$

By using the Comprehensive Li-Samuel ranking (3) the TrIFNs are converted into crisp numbers and the pair of intuitionistic fuzzy LPP are transformed into classic LPP by defuzzification method.

$$\max LS_{\lambda}(\tilde{X})$$
$$s.t\begin{cases}\sum_{i=1}^{m}LS_{\lambda}(\tilde{m}_{ij})w_i \geq LS_{\lambda}\left(\tilde{X}\right),\ j=1,2,\dots,n \\ \sum_{i=1}^{m}w_i=1, w_i\geq 0, i=1,2,\dots,m\end{cases}$$

and

$$\min LS_{\lambda}(\tilde{Y})$$
$$s.t\begin{cases}\sum_{j=1}^{n}LS_{\lambda}(\tilde{m}_{ij})\omega_j \leq LS_{\lambda}(\tilde{Y}), i=1,2,\dots,m \\ \sum_{j=1}^{n}\omega_j=1, \omega_j\geq 0,\ j=1,2,\dots,n\end{cases}$$

The above LPP can be expanded into the following form,

$$\max X$$
$$s.t\begin{cases}\sum_{i=1}^{m}\left(\frac{1}{\sqrt{3}}\right)\left[\frac{(m_1)_{ij}+2(m_2)_{ij}+(m_3)_{ij}}{2}\right][\lambda u_{\tilde{m}_{ij}}+(1-\lambda)\left(1-v_{\tilde{m}_{ij}}\right)]w_i \geq X,\ j=1,2,\dots,n \\ \sum_{i=1}^{m}w_i=1, w_i\geq 0, i=1,2,\dots,m\end{cases} \tag{6}$$

and

$$\min Y$$
$$s.t\begin{cases}\sum_{j=1}^{n}\left(\frac{1}{\sqrt{3}}\right)\left[\frac{(m_1)_{ij}+2(m_2)_{ij}+(m_3)_{ij}}{2}\right][\lambda u_{\tilde{m}_{ij}}+(1-\lambda)\left(1-v_{\tilde{m}_{ij}}\right)]\omega_j \leq Y, i=1,2,\dots,m \\ \sum_{j=1}^{n}\omega_j=1, \omega_j\geq 0,\ j=1,2,\dots,n\end{cases} \tag{7}$$

where X, Y, w_i and ω_j are decision variables; $\lambda \in [0,1]$ is a parameter. For a specific λ, the optimal solutions of the pair of linear programming problem are the optimal strategies of the TrIFN two-person zero-sum games.

IV. ALGORITHM FOR MADM PROBLEM UNDER TRIFN WITH DEFUZZIFICATION METHOD

- Step 1: Consider a normalized decision matrix $\tilde{D}$ where each alternative $A_i (i = 1, 2, ..., m)$ is evaluated against each attribute $C_j (j = 1, 2, ..., n)$ by the decision maker in terms of TrIFNs and the decision maker weights and attribute weights are unknown.
- Step 2: Construct a TrIFN two person zero sum game with A_i and C_j as the set of all pure strategies for player I and player II and $\tilde{D}$ be the payoff matrix whose entries are in TrIFNs. The TrIFN matrix game is denoted by $\tilde{\Gamma} = (I, II, w, \omega, \tilde{D})$ where $w = \left\{(w_1, w_2, ...w_m) \mid \sum_{i=1}^{m} w_i = 1, w_i \geq 0\right\}$ be the mixed strategies of player I and $\omega = \left\{(\omega_1, \omega_2, ...\omega_n) \mid \sum_{j=1}^{n} \omega_j = 1, \omega_j \geq 0\right\}$ be the mixed strategies of player II.
- Step 3: Construct a pair of TrIFN linear programming problems from the TrIFN two-person zero-sum game by using the eqs. (4) and (5).
- Step 4: Convert the pair of TrIFN LPPs into crisp LPPs using the proposed Comprehensive Li-Samuel ranking method using eqs. (6) and (7).
- Step 5: Solve the pair of LPPs to obtain the optimal mixed strategies.
- Step 6: Find the collective overall TrIFN of the alternatives by calculating the expected score of TrIFN in which the optimal mixed strategies is used as the attribute weights.
- Step 7: Rank all the alternatives using the proposed Comprehensive Li-Samuel ranking method with neutral preference level $\lambda = 1/2$.
- Step 8: Select the alternative with highest ranking value as the best alternative.

V. NUMERICAL ILLUSTRATION

A numerical illustration is given here to demonstrate the process of the proposed method. A selection of video monitoring system for a school campus is taken as a decision problem in [22]. The same decision problem is considered here in order to give a comparative study of the proposed method. The normalized decision matrix $\tilde{D}$ with alternatives A_1, A_2, A_3 and A_4, and with attributes C_1, C_2, C_3, C_4 and C_5 is given below:

Table 1: Normalized Decision Matrix

$\tilde{D}$	C_1	C_2	C_3	C_4	C_5
A_1	(0.3,0.4,0.9; 0.6,0.1)	(0.3,0.5,0.7; 0.5,0.4)	(0.5,0.7,0.8; 0.5,0.2)	(0.6,0.8,0.9; 0.7,0.3)	(0.4,0.5,0.6; 0.5,0.3)
A_2	(0.4,0.5,1; 0.5,0.2)	(0.5,0.6,0.7; 0.4,0.2)	(0.4,0.5,0.6; 0.5,0.1)	(0.6,0.7,0.8; 0.6,0.2)	(0.3,0.4,1; 0.6,0.4)
A_3	(0.2,0.4,0.5; 0.5,0.3)	(0.6,0.8,1; 0.7,0.1)	(0.4,0.6,0.8; 0.5,0.1)	(0.4,0.6,1; 0.4,0.4)	(0.4,0.5,0.6; 0.6,0.2)
A_4	(0.3,0.4,0.8; 0.6,0.1)	(0.4,0.5,0.6; 0.5,0.2)	(0.6,0.8,1; 0.4,0.3)	(0.4,0.6,0.8; 0.6,0.2)	(0.5,0.6,0.7; 0.6,0.3)

The conditions for attribute weights given by the decision maker are:

$\omega_1 + \omega_2 < 0.4;\ 2\omega_2 > \omega_1;\ \omega_3 > 0.05;\ \omega_4 > 2\omega_3;\ \omega_5 > 0.05.$

Step 2: The above decision matrix is transformed into $\tilde{\Gamma} = \left(I, II, w, \omega, \tilde{D}\right)$ a two-person zero-sum game with alternatives A_i $(i = 1, 2, 3, 4)$ and attributes C_j (j=1,2,…,5) as pure strategies. $w = \left\{\left(w_1, w_2, \ldots w_m\right) | \sum_{i=1}^{m} w_i = 1, w_i \geq 0\right\}$ and $\omega = \left\{\left(\omega_1, \omega_2, \ldots \omega_n\right) | \sum_{j=1}^{n} \omega_j = 1, \omega_j \geq 0\right\}$ are mixed strategies of decision maker and attribute. The decision matrix $\tilde{D} = (\tilde{m}_{ij})_{m \times n}$ represents the payoff matrix.

Step 3: The TrIFN matrix game is converted to a pair of TrIFN linear programming problem given below:

$$\max \tilde{X}$$

$$s.t \begin{cases} \tilde{m}_{11} w_1 + \tilde{m}_{21} w_2 + \tilde{m}_{31} w_3 + \tilde{m}_{41} w_4 \geq \tilde{X}, \\ \tilde{m}_{12} w_1 + \tilde{m}_{22} w_2 + \tilde{m}_{32} w_3 + \tilde{m}_{42} w_4 \geq \tilde{X}, \\ \tilde{m}_{13} w_1 + \tilde{m}_{23} w_2 + \tilde{m}_{33} w_3 + \tilde{m}_{43} w_4 \geq \tilde{X}, \\ \tilde{m}_{14} w_1 + \tilde{m}_{24} w_2 + \tilde{m}_{34} w_3 + \tilde{m}_{44} w_4 \geq \tilde{X}, \\ \tilde{m}_{15} w_1 + \tilde{m}_{25} w_2 + \tilde{m}_{35} w_3 + \tilde{m}_{45} w_4 \geq \tilde{X}, \\ \sum_{i=1}^{4} w_i = 1, w_i \geq 0, i = 1, 2, \ldots, 4 \end{cases}$$

$$\min \tilde{Y}$$

$$s.t \begin{cases} \tilde{m}_{11} \omega_1 + \tilde{m}_{12} \omega_2 + \tilde{m}_{13} \omega_3 + \tilde{m}_{14} \omega_4 + \tilde{m}_{15} \omega_5 \leq \tilde{Y}, \\ \tilde{m}_{21} \omega_1 + \tilde{m}_{22} \omega_2 + \tilde{m}_{23} \omega_3 + \tilde{m}_{24} \omega_4 + \tilde{m}_{25} \omega_5 \leq \tilde{Y}, \\ \tilde{m}_{31} \omega_1 + \tilde{m}_{32} \omega_2 + \tilde{m}_{33} \omega_3 + \tilde{m}_{34} \omega_4 + \tilde{m}_{35} \omega_5 \leq \tilde{Y}, \\ \tilde{m}_{41} \omega_1 + \tilde{m}_{42} \omega_2 + \tilde{m}_{43} \omega_3 + \tilde{m}_{44} \omega_4 + \tilde{m}_{45} \omega_5 \leq \tilde{Y}, \\ \omega_1 + \omega_2 < 0.4; 2\omega_2 > \omega_1; \omega_3 > 0.05; \omega_4 > 2\omega_3; \omega_5 > 0.05 \\ \sum_{j=1}^{5} \omega_j = 1, j = 1, 2, \ldots, 5 \end{cases}$$

Step 4: Applying the Comprehensive Li-Samuel ranking method for $\lambda = 0.5$ as a decision parameter to convert the pair TrIFN linear programming to classic linear programming problems are given below:

$$\max X$$

$$s.t \begin{cases} 0.4330 w_1 + 0.4503 w_2 + 0.2598 w_3 + 0.4114 w_4 \geq X, \\ 0.3175 w_1 + 0.4157 w_2 + 0.7390 w_3 + 0.3753 w_4 \geq X, \\ 0.5066 w_1 + 0.4041 w_2 + 0.4850 w_3 + 0.5081 w_4 \geq X, \\ 0.6264 w_1 + 0.5658 w_2 + 0.3753 w_3 + 0.4850 w_4 \geq X, \\ 0.3464 w_1 + 0.3637 w_2 + 0.4041 w_3 + 0.4503 w_4 \geq X, \\ \sum_{i=1}^{4} w_i = 1, w_i \geq 0, i = 1, 2, \ldots, 4 \end{cases}$$

$$\min Y$$
$$s.t\begin{cases} 0.4330\omega_1 + 0.3175\omega_2 + 0.5066\omega_3 + 0.6264\omega_4 + 0.3464\omega_5 \le Y, \\ 0.4503\omega_1 + 0.4157\omega_2 + 0.4041\omega_3 + 0.5658\omega_4 + 0.3637\omega_5 \le Y, \\ 0.2598\omega_1 + 0.7390\omega_2 + 0.4850\omega_3 + 0.3753\omega_4 + 0.4041\omega_5 \le Y, \\ 0.4114\omega_1 + 0.3753\omega_2 + 0.5081\omega_3 + 0.4850\omega_4 + 0.4503\omega_5 \le Y, \\ \omega_1 + \omega_2 < 0.4; 2\omega_2 > \omega_1; \omega_3 > 0.05; \omega_4 > 2\omega_3; \omega_5 > 0.05 \\ \sum_{j=1}^{5} \omega_j = 1,\ j = 1,2,\dots,5 \end{cases}$$

Step 5: By solving the above problems we can obtain the optimal strategy (w,ω), where $w = (0, 0.3679, 0.0690, 0.5631)$ and $\omega = (0.2184, 0.1816, 0.05, 0.1, 0.45)$.

Step 6: Calculate the expected score of the alternatives by using the decision maker weight w and attribute weight ω which are given below:

$\tilde{A}_1 = (0,0,0;0.5,0.4)$,

$\tilde{A}_2 = (0.1446, 0.1814, 0.3331; 0.4, 0.4)$,

$\tilde{A}_3 = (0.0271, 0.0378, 0.0483; 0.4, 0.4)$,

$\tilde{A}_4 = (0.2439, 0.3086, 0.4103; 0.4, 0.3)$.

Step 7: Ranking all the alternatives with Comprehensive Li-Samuel ranking method for $\lambda = 0.5$ are given below:

$R_{0.5}\left(\tilde{A}_1\right) = 0;$

$R_{0.5}\left(\tilde{A}_2\right) = 0.1213;$

$R_{0.5}\left(\tilde{A}_3\right) = 0.0218;$

$R_{0.5}\left(\tilde{A}_4\right) = 0.2019.$

Step 8: The order of the alternatives according to the proposed ranking method is $A_4 > A_2 > A_3 > A_1$. Hence the alternative with the greatest value A_4 is selected as the best.

Table 2: Comparison of proposed method with existing methods.

Model	**Optimal solution of the LPP**	**Ranking**
Xu et al. [22]	Max X=0.2879 $w = (0, 0.46, 0.10, 0.44)$ Min Y=0.3037 $\omega = (0.2667, 0.1333, 0.05, 0.1992, 0.3508)$	$A_4 > A_2 > A_3 > A_1$
Comprehensive Li-Samuel ranking	Max X=0.4152 $w = (0, 0.3679, 0.0690, 0.5631)$ Min Y=0.4345 $\omega = (0.2184, 0.1816, 0.05, 0.1, 0.45)$	$A_4 > A_2 > A_3 > A_1$
Comprehensive Li-Samuel ranking coupled with Xu et al. [22]	Max X=0.3484 $w = (0, 0.3962, 0.2154, 0.3883)$ Min Y=0.3578 $\omega = (0.2667, 0.1333, 0.05, 0.1982, 0.3518)$	$A_4 > A_2 > A_3 > A_1$

In Table 2, the result of MADM proplem using Xu et al. [22] method, the Comprehensive Li-Samuel ranking method and Comprehensive Li-Samuel ranking coupled with Xu et al. [22] method are observed to be consistent. The defuzzification in the Comprehensive Li-Samuel ranking method is done by converting each TrIFN co-efficient in LPP into crisp values, whereas in Comprehensive Li-Samuel ranking coupled with Xu et al. [22] method, the defuzzification is done by taking the minimum of membership degree and maximum of non-membership in each constraint of the LPP.

Table 3: Computations with Comprehensive Li-Samuel Ranking

Sl.no.	λ	Ranking
1	0	$A_4 > A_2 > A_3 > A_1$
2	0.1	$A_4 > A_2 > A_3 > A_1$
3	0.2	$A_4 > A_2 > A_3 > A_1$
4	0.3	$A_4 > A_2 > A_3 > A_1$
5	0.4	$A_4 > A_2 > A_3 > A_1$
6	0.5	$A_4 > A_2 > A_3 > A_1$
7	0.6	$A_4 > A_2 > A_3 > A_1$
8	0.7	$A_4 > A_2 > A_3 > A_1$
9	0.8	$A_4 > A_2 > A_3 > A_1$
10	0.9	$A_4 > A_2 > A_3 > A_1$
11	1	$A_4 > A_2 > A_3 > A_1$

VI. DISCUSSION

In order to overcome the limitation of the Li-Samuel ranking method, it is improved to consider both membership and non-membership functions which is called Comprehensive Li-Samuel ranking. Some properties and theorems are proved for the Comprehensive Li-Samuel ranking method are given. An algorithm for MADM problem under TrIFN with the Comprehensive Li-Samuel ranking method is proposed in which the decision matrix is considered as a TrIF two-person zero-sum game and solved using linear programming techniques. The TrIF two-person zero-sum game is solved by constructing a pair of TrIF linear programming problems then the TrIFNs are transformed into crisp values using the proposed Comprehensive Li-Samuel ranking method with neutral preference level $\lambda = 1/2$ then the pair of LPPs are solved and the optimal solutions are taken as attribute weights with which the collective overall TrIFN of alternatives are found using expected score. Finally the alternatives are ranked using the proposed Comprehensive Li-Samuel ranking method and the best alternative is selected. In the numerical illustration the same decision problem from [22] is considered here to give a comparative study of the proposed method. The result comparison between the current method and the Xu et al. [22] is given in Table 2. In that comparison all the result shows that A_4 is the best alternative even the ranking orders are the same but the optimal solution of the LPP constructed by the proposed Comprehensive Li-Samuel ranking method gives the maximum value of X which is greater than the maximum X value given by the LPP by Xu et al. [22]. Further the Xu et al. [22] method uses only the minimum membership value and maximum membership value of the matrix game while constructing the LPP whereas in the proposed method Comprehensive Li-Samuel ranking the membership and non-membership value of each TrIFN element in the matrix game is considered while constructing the LPP. It is shown in Table 3 that for different value of $\lambda \in [0,1]$ same order of ranking $A_4 > A_2 > A_3 > A_1$ occurs. In Xu et al. [22] for $0 \le \lambda < 0.5$ the alternative A_4 obtains the highest value, for $0.5 \le \lambda < 1$ the alternative A_2 obtains the highest value.

Even though the order of ranking in all the above discussed methods are same, the maximum value for the optimal solution is obtained from the newly proposed method.

VII. CONCLUSION

In this article, the Li-Samuel ranking method is improved and defined Comprehensive Li-Samuel ranking method. Then the process of transforming the decision matrix of the MADM problem into a two-person zero-sum matrix game is defined. Then the matrix game is solved by converting it into a pair of TrIFN linear programming problems and the problems are changed into crisp liner programming problem using the proposed Comprehensive Li-Samuel ranking method. The optimal solution of the linear programming problems are the optimal mixed strategies of the matrix game. Then the optimal mixed strategies are used as decision maker's weights to find the expected score of the alternatives. The alternatives are ranked using the proposed Comprehensive Li-Samuel ranking method. The alternative with the highest ranking is selected as the best alternative. A numerical illustration of a video monitoring system selection problem is solved using the proposed MADM algorithm and the results are compared with the existing methods presented in the literature. The comparative study reveals the effectiveness and applicability of the proposed methods and algorithms.

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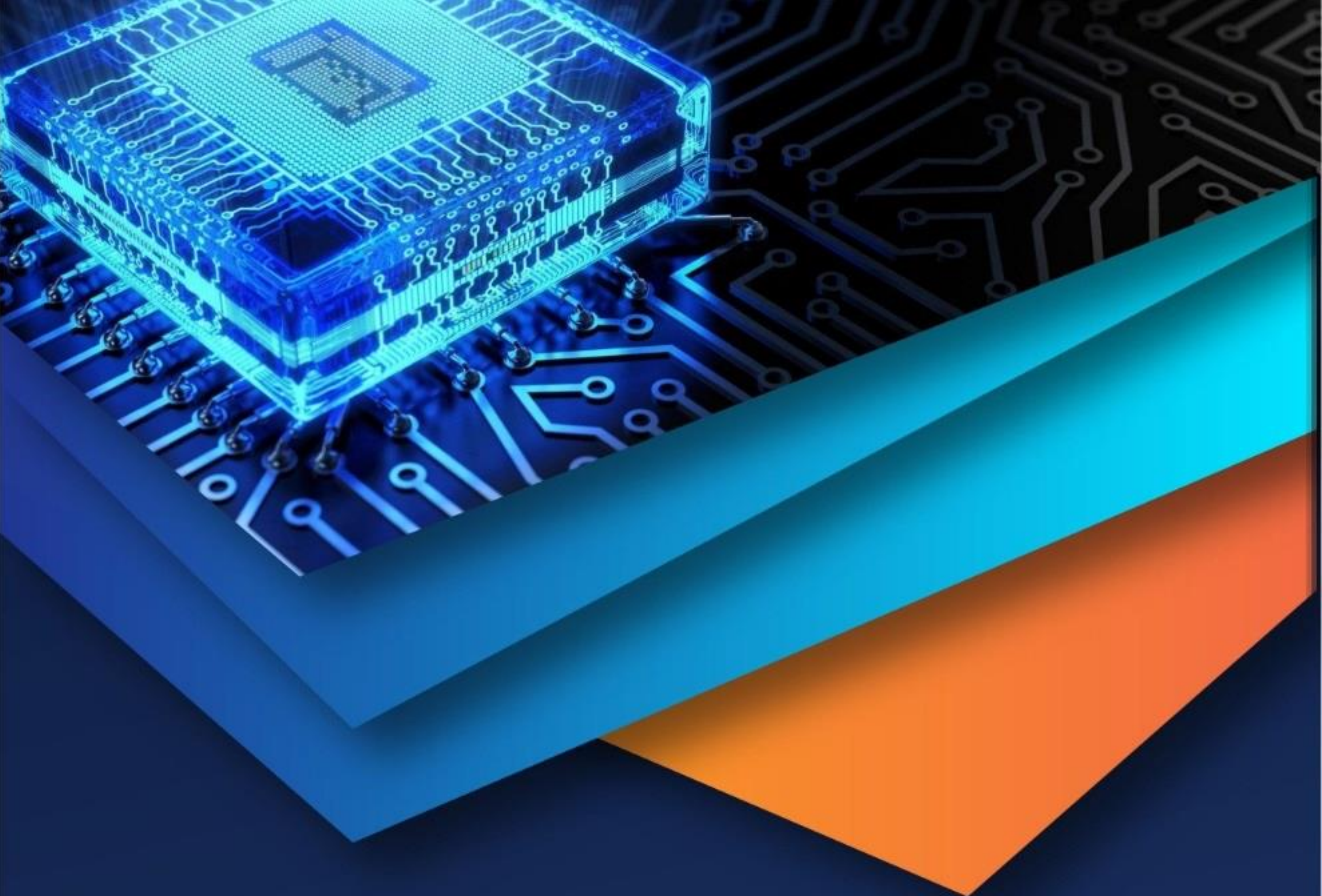

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