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Case study of Stress Calculation using Stress Superposition Method for Linear Analysis

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Abstract: Stress is one of the critical designing parameters in structural analysis of any mechanical assembly. Calculation of stress and corresponding lead time also has significant role while performing any FEM analysis. This paper explores the method of calculation of stresses using Superposition principle and its applicability when multiple load cases needs to be analyzed. This paper also describes the validation procedure of the FEA results to show the correctness of the stress superposition method against actual FEM analysis.
Keywords: Superposition principle, Hand calculation, FEM, linear Analysis, Rigid body elements

I. INTRODUCTION

The term "Stress" indicates the resistance of a body to deform under the action of an external force. In FE analysis, analytical solvers perform calculation of stresses under given loading conditions. For every analysis considerable time is taken by the system hardware to perform these actions. In many cases multiple iterations need to be performed to reach a desirable result. In such scenario's total analysis costs more and due to which a significant time is required to finish the tasks. As the time plays considerable role, method need to be build/implemented to reduce cycle time. In this paper stress superposition method is proposed as a method which can be used to estimate stress of multiple load cases of a linear FEM. A case study is performed where application of superposition method is shown and stress obtained using this method is validated against actual FEM analysis. It is shown as how the stresses for any new mechanical loads can be obtained and superposed with stresses obtained from other loading type such as pressure and thermals. In this paper to show the application of superposition method as an example firstly, base FEM is created and analysed with unit Forces (Fx, Fy & Fz) and Moments (Mx, My & Mz) in all of the directions one by one separately. Also, surface Pressure and Thermal alone analysis is conducted separately. From the Unit load results, component stresses like Sx, Sy, Sz, Txy, Tyz & Txz are extracted. Similarly, component stresses are listed from Surface pressure and Thermal alone analysis. Next steps in the process is stress extrapolation for new loads and then calculation of Principal stresses like Max Principal Stress (S1) & Min Principal Stress (S3) by using Superposition principle formulae. To ensure the correctness of this method, actual FEM analysis with considered forces, moments, pressure and thermals is conducted.

A. Unit Load Details

Unit loads here implies the example load such as 1000lb (force) and 1000lb-in (moment) which can create considerable amount of stress. Unit Forces and Moments are applied as below in Cartesian coordinate system (CSYS, 0).
Unit load Force (1000lb) applied in X-direction (i.e. +Fx & -Fx)
1) Unit load Force (1000lb) applied in Y-direction (i.e. Fy & -Fy)
2) Unit load Force (1000lb) applied in Z-direction (i.e. +Fz & -Fz)
3) Unit load Moment (1000lb-in) applied in X-direction (i.e. +Mx & -Mx)
4) Unit load Moment (1000lb-in) applied in Y-direction (i.e. +My & -My)
5) Unit load Moment (1000lb-in) applied in Z-direction (i.e. +Mz & -Mz)
Surface pressure and uniform temperature is also applied as below.

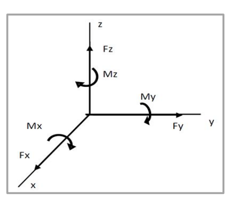

Fig. 1 Forces & Moments representation
(https://www.sensy.com/en/technology/load-cell/force-measurement)

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6) Surface pressure (10psi)
7) Uniform Thermals (100°F)

B. Scale Factor Calculations

The term “Scale factor” indicates the amount of load increment or decrement with respect to Unit load that applied during initial phase.

$$\text{Scale factor} = \frac{\text{Actual load}}{\text{Unit load}}$$

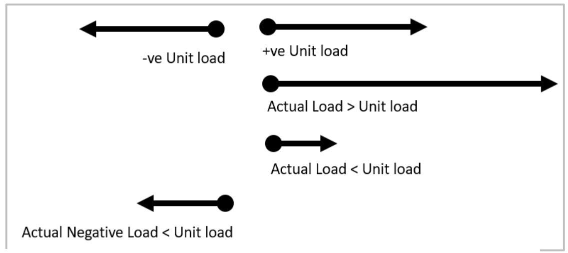

Fig. 2 Load consideration wrt to Unit load details
(Open source)

C. Superposition Principle

The superposition principle states that, for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually. Based on the definition of the superposition, we can claim that any system (function) which respects the principle of superposition is a linear system (function) the superposition principle applies to all linear systems (functions).

For example: A system, defined by the function f(x), is linear if the following relationship is true:

$$f(x_1) = y_1$$
$$f(x_2) = y_2$$
$$f(x_1 + x_2) = y_1 + y_2$$

The principle of superposition can be used to solve problems in mathematics, control systems, electronics and physics.

D. Principal Stresses Calculations

The state of stresses at a point is completely defined by normal and shear stress components in reference to an orthogonal coordinate system XYZ. In general, the values of the stress components change if the coordinate system is rotated. At a certain orientation (X’Y’Z’), all shear stresses vanish and the state of stresses is completely defined by 3 normal stress components. These 3 normal stress components are referred to as principal stresses and the corresponding reference axes (X’Y’Z’) are referred to as principal axes.

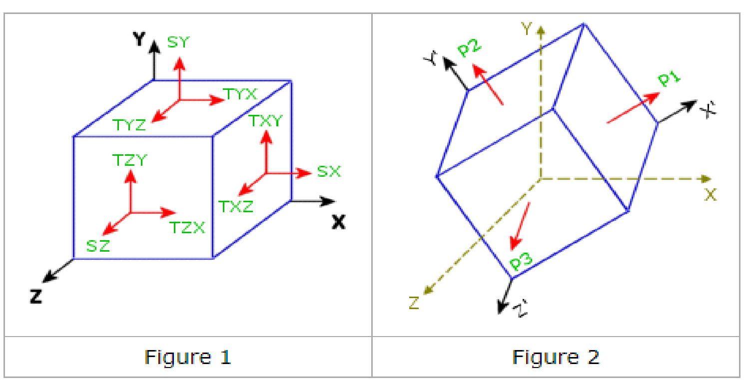

Fig. 3 Directional Normal & Shear stresses on plane
(https://help.solidworks.com/2021/english/SolidWorks/cworks/c_Principal_Stresses_Definition.htm)

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Below are the principal stress calculations formulae using component stresses.

$$I_1 = Sx + Sy + Sz$$

$$I_2 = Sx * Sy\ +\ Sy * Sz\ +\ Sy * Sz\ - Txy^2\ -\ Tyz^2\ -\ Tzx^2$$

$$I_3 = Sx * Sy * Sz\ - Sx * Tyz^2 - Sy * Tzx^2 - Sz * Txy^2 + 2 * Txy^2 * Tyz^2 * Tzx^2$$

$$\Phi = \frac{1}{3} * \cos^{-1}\left(\frac{2 * I_1^3 - 9 * I_1 * I_2 + 27 * I_3}{2 * (I_1^2 - 3 * I_2)^{3/2}}\right)$$

Principal stresses calculations formulae:

Principal Stress, P1 : $\frac{I_1}{3} + \frac{2}{3}\left(\sqrt{I_1^2 - 3 * I_2}\right) * \cos\Phi$

Principal Stress, P2 : $\frac{I_1}{3} + \frac{2}{3}\left(\sqrt{I_1^2 - 3 * I_2}\right) * \cos\left(\Phi - \frac{2 * \Pi}{3}\right)$

Principal Stress, P3 : $\frac{I_1}{3} + \frac{2}{3}\left(\sqrt{I_1^2 - 3 * I_2}\right) * \cos\left(\Phi - \frac{4 * \Pi}{3}\right)$

(Source: https://en.wikiversity.org/wiki/Elasticity/Principal_stresses)

II. SCHEMATIC FLOW DIAGRAM

Below is the schematic flow diagram for superposition principle-based stress extraction method.

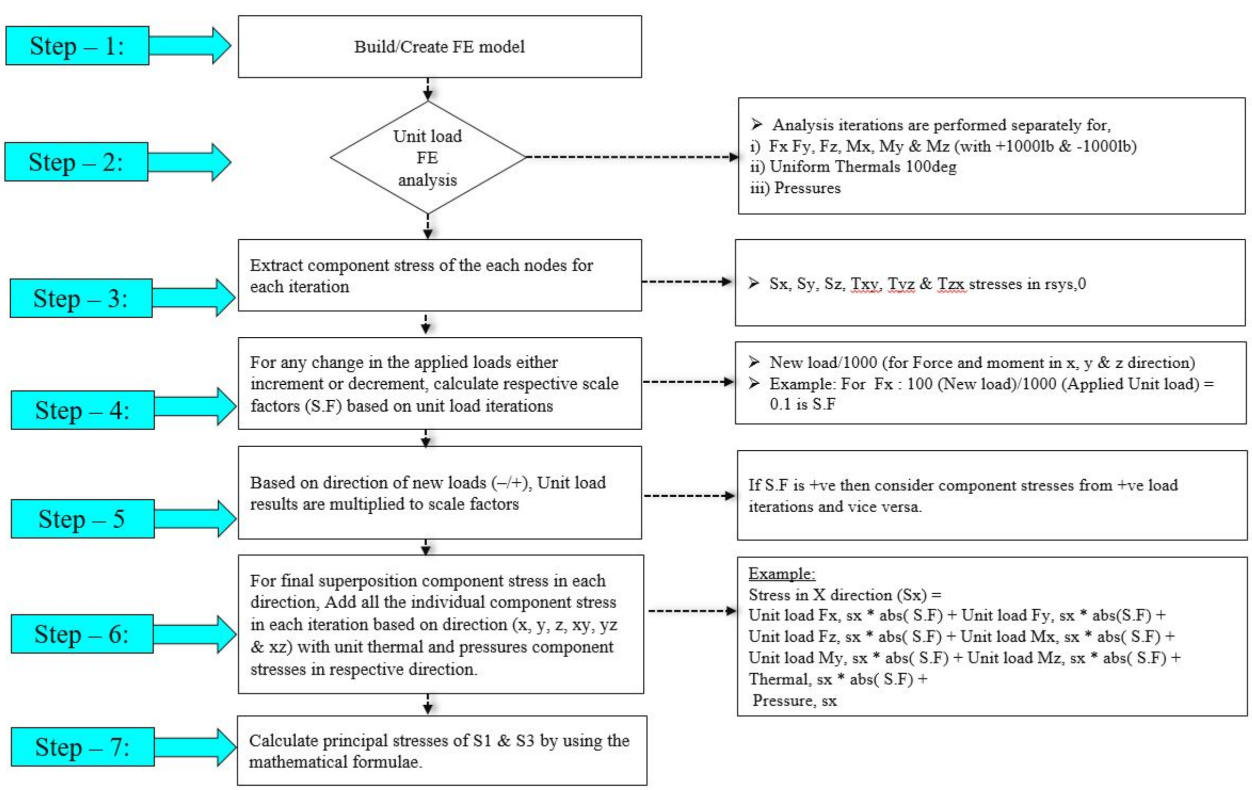

Fig. 4 Flow chart of superposition principle results extraction
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III. CASE STUDY

A. Consideration of FE Model

A simple casing geometry is considered for building finite element model as shown below. FE model is built with hexahedral elements with element type of Solid45 in ANSYS Software. Good quality mesh is maintained to capture all the features.

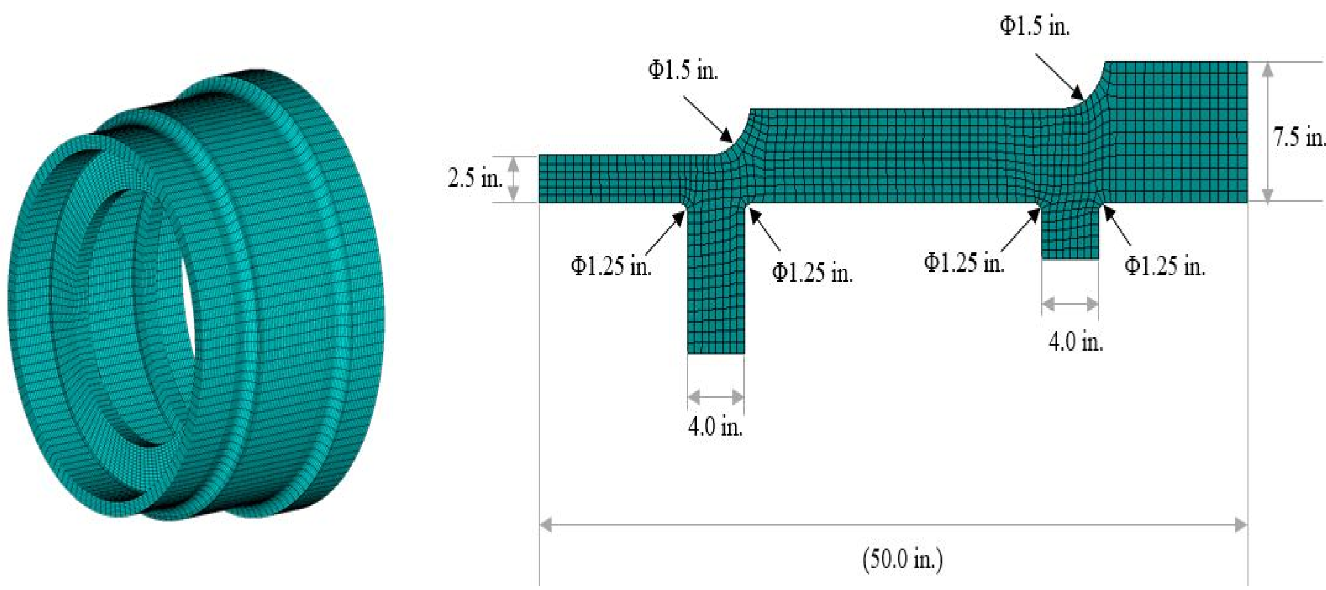

Fig. 5 Geometry & FE mesh details of case study model
(https://cad.emachineshop.com/#/)

B. Material Properties Details

Basic material properties are applied to perform the analysis. Material of 304 stainless steel is chosen & applied.
Below are the mechanical & thermal properties of 304 stainless steel material. Reference temperature of 70°F applied in the model.

S.NO	PROPERTY	SYMBOL	VALUE	UNITS
I	Density	ρ	0.00074793	lb/in³
II	Young's Modulus	E	2.8E+007	psi
III	Poisson's ratio	υ	0.29	No units
IV	Thermal expansion secant coefficient	α	9.61E-06 @ 212 F 9.89E-06 @ 599 1.04E-05 @ 1200	In/in-°F

TABLE 1: Material properties details of 304 Stainless Steel
(http://matweb.com/search/datasheet.aspx?matguid=abc4415b0f8b490387e3c922237098da&ckck=1)

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C. Boundary Condition & Loads Application

Load is applied to the model by using RBE3 (Rigid body elements). Master node element is created exactly at the center of the flanges as shown in below figure and flange radial face nodes are connected using RBE3. Unit loads in each direction is applied separately in each iteration at the respective Master node. Also, Thermals & Pressure is applied as separate iteration to compute the component stresses due to these loads. The aft portion of the model is constrained in all DOF (ux, uy & uz).

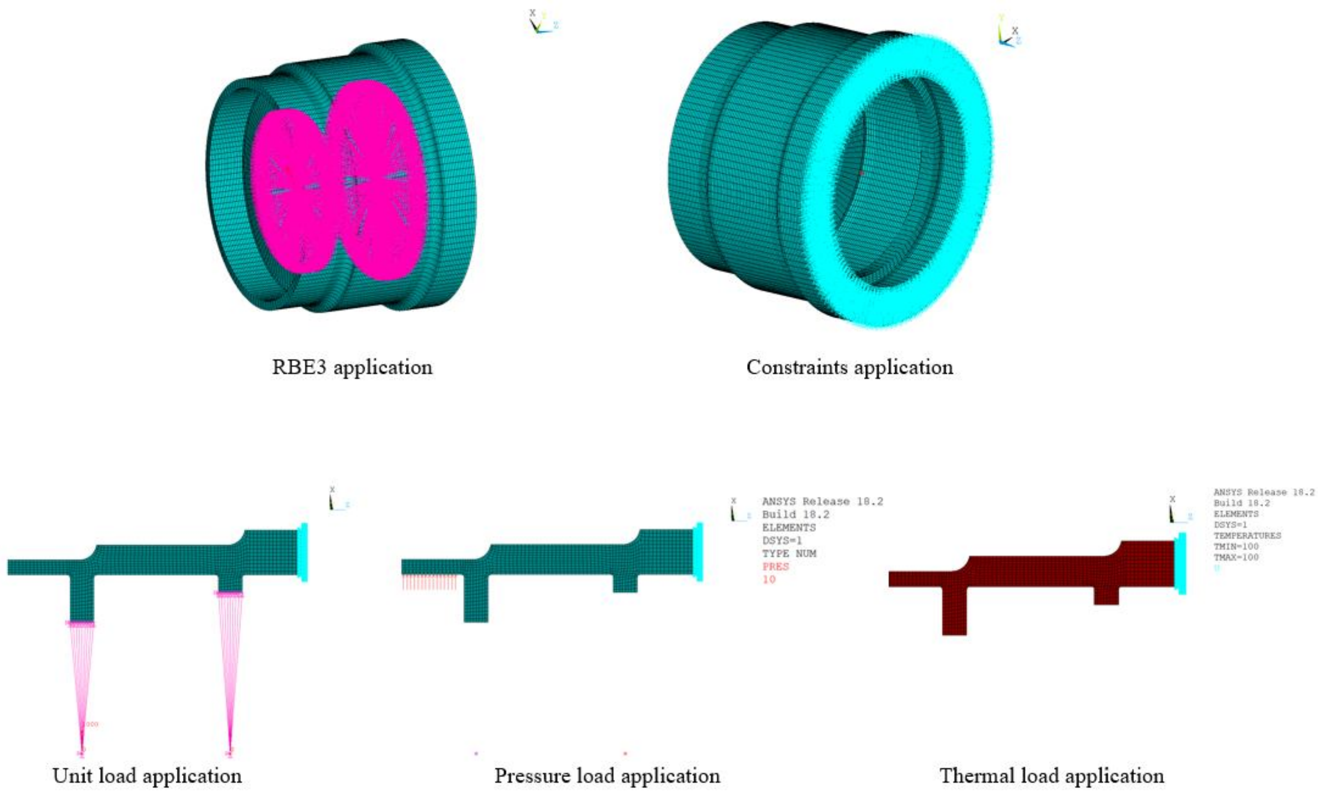

Fig. 6 Boundary conditions & Loading details
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D. Extraction of Component Stresses for Unit Load Iterations

The component stresses (Sx, Sy, Sz, Txy, Tyz & Txz) are extracted for all the unit load iterations and tabulated.
In below figure, as an example stress details are shown for one node (number 40271).

Flange - 1 (Aft flange), Node 40271							
Component	Load	Sx (in "psi")	Sy (in "psi")	Sz (in "psi")	Sxy (in "psi")	Syz (in "psi")	Sxz (in "psi")
Fx	+1000lb	3.02859	-0.04533	5.33401	1.05123	1.07140	3.11182
	-1000lb	-3.02860	0.04533	-5.33401	-1.05123	-1.07140	-3.11183
Fy	+1000lb	0.87817	0.09115	1.73313	0.48730	0.16251	1.07140
	-1000lb	-0.87817	-0.09115	-1.73313	-0.48730	-0.16251	-1.07140
Fz	+1000lb	12.58596	5.65873	12.89059	2.51646	2.88897	8.89134
	-1000lb	-12.58597	-5.65873	-12.89054	-2.51647	-2.88896	-8.89132
Mx	+1000lb-in	-0.41311	-0.03240	-0.32120	0.08254	0.10048	-0.27580
	-1000lb-in	0.41311	0.03240	0.32120	-0.08254	-0.10048	0.27580
My	+1000lb-in	0.91409	0.45706	0.98856	0.23778	0.27580	0.65872
	-1000lb-in	-0.91409	-0.45706	-0.98856	-0.23778	-0.27580	-0.65872
Mz	+1000lb-in	0.01713	-0.01713	0.00000	-0.02358	-0.01961	0.00637
	-1000lb-in	-0.01713	0.01713	0.00000	0.02358	0.01961	-0.00637

Flange - 2 (FWD flange), Node 40271							
Component	Load	Sx (in "psi")	Sy (in "psi")	Sz (in "psi")	Sxy (in "psi")	Syz (in "psi")	Sxz (in "psi")
Fx	+1000lb	-0.07306	-0.05654	-0.19776	-0.02333	-0.04753	-0.09974
	-1000lb	0.07306	0.05654	0.19776	0.02333	0.04753	0.09974
Fy	+1000lb	-0.05178	0.00966	-0.06426	0.03101	0.03111	-0.04753
	-1000lb	0.05178	-0.00966	0.06426	-0.03101	-0.03111	0.04753
Fz	+1000lb	-0.11479	-0.07232	-0.24090	-0.01543	-0.04249	-0.13077
	-1000lb	0.11479	0.07232	0.24090	0.01543	0.04249	0.13077
Mx	+1000lb-in	0.00350	-0.00052	0.00387	-0.00250	-0.00234	0.00298
	-1000lb-in	-0.00350	0.00052	-0.00387	0.00250	0.00234	-0.00298
My	+1000lb-in	-0.00437	-0.00481	-0.01190	-0.00112	-0.00298	-0.00585
	-1000lb-in	0.00437	0.00481	0.01190	0.00112	0.00298	0.00585
Mz	+1000lb-in	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	-1000lb-in	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Pressure, Node 40271						
Pressure	Sx (in "psi")	Sy (in "psi")	Sz (in "psi")	Sxy (in "psi")	Syz (in "psi")	Sxz (in "psi")
10psi	52.33386	67.17172	85.22684	-5.39017	16.01065	49.27570

Temperature, Node 40271						
Temperature	Sx (in "psi")	Sy (in "psi")	Sz (in "psi")	Sxy (in "psi")	Syz (in "psi")	Sxz (in "psi")
100degF	3.03250	138.08325	-70.83115	-49.06006	-13.96244	-42.97197

Fig. 7 Unit load results for Node 40271
(This image does not contain any technical data)

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E. Calculation of Principal stresses by using superposition principle:

To calculate the principal stresses by using superposition principle, below steps are followed.

1) *New load Data:* Below table shows as an example of new loads. Both positive and negative directions are considered. The forces and moments are considered at each flange master element node. Uniform thermal and pressure load are also considered.

FLANGE - 1				**FLANGE - 2**		
Component	Load	Units		Component	Load	Units
Fx	80000	lb		Fx	-1500	lb
Fy	-500	lb		Fy	4000	lb
Fz	120000	lb		Fz	-5200	lb
Mx	-3700	lb - in		Mx	800	lb - in
My	2100	lb - in		My	-6600	lb - in
Mz	9000	lb - in		Mz	1200	lb - in
Thermals	100	°F		Thermals	100	°F
Pressure	10	psi				

TABLE 2: Load details for Flange 1 & Flange 2
(This table does not contain any technical data)

2) *Scale factor Calculations wrt to unit Loads:* Here the purpose of the superposition principle is to calculate the effect of load variation for linear models.

Whenever there is any change in applied load either increment or decrement, respective load scale can be calculated by using below formula.

$$\text{Scale factor} = \frac{\text{Actual load}}{\text{Unit load}}$$

Here unit load means 1000lb (force) or 1000lb-in (moment).

Scale factors are calculated for Table2 loads by dividing with considered unit load as 1000lb or 1000lb-in and scale factors are as below.

FLANGE - 1			**FLANGE - 2**	
Component	Scale factor		Component	Load
Fx	80.0		Fx	-1.5
Fy	-0.5		Fy	4.0
Fz	120.0		Fz	-5.2
Mx	-3.7		Mx	0.8
My	2.1		My	-6.6
Mz	9.0		Mz	1.2

TABLE 3: Scale factor calculation wrt to Unit load
(This table does not contain any technical data)

3) *Summation of Component Stresses from Different Loads:* Calculation of principal stresses involves summation of the component stresses wrt to scale factors & corresponding direction. *i.e. positive or negative.*

If the scale factor is positive then results data will be considered from positive unit loads data and if negative then results data will consider from negative unit load data as shown below.

<u>Example:</u> From the section E.A.2 for Flange - 1,

Component stress in X direction (Sx) due to applied forces and moments =

(+ve Unit load Fx), sx * S.F + (-ve Unit load Fy), sx * abs (S.F) +

(+ve Unit load Fz), sx * abs (S.F) + (-ve Unit load Mx), sx * abs (S.F) +

(+ve Unit load My), sx * abs (S.F) + (+ve Unit load Mz), sx * abs (S.F)

In above example “Fy” & “Mx” are negative hence loads are considered from negative unit loads table.

In the similar way component stresses Sy, Sz, Txy, Tyz & Txz are calculated both for Flange - 1 and Flange - 2.

Calculation for the total component stress of the model due to all applied loads is as below:
Total Component stress Sx = Component stress Sx of Flange 1 + Component stress Sx of Flange 2 +
Thermal stress, Sx + Pressure stress, Sx
Similarly, total component stresses for Sy, Sz, Txy, Tyz & Txz directions can be calculated for all nodes of the FEM.

4) Calculation of Principal Stresses

By using the formulae stated in section D of page3, both max and min principal stresses are calculated.
By substituting the total component stresses Sx, Sy, Sz, Txy, Tyz & Txz in the formulae, Principal stresses are found as below.
Max Principal stress (P1) = **4436.55 psi**
Min Principal stress (P3) = **695.26 psi**
These calculated stresses will be compared against FEM analysis stress for validation.

F. Validation of calculated results with Ansys analysis results:

In the FE model all of the loads that are stated in section E.A.1 are directly applied along with Thermals and Pressures. Reference temperature of 70°F is used similar to Unit load iterations. Updated FE model is analysed in Ansys to validate the calculated results.

1) Loads updation & Boundary condition detail of validation model:

Below figure shows all of the applied loads.

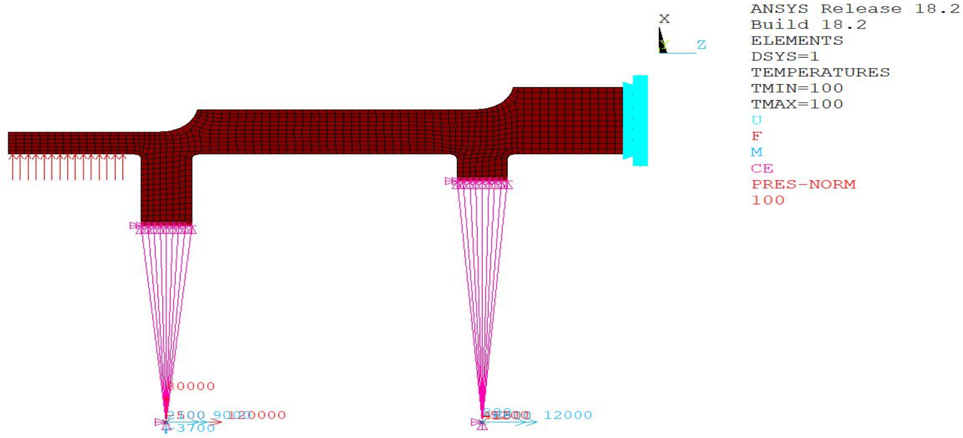

Fig. 8 Boundary condition & Loading details of validation run
(This image does not contain any technical data)

2) Displacements details for validation:

Below figure shows the displacement details for validation run. Result coordinate system used is global Cartesian.

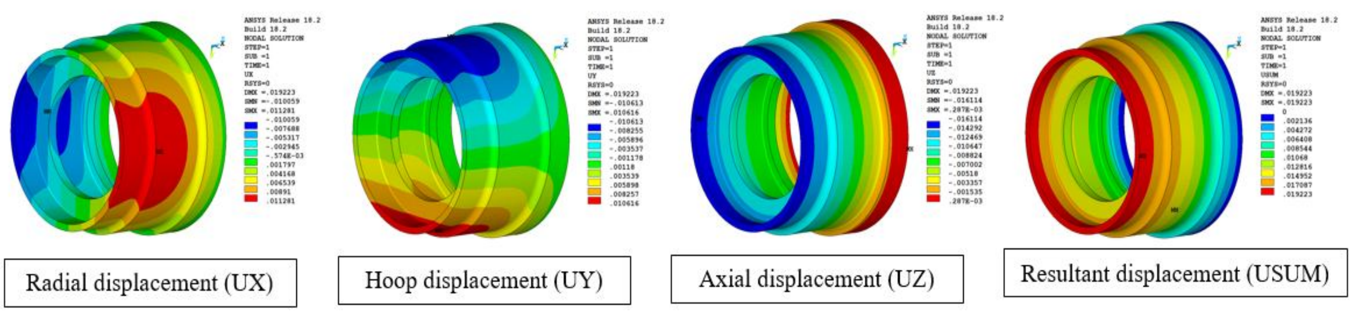

Fig. 9 Displacement plots
(This image does not contain any technical data)

3) Component stress details of validation model:

Below figure shows the component stresses details for validation run. Result coordinate system used is global Cartesian. Annotation in the plots are shown for stress at Node number 40271.

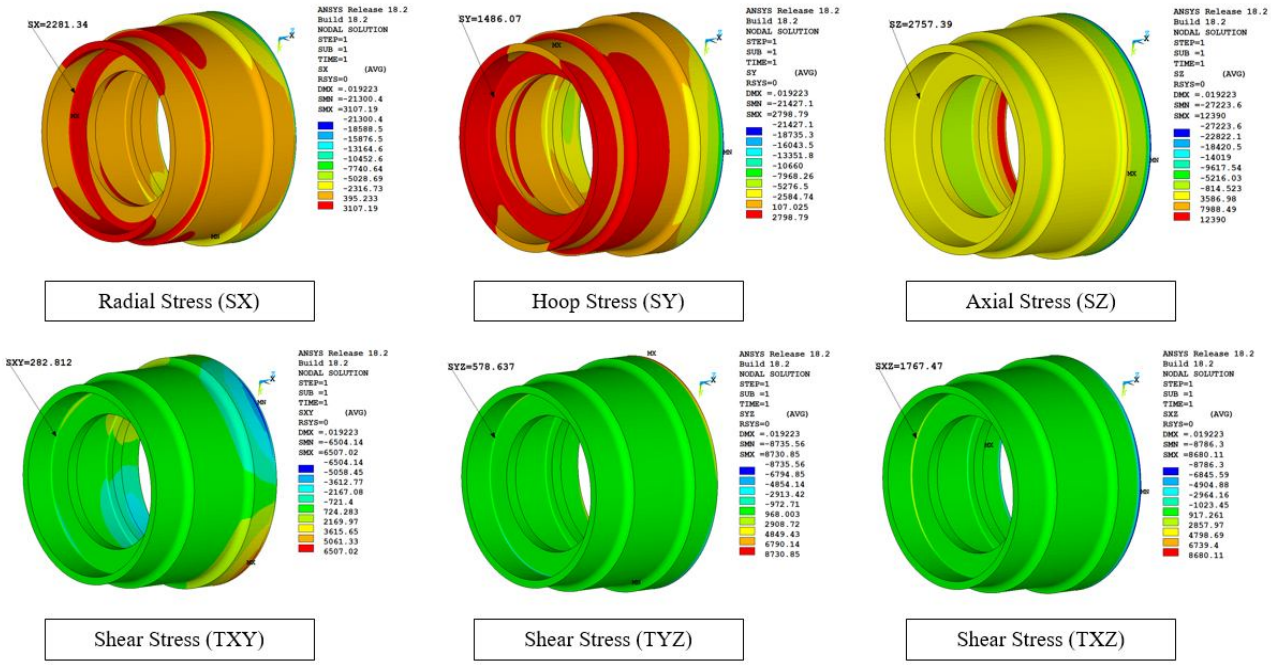

Fig. 10 Component stress plots
(This image does not contain any technical data)

4) Principal stresses details of validation model:

Below figure shows the Max and Min principal stress details for validation run. Annotation in the plots are shown for stress at Node number 40271.

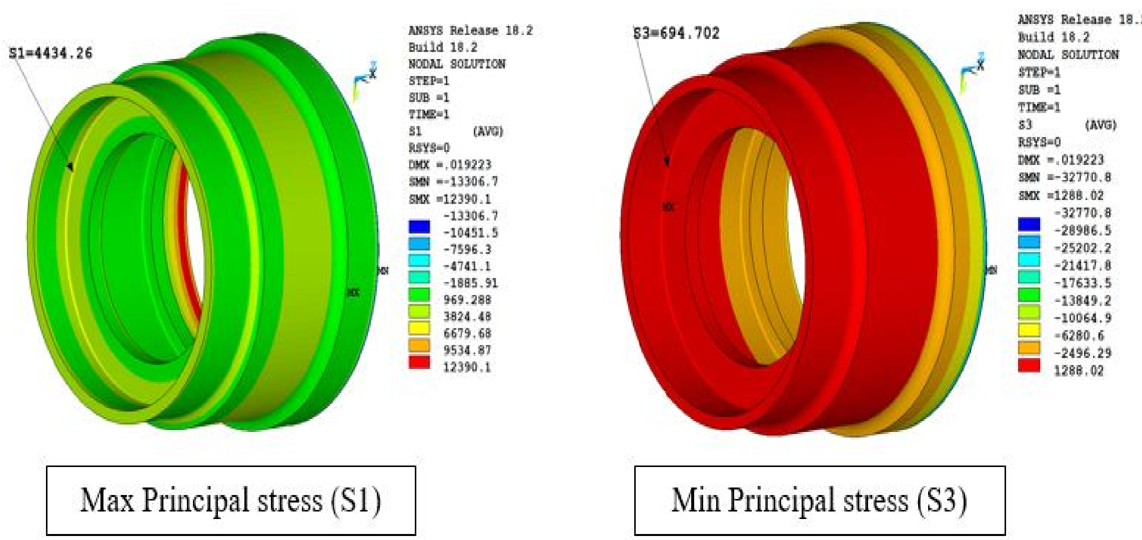

Fig. 11 Maximum Principal Stress (S1) & Minimum Principal Stress (S3) details for node 40271
(This image does not contain any technical data)

Below summary table shows the difference between stress calculated using actual FEM analysis & superposition approach.
As per the results, Stress obtained using superposition approach matches closely with that using actual FEM analysis.

Stress	Superposition Approach	Actual FEM Analysis	% difference of stress between Superposition approach and actual FEM Analysis
Max Principal stress, S1 (in "psi")	4436.55	4434.26	-0.05%
Min Principal stress, S3 (in "psi")	695.26	694.70	-0.08%

TABLE 4: Comparison of Validation run vs Superposition results
(This table does not contain any technical data)

IV. CONCLUSIONS

From the case study, superposition results and actual FEM analysis closely matched and the difference is less than 0.1%.
Hence superposition principle can be used for estimating stress for any linear analysis even with combination of other load such as pressure and thermal.
Thus, initially only FEM need to be analysed using unit loads to generate component stress profile and later for multiple iterations where the forces and moments would change, Stress at any location can be estimated using stress superposition approach without analysing FEM every time. This will save lot of lead time and need of performing FEM analysis for multiple load cases.

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[3] https://www.sensy.com/en/technology/load-cell/force-measurement
[4] https://help.solidworks.com/2021/english/SolidWorks/cworks/c_Principal_Stresses_Definition.htm
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