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Some Structures of Neutrosophic Fuzzy Relations over Neutrosophic Fuzzy Sets

P. Geethanjali[1], V. Shalini[2]
[1]Assistant Professor, [2]PG Scholar, PG and Research Department of Mathematics, Cauvery College for Women(Autonomous), Trichy-18

***Abstract:** Neutrosophic fuzzy relations are crucial in real-world situations when uncertainties exist regarding the truth, indeterminacy, and falsity membership degrees of an element. The topic is attracting a lot of attention from academics these days since it makes use of neutrosophic fuzzy relations, which facilitate and expedite decision-making. For instance, neutrosophic fuzzy relations over neutrosophic fuzzy sets are defined in this paper. The neutrosophic fuzzy relations over neutrosophic fuzzy sets are characterized in terms of several features. Finally, a few properties of the fuzzy relations over a neutrosophic fuzzy set are investigated.*
***Keywords:** Neutrosophic Fuzzy Sets, Neutrosophic Fuzzy Relations, Neutrosophic Fuzzy Inverse Relations.*

I. INTRODUCTION

L.A. Zadeh [25] invented fuzzy set theory, and Atanassov [1] developed the intuitionistic fuzzy set and fuzzy set extension. An element can include membership degrees of truth, indeterminacy, and falsity, since it is an extension of an intuitionistic fuzzy set. Nevertheless, neutrosophic fuzzy sets have been the subject of current attention for a number of educators. Since the neutrosophic fuzzy set was developed, it has been recognized as a powerful mathematical method that functions well in situations where there are more yeses, absence, no, and rejection replies in human perspectives. Kaufmann[18] and Zadeh[26] were the next to construct fuzzy relations. Kalaiarasi and Geethanjali [14][15] have also expressed fuzzy graphs concept.

Several authors have also looked into it, such as Zimmerman [27] and Klir and Yaun [19]. Many academics have now used it extensively in other fields, such as fuzzy reasoning, fuzzy control, fuzzy comprehensive evaluation[21][11][9], medical diagnostics, clustering analysis, and decision-making [2] [5] [24] [22]. Bruillo and Bustince gave the definition of intuitionistic fuzzy relations[4][3] and discussed some of its features. More study on intuitionistic fuzzy relations and their composition operation was done in 2005 by Lei et al[20]. In 2005, it was also shown that there are fourteen intuitionistic fuzzy relations and how they are composed. Yang developed the idea of intuitionistic fuzzy relation[23] kernels and closures in addition to demonstrating fourteen intuitionistic fuzzy relation theorems. Kalaiarasi and Mahalakshmi [16][17] gave the definition of strong fuzzy graph and discussed some of its features. Kalaiarasi and Divya[12][13] have also expressed fuzzy graph concept.

B.C. Cuong established the idea of neutrosophic fuzzy relations and looked into some of its related properties[6][7]. In this paper, we define the neutrosophic fuzzy relation over the neutrosophic fuzzy set. Examples of various processes utilizing this neutrosophic fuzzy relation are given.

This article is organized as follows: Section 2 presents some preliminary findings that are necessary to comprehend the remainder of the article. In section 3, some structural features of neutrosophic fuzzy relations over neutrosophic fuzzy sets are illustrated. Section 4 describes some properties of neutrosophic fuzzy relations in neutrosophic fuzzy sets. Lastly, a few characteristics of the neutrosophic fuzzy relations over a neutrosophic fuzzy set are explored.

II. PRELIMINARIES

A. Definition 2.1[25]

Let X be non-empty set. A set A in X is given by

$$A = \{ (x, \mu_A(x)) : x \in X \},$$

Where $\mu_A : X \to [0,1]$.

B. Definition 2.2[1]

An intuitionistic fuzzy set A in X is given by

$$A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \},$$

Where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, with the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1; \forall x \in X$$

The values $\mu_A(x)$ and $\nu_A(x)$ represent the membership degree and non-membership degree respectively of the elements X to the set A.

For any intuitionistic fuzzy set A on the universal set X, for $x \in X$

$$\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$$

is called the hesitancy degree (or intuitionistic fuzzy index) of an element X in A.It is the degree of indeterminacy membership of the element X whether belonging to A or not.

Obviously, $0 \le \pi_A(x) \le 1$ for any $x \in X$.

Particularly, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is always valid for any fuzzy set A on the universal set X.

The set of all intuitionistic fuzzy sets in X will be denoted by $IFS(X)$.

C. Definition 2.3

A neutrosophic fuzzy set A_{NS} on a universe of discourse X is of the form

$$A_{NS} = \{ (x, T_{A_{NS}}(x), I_{A_{NS}}(x), F_{A_{NS}}(x)) : x \in X \},$$

Where $T_{A_{NS}}(x) \in [0,1]$ is called the degree of truth-membership of X in A_{NS}

$I_{A_{NS}}(x) \in [0,1]$ is called the degree of indeterminacy-membership of X in A_{NS}

$F_{A_{NS}}(x) \in [0,1]$ is called the degree of falsity-membership of X in A_{NS}, and where

$T_{A_{NS}}(x)$, $I_{A_{NS}}(x)$ and $F_{A_{NS}}(x)$ satisfy the following condition :

$$0 \le T_{A_{NS}}(x) + I_{A_{NS}}(x) + F_{A_{NS}}(x) \le 3; \forall x \in X$$

Here $3 - (T_{A_{NS}}(x) + I_{A_{NS}}(x) + F_{A_{NS}}(x)); \forall x \in X$ is called the degree of refusal membership of X in A_{NS}.

The set of all neutrosophic fuzzy sets in X will be denoted by $NFS(X)$.

D. Definition 2.4

Let $P, Q \in NFS(X)$, then the subset, equality, the union, intersection and complement are defined as follows:

1) $P \subseteq Q$ iff $\forall x \in X, T_P(x) \le T_Q(x), I_P(x) \le I_Q(x)$ and $F_P(x) \ge F_Q(x)$
2) $P = Q$ iff $\forall x \in X, T_P(x) = T_Q(x), I_P(x) = I_Q(x)$ and $F_P(x) = F_Q(x)$
3) $P \cup Q = \{ (x, \max(T_P(x), T_Q(x)), \min(I_P(x), I_Q(x)), \min(F_P(x), F_Q(x))) : x \in X \}$
4) $P \cap Q = \{ (x, \min(T_P(x), T_Q(x)), \min(I_P(x), I_Q(x)), \max(F_P(x), F_Q(x))) : x \in X \}$
5) $P^c = \{ (x, T_P(x), I_P(x), F_P(x)) : x \in X \}$.

E. Definition 2.5

Let A, B and C be ordinary non-empty sets. A neutrosophic fuzzy relation R is a neutrosophic fuzzy subset of $A \times B$ i.e. R given by

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$$R=\{((a,b),T_R(a,b),I_R(a,b),F_R(a,b)):a\in A,b\in B\}$$

Where $T_R : A\times B\to[0,1]$, $I_R : A\times B\to[0,1]$, $F_R : A\times B\to[0,1]$ satisfying the condition $0\le T_R(a,b)+I_R(a,b)+F_R(a,b)\le 3$ for every $(a,b)\in(A\times B)$.

The set of all neutrosophic fuzzy relations in $A\times B$ will be denoted by $NFR(A\times B)$.

F. *Definition 2.6*

Let R and P be two neutrosophic fuzzy relations between A and B ,for every $(a,b)\in A\times B$ we define

1) $R\le P \Leftrightarrow (T_R(a,b)\le T_P(a,b))$ and $(I_R(a,b)\le I_P(a,b))$ and $(F_R(a,b)\ge F_P(a,b))$
2) $R\vee P=\{(a,b),T_R(a,b)\vee T_P(a,b),I_R(a,b)\wedge I_P(a,b),F_R(a,b)\wedge F_P(a,b):a\in A,b\in B\}$
3) $R\wedge P=\{(a,b),T_R(a,b)\wedge T_P(a,b),I_R(a,b)\wedge I_P(a,b),F_R(a,b)\vee F_P(a,b):a\in A,b\in B\}$
4) $R^c=\{((a,b),T_R(a,b),I_R(a,b),F_R(a,b)):a\in A,b\in B\}$

Here, $\vee$ and $\wedge$ denote the maximum and minimum operators respectively.

G. *Definition 2.7*

Let $R\in NFR(A\times B)$ and $S\in NFR(B\times C)$. Then the composition of R and S is the NFR from A to C defined as

$$R\circ S=\{((a,c),T_{R\circ S}(a,c),I_{R\circ S}(a,c),F_{R\circ S}(a,c)):a\in A,c\in C\}$$

where $T_{R\circ S}(a,c)=\vee_{b\in B}\{T_S(a,b)\wedge T_R(b,c)\}$,

$I_{R\circ S}(a,c)=\wedge_{b\in B}\{I_S(a,b)\wedge I_R(b,c)\}$ and

$F_{R\circ S}(a,c)=\wedge_{b\in B}\{F_S(a,b)\vee F_R(b,c)\}$.

III. SOME STRUCTURES OF NEUTROSOPHIC FUZZY RELATIONS

A. *Definition 3.1*

Let $R\in NFR(A\times B)$.We define the inverse relation R^{-1} between B and A :

$$T_{R^{-1}}(b,a)=T_R(a,b),\ I_{R^{-1}}(b,a)=I_R(a,b),\ F_{R^{-1}}(b,a)=F_R(a,b),\ \forall(a,b)\in A\times B.$$

B. *Theorem 3.1*

Let $R\in NFR(P\times Q)$ be a neutrosophic fuzzy relation. Then $(R^{-1})^{-1}=R$.

Proof:

By the definition of inverse relation, we have

$$T_{R^{-1}}(b,a)=T_R(a,b),\ \ I_{R^{-1}}(b,a)=I_R(a,b),\ \ F_{R^{-1}}(b,a)=F_R(a,b).$$

Now, $T_{(R^{-1})^{-1}}(b,a)=T_{R^{-1}}(a,b)=T_R(b,a)$

$I_{(R^{-1})^{-1}}(b,a)=I_{R^{-1}}(a,b)=I_R(b,a)$ and

$F_{(R^{-1})^{-1}}(b,a)=F_{R^{-1}}(a,b)=F_R(b,a)$.

Hence $(R^{-1})^{-1}=R$.

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C. Theorem 3.2

Let $R_1, R_2 \in NFR(P \times Q)$ be two neutrosophic fuzzy relations. Then

1) $(R_1 \cup R_2)^{-1} = R_1^{-1} \cup R_2^{-1}$.
2) $(R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1}$.

Proof:

1) By the definition of inverse relation, we have
$T_{R_1^{-1}}(b,a) = T_{R_1}(a,b), I_{R_1^{-1}}(b,a) = I_{R_1}(a,b)$, $F_{R_1^{-1}}(b,a) = F_{R_1}(a,b)$ and $T_{R_2^{-1}}(b,a) = T_{R_2}(a,b)$,
$I_{R_2^{-1}}(b,a) = I_{R_2}(a,b)$, $F_{R_2^{-1}}(b,a) = F_{R_2}(a,b)$.

Therefore,

$$
\begin{aligned}
T_{(R_1 \cup R_2)^{-1}}(b,a) &= T_{R_1 \cup R_2}(a,b) \\
&= \max\{T_{R_1}(a,b), T_{R_2}(a,b)\} \\
&= \max\{T_{R_1^{-1}}(b,a), T_{R_2^{-1}}(b,a)\} \\
&= T_{R_1^{-1} \cup R_2^{-1}}(b,a)
\end{aligned}
$$

$$
\begin{aligned}
I_{(R_1 \cup R_2)^{-1}}(b,a) &= I_{R_1 \cup R_2}(a,b) \\
&= \min\{I_{R_1}(a,b), I_{R_2}(a,b)\} \\
&= \min\{I_{R_1^{-1}}(b,a), I_{R_2^{-1}}(b,a)\} \\
&= I_{R_1^{-1} \cup R_2^{-1}}(b,a)
\end{aligned}
$$

And
$$
\begin{aligned}
F_{(R_1 \cup R_2)^{-1}}(b,a) &= F_{R_1 \cup R_2}(a,b) \\
&= \min\{F_{R_1}(a,b), F_{R_2}(a,b)\} \\
&= \min\{F_{R_1^{-1}}(b,a), F_{R_2^{-1}}(b,a)\} \\
&= F_{R_1^{-1} \cup R_2^{-1}}(b,a)
\end{aligned}
$$

Hence $(R_1 \cup R_2)^{-1} = R_1^{-1} \cup R_2^{-1}$.

Similarly 2),

Hence $(R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1}$.

IV. NEUTROSOPHIC FUZZY RELATION IN A NEUTROSOPHIC FUZZY SET

A. Definition 4.1

The relation $R \in NFR(A \times A)$ is called:

1) Reflexive if $T_R(a,a) = 1, I_R(a,a) = 0$ and $F_R(a,a) = 0; \forall a \in A$.
2) Anti-reflexive if $T_R(a,a) = 0, I_R(a,a) = 0$ and $F_R(a,a) = 1; \forall a \in A$.

B. Definition 4.2

A NFR, $R(A \times A)$ is reflexive of order (α, γ, β) if $T_R(a,a) = \alpha$, $I_R(a,a) = \gamma$ and $F_R(a,a) = \beta; \forall a \in A$ and $\alpha + \gamma + \beta \leq 1$.

C. Theorem 4.1

Let $R \in NFR(X \times X)$, then R is reflexive iff R^c is anti-reflexive.

Proof:

Let R is reflexive. Then we have,

$$T_R(a,a)=1,\ I_R(a,a)=0 \text{ and } F_R(a,a)=0$$

From the definition of complement relation, we have

$$T_{R^c}(a,a)=T_R(a,a),\ I_{R^c}(a,a)=I_R(a,a) \text{ and } F_{R^c}(a,a)=F_R(a,a)$$

Which implies ,

$$T_{R^c}(a,a)=0,\ I_{R^c}(a,a)=0 \text{ and } F_{R^c}(a,a)=1.$$

Thus, R^c is anti-reflexive.

Conversely, let R^c is anti-reflexive. Then

$$T_{R^c}(a,a)=0,\ I_{R^c}(a,a)=0 \text{ and } F_{R^c}(a,a)=1.$$

From the definition of complement relation, we have

$$T_{R^c}(a,a)=T_R(a,a),\ I_{R^c}(a,a)=I_R(a,a) \text{ and } F_{R^c}(a,a)=F_R(a,a)$$

Which implies,

$$T_R(a,a)=1,\ I_R(a,a)=0 \text{ and } F_R(a,a)=0.$$

Hence proved

D. Definition 4.3

A neutrosophic fuzzy relation $R(A \times A)$ is symmetric if

$$T_R(a,b)=T_R(b,a),\ I_R(a,b)=I_R(b,a) \text{ and } F_R(a,b)=F_R(b,a); \forall a,b \in A.$$

E. Theorem 4.2

If R is symmetric, then so is R^{-1}.

Proof:

We know,

$$\begin{aligned} T_{R^{-1}}(a,b) &= T_R(b,a) \\ &= T_R(a,b) \text{ since } R \text{ is symmetric} \\ &= T_{R^{-1}}(b,a) \text{ by definition of } R^{-1} \end{aligned}$$

$$\begin{aligned} I_{R^{-1}}(a,b) &= I_R(b,a) \\ &= I_R(a,b) \text{ since } R \text{ is symmetric} \\ &= I_{R^{-1}}(b,a) \text{ by definition of } R^{-1} \end{aligned}$$

$$\begin{aligned} F_{R^{-1}}(a,b) &= F_R(b,a) \\ &= F_R(a,b) \text{ since } R \text{ is symmetric} \\ &= F_{R^{-1}}(b,a) \text{ by definition of } R^{-1} \end{aligned}$$

F. Definition 4.4

Let $R(A\times A)$ be a neutrosophic fuzzy relation. Then R is transitive $R\circ R\subseteq R$.

G. Theorem 4.3

If R is transitive ,then so is R^{-1}.

Proof:

We know,

$$\begin{aligned}
T_{R^{-1}}(a,b) &= T_R(b,a)\\
&\geq T_{R\circ R}(b,a)\\
&= \vee_{c\in A}\{T_R(b,c)\wedge T_R(c,a)\}\\
&= \vee_{c\in A}\{T_{R^{-1}}(a,c)\wedge T_{R^{-1}}(c,b)\}\\
&= T_{R^{-1}\circ R^{-1}}(a,b)
\end{aligned}$$

$$\begin{aligned}
I_{R^{-1}}(a,b) &= I_R(b,a)\\
&\geq I_{R\circ R}(b,a)\\
&= \wedge_{c\in A}\{I_R(b,c)\wedge I_R(c,a)\}\\
&= \wedge_{c\in A}\{I_{R^{-1}}(a,c)\wedge I_{R^{-1}}(c,b)\}\\
&= I_{R^{-1}\circ R^{-1}}(a,b)
\end{aligned}$$

$$\begin{aligned}
F_{R^{-1}}(a,b) &= F_R(b,a)\\
&\leq F_{R\circ R}(b,a)\\
&= \wedge_{c\in A}\{F_R(b,c)\vee F_R(c,a)\}\\
&= \wedge_{c\in A}\{F_{R^{-1}}(a,c)\vee F_{R^{-1}}(c,b)\}\\
&= F_{R^{-1}\circ R^{-1}}(a,b)
\end{aligned}$$

So, $R^{-1}\circ R^{-1}\subseteq R^{-1}$.

Hence the thoerem is proved.

V. CONCLUSION

The analysis of mathematics with uncertainty has expanded recently in the area of neutrosophic fuzzy set theory, which considers an object's degrees of truth, falsity, and indeterminacy. In this paper we study different concepts like reflexivity, symmetry, and transitivity of a neutrosophic fuzzy relation are specified over a neutrosophic fuzzy set. Finally, a few properties of the fuzzy relations over a neutrosophic fuzzy set are explored.

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