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Special Dio 3-Tuples Involving Square Pyramidal Numbers

Dr. C. Saranya[1], B. Achya[2]
[1]Assistant Professor, [2]PG student, PG and Research Department of Mathematics, Cauvery College for Women,(Autonomous) (Affiliated to Bharathidasan University), Trichy-18.

***Abstract:** In this communication, we accomplish special dio 3-tuples comprising of square pyramidal numbers such that the product of any two members of the set subtracted by their sum and increased by a polynomial with integer coefficients is a perfect square.*
***Keywords:** Special dio 3-tuples, Pyramidal number, perfect square, square pyramidal number.*

NOTATION: p_n^4: square pyramidal number of rank n.

I. INTRODUCTION

In Mathematics, a Diophantine equation is a polynomial equation, ordinarily in at least two questions, to such an extent that solitary the whole number arrangements are looked for or examined (a whole number arrangement is an answer to such an extent that all the questions take whole number values).The word Diophantine alludes to the Hellenistic mathematician of the third century, Diophantus of Alexandria, who made an investigation of such conditions and was one of the principal mathematician to bring imagery into variable based math. The numerical investigation of Diophantine issues that Diophantus started is currently called Diophantine analysis. While singular conditions present a sort of confuse and have been considered from the beginning of time, the definition of general hypotheses of Diophantine conditions (past the hypothesis of quadratic structures) was an accomplishment of the twentieth century.

In [1-5], hypothesis of numbers were talked about. In [6-14], Diophantine triples with the property $D(n)$ for any integer n and furthermore for any straight polynomials were talked about and Dio triples for different numbers are constructed. In this paper, we exhibit special dio 3-tuples (a, b, c) involving square pyramidal number such that the product of any two elements of the set subtracted by their sum and increased by a polynomial with integer coefficients is a perfect square.

II. BASIC DEFINITION

A set of three different polynomials with integer coefficients (a_1, a_2, a_3) is said to be a special dio 3-tuple with property D(n) if $a_i * a_j$ - $(a_i + a_j) + n$ is a perfect square for all $1 \leq i < j \leq 3$,where n may be non-zero integer or polynomial with integer coefficients.

III. METHOD OF ANALYSIS

A. Section-A

Construction of the special dio-3 tuples involving square pyramidal number of rank n and $n-1$:

Let $a = 6p_n^4$ and $b = 6p_{n-1}^4$ be square pyramidal numbers of rank n and $n-1$ respectively, such that

$$ab - (a+b) + n^4 + 4n + 1 = \alpha^2 \tag{1}$$

Equation (1) is a perfect square, where $\alpha = 2n^3 - n - 1$

Let c be non zero-integer such that,

$$ac - (a+c) + n^4 + 4n + 1 = \beta^2 \tag{2}$$

$$bc - (b+c) + n^4 + 4n + 1 = \gamma^2 \tag{3}$$

Solving (2) & (3)$\Rightarrow -c(b-a)+(b-a)(n^4+4n+1)=b\beta^2-a\gamma^2$ (4)

(3) - (2) $\Rightarrow \gamma^2-\beta^2 = c(b-a)-(b-a)$

Therefore (4) becomes,

$(b-1)\beta^2-(a-1)\gamma^2=(b-a)(n^4+4n)$

Setting $\beta = x+(a-1)y$ and $\gamma = x+(b-1)y$,

$$\Rightarrow (b-1)(x+(a-1)y)^2-(a-1)(x+(b-1)y)^2=(b-a)(n^4+4n)$$

$$\Rightarrow x^2=(ab-a-b+1)y^2+n^4+4n \quad (5)$$

Now put $y=1$,

$\Rightarrow x=(2n^3-n-1)$

The initial solution of (5) is given by,

$x_0=(2n^3-n-1),\ \ y_0=1$

Since, $\beta = x+(a-1)y$ and $\gamma = x+(b-1)y$,

we obtain that, $\beta = 4n^3+3n^2-2$

Therefore, the equation (2) becomes,

(2) $\Rightarrow ac-(a+c)+n^4+4n+1=\beta^2$

$\Rightarrow c(2n^3+3n^2+n-1)-(2n^3+3n^2+n)+n^4+4n+1=\beta^2$

So that, $c = 8n^3-3$

Hence, $c=2(a+b)-4n-3$

Therefore, the triples

$$\{a,b,(2(a+b-4n-3))\}=\{6p_n^4,6p_{n-1}^4,(2(6p_n^4+6p_{n-1}^4-4n-3))\}$$ is a

Diophantine triples with the property $D(n^4+4n+1)$.

Some numerical examples are given below in the following table.

Table 1

n	Special dio 3-tuples	$D(n^4+4n+1)$
1	(6,0, 5)	6
2	(30,6,61)	25
3	(84,30,213)	94

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B. Section B

Construction of the special dio-3 tuples involving square pyramidal number of rank n and $n-2$:

Let $a = 6p_n^4$ and $b = 6p_{n-2}^4$ be square pyramidal numbers of rank n and $n-2$ respectively, such that

$$ab-(a+b)+6n^3-9n^2+12n-2=\alpha^2 \quad (6)$$

Equation (6) is a perfect square, where $\alpha = 2n^3-3n^2-2n+2$

Let c be non zero-integer such that,

$$ac-(a+c)+6n^3-9n^2+12n-2=\beta^2 \quad (7)$$

$$bc-(b+c)+6n^3-9n^2+12n-2=\gamma^2 \quad (8)$$

Solving (7) & (8),

$$\Rightarrow -c(b-a)+(b-a)(6n^3-9n^2+12n-2)=b\beta^2-a\gamma^2 \quad (9)$$

(8) - (7) $\Rightarrow \gamma^2-\beta^2 = c(b-a)-(b-a)$

Therefore (9) becomes,

$$(b-1)\beta^2-(a-1)\gamma^2=(b-a)(6n^3-9n^2+12n-3)$$

Setting $\beta = x+(a-1)y$ and $\gamma = x+(b-1)y$,

$$\Rightarrow (b-1)(x+(a-1)y)^2-(a-1)(x+(b-1)y)^2=(b-a)(6n^3-9n^2+12n-3)$$

$$\Rightarrow x^2=(ab-a-b+1)y^2+6n^3-9n^2+12n-3 \quad (10)$$

Now put $y=1$,

$$\Rightarrow x=(2n^3-3n^2-2n+2)$$

The initial solution of (10) is given by,

$$x_0=(2n^3-3n^2-2n+2)\ ,\ y_0=1$$

Since, $\beta = x+(a-1)y$ and $\gamma = x+(b-1)y$,

we obtain that, $\beta = 4n^3-n+1$

Therefore, the equation (7) becomes,

(7) $\Rightarrow ac-(a+c)+6n^3-9n^2+12n-2=\beta^2$

So that, $c = 8n^3-12n^2+10n-3$

Hence, $c = 2(a+b)-18n+9$

Therefore, the triples

$\{a,b,(2(a+b))-18n+9)\} = \{6p_n^4, 6p_{n-2}^4, (2(6p_n^4+6p_{n-2}^4)-18n+9)\}$ is a

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Diophantine triples with the property $D(6n^3 - 9n^2 + 12n - 2)$.

Some numerical examples are given below in the following table.

Table 2

n	Special dio 3-tuples	$D(6n^3 - 9n^2 + 12n - 2)$
1	(6,0, 3)	7
2	(30,0,33)	34
3	(84,6,135)	115

C. Section C

Construction of the special dio-3 tuples involving square pyramidal number of rank n and $n-3$:

Let $a = 6p_n^4$ and $b = 6p_{n-3}^4$ be square pyramidal numbers of rank n and $n-3$ respectively, such that

$$ab - (a+b) + n^4 + 8n^3 - 42n^2 + 54n + 19 = \alpha^2 \qquad (11)$$

Equation (11) is a perfect square, where $\alpha = 2n^3 - 6n^2 - n + 7$

Let c be non zero-integer such that,

$$ac - (a+c) + n^4 + 8n^3 - 42n^2 + 54n + 19 = \beta^2 \qquad (12)$$

$$bc - (b+c) + n^4 + 8n^3 - 42n^2 + 54n + 19 = \gamma^2 \qquad (13)$$

Following the procedure as in Section B, we get

$$c = 8n^3 - 24n^2 + 36n - 17$$
$$\Rightarrow c = 2(a+b) - 40n + 43$$

Therefore, the triples

$\{a, b, (2(a+b)) - 40n + 43))\} = \{6p_n^4, 6p_{n-3}^4, (2(6p_n^4 + 6p_{n-3}^4) - 40n + 43)\}$ is

a special dio 3-tuple with the property $D(n^4 + 8n^3 - 42n^2 + 54n + 19)$.

Some numerical examples are given below in the following table.

Table 3

n	Special dio 3-tuples	$D(n^4 + 8n^3 - 42n^2 + 54 + 19)$
1	(6, -6, 3)	40
2	(30,0,23)	39
3	(84,0,91)	100

IV. CONCLUSION

In this paper, we construct the special dio 3-tuples involving square pyramidal numbers. One may search for other special dio 3-tuples for different numbers with suitable properties.

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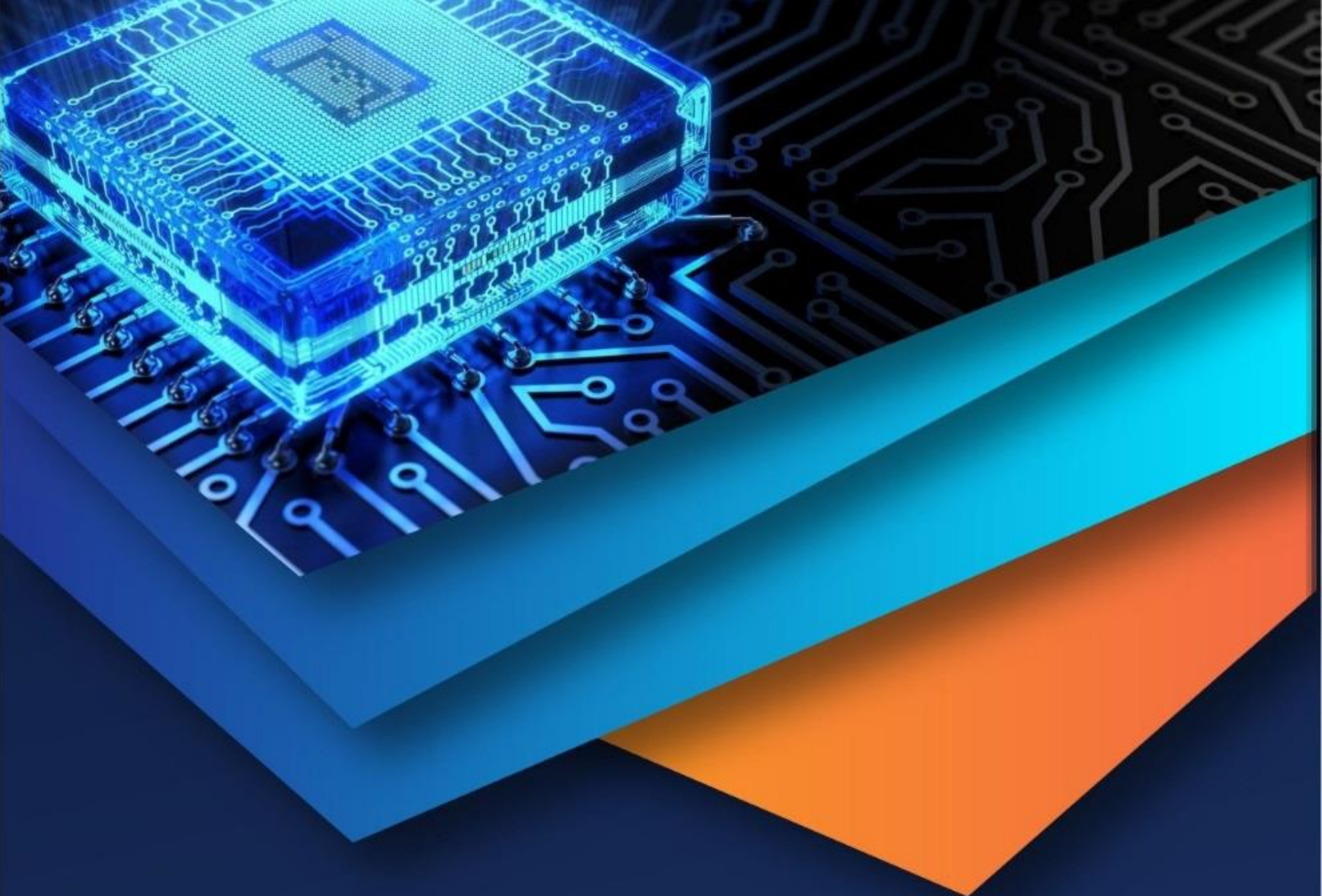

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