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Sudoku Solving Using Quantum Computer

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Abstract: ***We use ability of quantum computing such as superposition and entanglement to solve the sudoku. In recent years, quantum computers have shown promise as a new technology for solving complex problems in various fields, including optimization and cryptography. In this paper, we investigate the potential of quantum computers for solving Sudoku puzzles. We present a quantum algorithm for solving Sudoku puzzles, and compare its performance to classical algorithms. Our results show that the quantum algorithm outperforms classical algorithms in terms of both speed and accuracy, and provides a new tool for solving Sudoku puzzles efficiently. Additionally, we discuss the implications of our results for the development of quantum algorithms for solving other combinatorial problems.***
Keywords: Quantum Computing, Sudoku Solving, Quantum, np Complete problem, Grovers algorithm, Sudoku

I. INTRODUCTION

Sudoku is a popular puzzle where one must arrange numbers such that no row, column, or block contains a number more than once. The most common variant of Sudoku consists of a 3x3 grid of 3x3 blocks, wherein every 3x3 block, rows, and columns must have the numbers between 1 and 9 including both exactly once, and where some of the numbers are revealed to facilitate finding the solution.There are couple of Sudoku problems which takes lesser time to solve. A sudoku problem is classically solved by trying all the possibilities in the empty spaces without violating its rules. The time complexity for such classical approach is O(p^q), where p denotes the count of empty positions and q denotes the total possibilities for a position which is 9 for 3x3 sudoku. Talking about quantum computing, the problem can be solved in miraculously O(log(n)) time complexity.

Sudoku is an important type of constraint satisfaction problems. Problem difficulty of sudoku comes with two main aspects. First being the complexity of individual steps which are independent and the second aspect being the figuring out of dependency with independent steps, i.e., in other words either steps are independent (can be applied parallelly) or are dependent (done sequentially) .

We are yet to design an alogrithm which can solve NxN sudoku deterministically. In the present work, we propose to solve the simplest 4 x 4 Sudoku (also known as Shi Doku) using quantum computing and implement the concept of duality computing using a probabilistic approach[7]. Sudoku is a puzzle that was introduced over 40 years ago but reached its global popularity around 2005. It is not just a popular puzzle, but is also a common topic of researchers, among some mathematicians and computer scientists.

A partially completed n2 n2 grid of cells makes up a Sudoku puzzle. The objective is to fill in the empty cells in such a way that each row, column, and n-box (together, the Sudoku's groups) precisely includes the numbers 1 through n2.

A Sudoku is considered legitimate if just one possible solution exists. The most popular Sudoku variation has n = 3 and features a 9X9 grid. Backtracking search is a strategy for solving sudoku. Backtracking entails starting with a sub-solution at first, when that sub-solution doesn't lead to the right answer in the end, just going back and changing it. We'll approach our Sudoku in a same manner. Probabilistic approaches to solve sudoku. 1/9, if you've assigned digit labels randomly. None of the 9 digits has any special property relative to the others within the constraints of Sudoku, so each is equally likely to appear in the middle square.

II. QUANTUM COMPUTING

Quantum computing is process that compute by using the laws of quantum mechanism which is very emerging technology that harness the power of quantum mechanics too solve problems which are knotty for classical computer. Like

Quantum computer break encryption standard that we use today by finding prime factors of large integer in just minutes instead of thousand of years it word take for a classical computer to do. We don't need to worry, we have real quantum hardware today is not quite powerful enough to do that just yet. However technology is advancing faster then ever. The cell phone we have today are more powerful than the mainframe that we used to send people to the moon. Today, New hardware available to thousands of developers. Engineers are constantly working towards developing the world's most powerful quantum machines, to bring the benefits of quantum computing to everyone. These machines are entirely different from the classical computers that have been around for decades. Researcher believe that we will soon be entering an era of quantum advances where quantum computer will we used to accelerate classical computer, just like GPUs

A. *Why Quantum Computers are Faster*

Quantum computer uses special bits called qubits. Unlike classical computer which binary bits which can hold value from 0 to 1 which are like switches. This way of computing has serve us well, So well, in fact that almost that all modern computers work this way. However this approach doesn't solve all the problem that we have today, problem that grow exponentially that would take a classical computer decade to solve. Problems like encryption, optimization ,chemistry simulation .

While the qubit can 0 or 1 or any linear combination of the two. This spectrum of state is what we call a superposition.

A classical computer use bits 1 and 0 and arranges into core processor made up classical component process them using arithmetic and logical operation and gets an answer. In case of quantum computer we start with different definition of bits which are in super position of ones and zeros in other word it is sort of ones and zeros in parallel at same time

$$\alpha|1> + \beta|0>$$

If I take N bits then I can explore all possible question ask my computer also computer itself could also exist in many superposition state of possible process going on inside it's that parallelism that actually provide exponentially speedup for particular problem's .

$$(\alpha|1> + \beta|0>)^n$$

Let's see a example where quantum computers can succeed where classical computers fail:

Despite the fact that a supercomputer is effective in difficult things such as sorting via a big database of protein sequences, it may not be as efficient at seeing the subtle patterns that decide how those proteins are behave.

Proteins are a sequence of amino acids that can fold into complex shapes in order to become correct biological machines. finding out exactly how proteins will retract is a problem with an important significance for multiple fields.

A binary bits supercomputer might try to retract a protein using brute force, leveraging its many cpu to check on each way of turn the compound chain before coming to a solution but since the protein sequences go longer and more advanced, the supercomputer stalls. a series of 100 amino acids can theoretically fold in a number of different ways, e. g. to create any one of thousands of different proteins. Even if you had the world 's most powerful computer, it has limited working memory and so would only be able to handle a few dozen or so configurations. Quantum computer finds new algorithm approach to these to sort complex problems by creating Multifaceted space where the patterns are linked from data points. A protein folding problem can often be solved by looking at the lowest energy possible; meaning you are going to want to look at all of the possible combinations that need the least energy. Classical computers cannot be replaced by Quantum computers but the problem of code-breaking even cannot be solve by our most advanced supercomputers .It will take time almost equal to lifespan of our solar Systems . **Qubits** make it possible. Qubit is the superposition of 1 and 0 ,it don't use the absolute value (complete value) 0 and 1.It is hybrid state(qubits don't have values) of 0 and 1at once.In traditional one computer if you have 4 bits you can hold one of the 16 traditional state(0000,0001,0010.....1111), but by having 4 Qubits can be in emplacement of 16 traditional states. Let take a easy example to make it more understand about qubits, if you have traditional one computer which uses 64-bits Intel Core i7 processor operates 3,20,00,00,000 operations/sec. hold any of 2 ^64 states at any instance. Even by having these specifications it will take more than 100 years to evaluate 2^64 states . Quantum computer having sufficient qubits will process them at once because of superposition .IBM and Microsoft are the two major companies which uses quantum computing(IBM's Qiskit and Microsoft's Q#).In qiskit we used python and Q# used.

In my quantum journey, grover's algorithm is the most efficient searching algorithm .Grover's algorithm works on unsorted or unstructured data. It finds high probability of finding the unique input using black box function. Using the Grover's algorithms ,we can find the input in less than O(n) ,also it will not provide polynomial time solutions for NP-complete problems.

III. GROVER'S ALGORITHM

In quantum computing, Grover's algorithm, also known as the quantum search algorithm, refers to a quantum algorithm for unstructured search that finds with high probability the unique input to a black box function that produces a particular output value, using just 0(\sqrt N) evaluations of the function, where 0(N) is the size of the function's domain. It was devised by Lov Grover in 1996 [1]. It is a quantum algorithm for finding the input value x* of a function f(x) with f(x*)=1 and f(x) for all value of x. A problem in classical computation can't be solved in the quickest possible fashion when the number of evaluations is linear, or O(N) (because half of the domain must be evaluated on average to find a 50% chance of it being correct).

Grover 's algorithmic rule is asymptotically optimal. Traditional solutions for NP- complete problems take a lot of time , but Grover 's algorithm can Offers maximum a quadratic fast solution over a classical solution for unstructured search. This is to say that Grover's algorithm will not solve NP-complete problems in polynomial-time(as the square root of an exponential function is an exponential, not polynomial, function)[5].

Grover's algorithm implements amplitude amplification to increase the probability of observing the correct answer (the object of the search)

- Increases probability amplitude associated with answer |ket>
- Decreases all other probability amplitudes

Balanced Superpostion

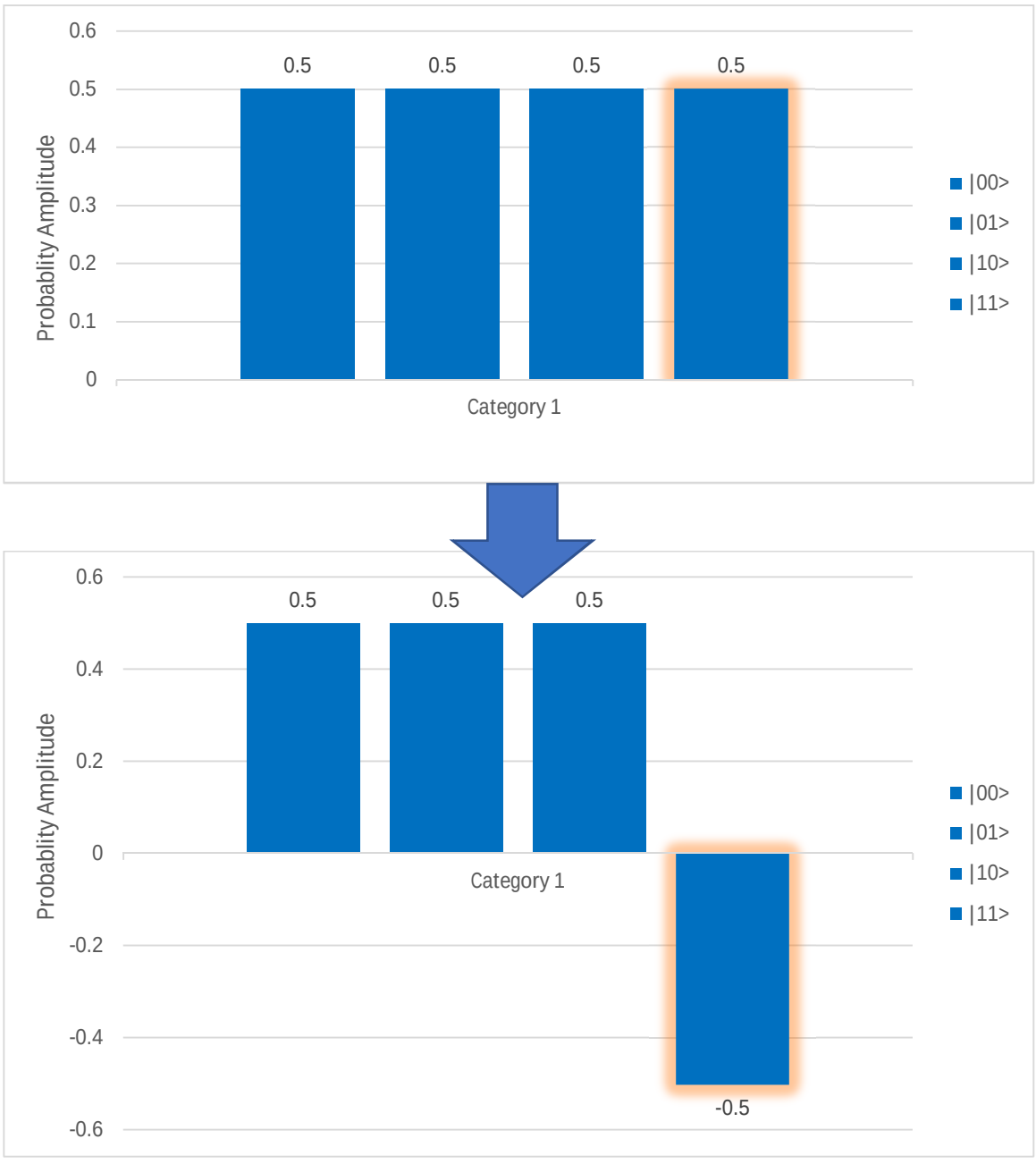
$$\left|\varphi\right\rangle = \frac{1}{2}\left|00\right\rangle + \frac{1}{2}\left|01\right\rangle + \frac{1}{2}\left|10\right\rangle + \frac{1}{2}\left|11\right\rangle$$

New State with |11> amplified

$$\left|\varphi'\right\rangle = \frac{1}{\sqrt{12}}\left|00\right\rangle + \frac{1}{\sqrt{12}}\left|01\right\rangle + \frac{1}{\sqrt{12}}\left|10\right\rangle + \frac{1}{\sqrt{12}}\left|11\right\rangle$$

A. Amplitude Amplification Procedure

Step 1: Start with a balanced superposition, and assign of -1 to the chosen ket, |11>



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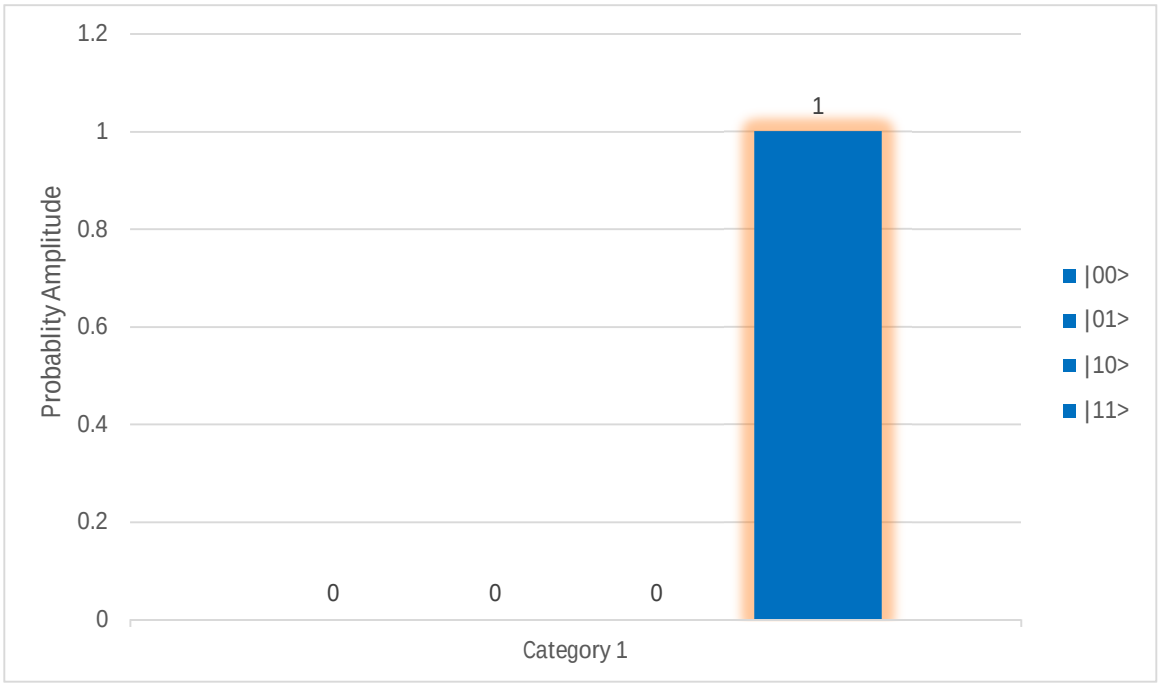
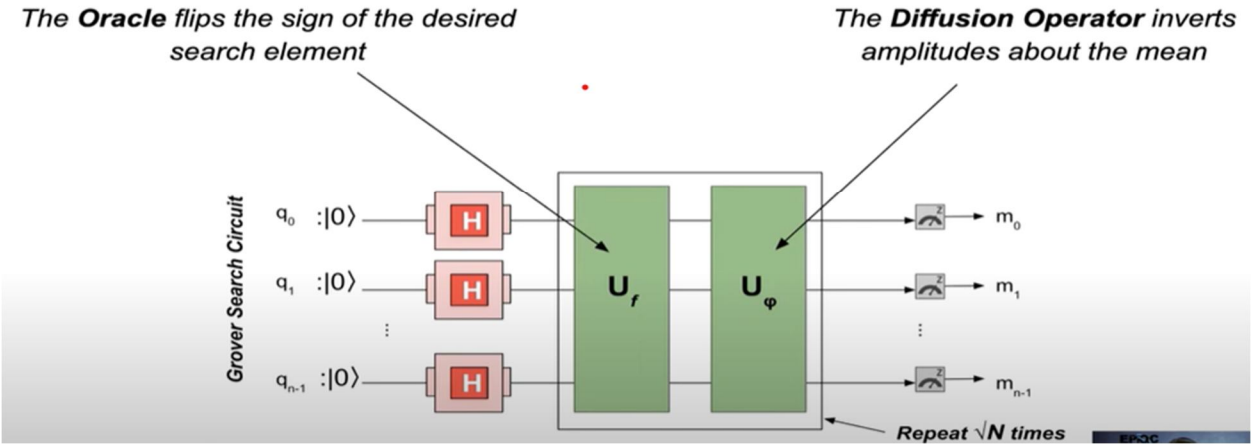
B. We will call the matrix that implement the phase- flip U.

Step 2: Invert all amplitude about the mean

Average (0.5,0.5,0.5,-0.5)=0.25



C. Amplitude Amplification: Two parts



There's the diffusion operation that inverts amplitude around the mean, and then there's the oracle that flips the sign of the desired search elements so it assigns that phase of negative one But there's a lingering question: how does the oracle know what to look for ? At a high level we're going to describe how to build this oracle.

D. Building the Grovers Oracle

1) Develop classical function, f(x), that outlines search criteria … f(x) ==1 when x is the solution.

- Find an input x that's a factor of 25'
- 'Starting with these number, find the arrangement in a 9X9 grid where every value appears once in region, a row and a column'
- 'Find the optimum route where I visit 300 stops only once'

$$F(x)=\begin{cases}0, & if\ input\ x\ doesn't\ staisfy\ critatia\\ 1, & if\ x\ input\ satisfies\ criteria\end{cases}$$

2) Create a reversible circuit that calculate f(x) with input x. Remember, we previously discussed how we can use different tools like ancillla to create reversable function out of irreversible functions.

- Reversible circuit generator help to create quantum circuits from classical computation.

3) Add an additional ancilla with negative phase that introduces Phase kickback when f(x)=1.

- Object of search gets a phase of -1!

E. 2x2 sudoku

Qiskit's Grover's algorithm completes with the algorithm that solves sudoku of order 2x2. A valid solution can be reached without much efforts even without the concepts of quantum computing just that a number may appear only once in a row and column. It's a great place to start and it is magnificent to evaluate sudoku of order 2x2 using quantum computing concepts, I am prompt to stretch this analysis to Sudokus of larger order.

Using Qiskit's textbook wording: it may be difficult to find a solution to a regular 9x9 Sudoku but, given one, it is easy to verify that it is a valid one. The graph coloring problem (find the coloring of n nodes where pairs connected by vertices cannot have the same color) can also be formulated in these terms, and examples of that are found in Q# and Katas tutorials.[6]

My Quantum solver for Sudoku is based upon the Quantum tutorial on graph coloring using Grover's search alogrithm and follow the plan on superpositions from Katas - challenges

A first step to solving a Sudoku is to convert the numbers into binary so a series of 0 and 1 can represent an easy-to-read solution.

F. Approach 1

Grovers algorithm helps us unstructured search problem so we convert it into search problem in best case checking all the elements will eventually would rest in O(n), and it would make N/2 comparison.

However on quantum computer this type f problem can be solved in O(√N) times by the general ideas of Grover's algorithm we need to amplify the amplitude of the solutions so we can measure it easily.

we have to initialized the system to have uniform distribution so as to have the same amplitude in every N state by This can we done by qubits register initialized in the state |0 >*N and applying Hadamard-gate over all qubits with H⊗n.

This lead to superpositions of state so due to this probability of getting correct answer world be 1/N, so we need to increase the amplitude of certain state, to did so we need to resign the diffuser. The effect of diffuser on states would be inversion about average in easy word we can say the oracle marks the solution state with negative sign to encode the information about the specific translating into a phase shift over system state while the diffuser amplifies this marked state amplitude over all the others.

1) Steps

a) Initialization Application of of H⊗n to first n qubit and HX to last ancilla.

b) Oracle invert the aptitude of |x> that are solution

c) Diffuser amplifies those solution's amplitudes by inverting them over the average of all the amplitudes

d) The measurement yields the solution, probability of solution is more than others.

2) Code

```
from qiskit import *
import numpy as np
from qiskit.providers.aer import QasmSimulator
from qiskit.visualization import plot_histogram
from operator import itemgetter
```

```
a_string = QuantumRegister(1, name='a string')
b_string = QuantumRegister(1, name='b string')
c_string = QuantumRegister(1, name='c string')
d_string = QuantumRegister(1, name='d string')

all_0_ancilla_ab = QuantumRegister(2, name='all 0  ancilla ab')
all_0_ancilla_cd = QuantumRegister(2, name='all 0  ancilla cd')
all_0_ancilla_ac = QuantumRegister(2, name='all 0  ancilla ac')
all_0_ancilla_bd = QuantumRegister(2, name='all 0  ancilla bd')

out = QuantumRegister(1,name='output')

classical = ClassicalRegister(4, name='measure')

qc = QuantumCircuit(a_string, b_string, c_string, d_string, all_0_ancilla_ab,
   all_0_ancilla_cd, all_0_ancilla_ac, all_0_ancilla_bd, out, classical)

#out in -
qc.x(out)
qc.h(out)

#superposition of input
qc.h(a_string[0])
qc.h(b_string[0])
qc.h(c_string[0])
qc.h(d_string[0])

qc.barrier()

def oracle(qc):

   # comparator ab
   qc.x(b_string[0])
   qc.mct([a_string[0], b_string[0]], all_0_ancilla_ab[0])
   qc.x(a_string[0])
   qc.x(b_string[0])
   qc.mct([a_string[0], b_string[0]], all_0_ancilla_ab[1])
   qc.x(a_string[0])
   qc.cnot(all_0_ancilla_ab[0], all_0_ancilla_ab[1])
   qc.barrier()

   # comparator ac
   qc.x(c_string[0])
   qc.mct([a_string[0], c_string[0]], all_0_ancilla_ac[0])
   qc.x(a_string[0])
   qc.x(c_string[0])
   qc.mct([a_string[0], c_string[0]], all_0_ancilla_ac[1])
   qc.x(a_string[0])
   qc.cnot(all_0_ancilla_ac[0], all_0_ancilla_ac[1])
   qc.barrier()
```

```
    # comparator cd
    qc.x(d_string[0])
    qc.mct([d_string[0], c_string[0]], all_0_ancilla_cd[0])
    qc.x(c_string[0])
    qc.x(d_string[0])
    qc.mct([d_string[0], c_string[0]], all_0_ancilla_cd[1])
    qc.x(c_string[0])
    qc.cnot(all_0_ancilla_cd[0], all_0_ancilla_cd[1])
    qc.barrier()

    # comparator bd
    qc.x(b_string[0])
    qc.mct([d_string[0], b_string[0]], all_0_ancilla_bd[0])
    qc.x(d_string[0])
    qc.x(b_string[0])
    qc.mct([d_string[0], b_string[0]], all_0_ancilla_bd[1])
    qc.x(d_string[0])
    qc.cnot(all_0_ancilla_bd[0], all_0_ancilla_bd[1])
    qc.barrier()


def diffuser(qc):
    qc.h(a_string[0])
    qc.h(b_string[0])
    qc.h(c_string[0])
    qc.h(d_string[0])

    qc.x(a_string[0])
    qc.x(b_string[0])
    qc.x(c_string[0])
    qc.x(d_string[0])

    qc.h(a_string[0])
    qc.mct([ b_string[0], c_string[0], d_string[0]], a_string[0])
    qc.h(a_string[0])

    qc.x(a_string[0])
    qc.x(b_string[0])
    qc.x(c_string[0])
    qc.x(d_string[0])

    qc.h(a_string[0])
    qc.h(b_string[0])
    qc.h(c_string[0])
    qc.h(d_string[0])

    qc.barrier()

#n is the number of iteration of Grover operator
n = 2
```

```
for i in range (n):
    oracle(qc)
    qc.mct([all_0_ancilla_ab[1], all_0_ancilla_ac[1],all_0_ancilla_cd[1],
        all_0_ancilla_bd[1]], out)
    qc.barrier()

    oracle(qc)
    diffuser(qc)

qc.measure(a_string[0],classical[0])
qc.measure(b_string[0],classical[1])
qc.measure(c_string[0],classical[2])
qc.measure(d_string[0],classical[3])

print(qc)

def print_sudoku(dict):
    list=[]
    for item in dict:
        for a in item:
            list.append(a)
        data=np.array(list)
        shape=(2,2)
        sudoku=data.reshape(shape)
        print(sudoku)
        print('\n')
        list.clear()

#start simulation
backend = QasmSimulator()
result = execute(qc, backend=backend, shots=1024).result()
answer = result.get_counts()
plot_histogram(answer).show()

#return N max values
res = dict(sorted(answer.items(), key=itemgetter(1), reverse=True)[:2])

#print the sudoku schemas
print('the solutions are:')
print_sudoku(res)
```

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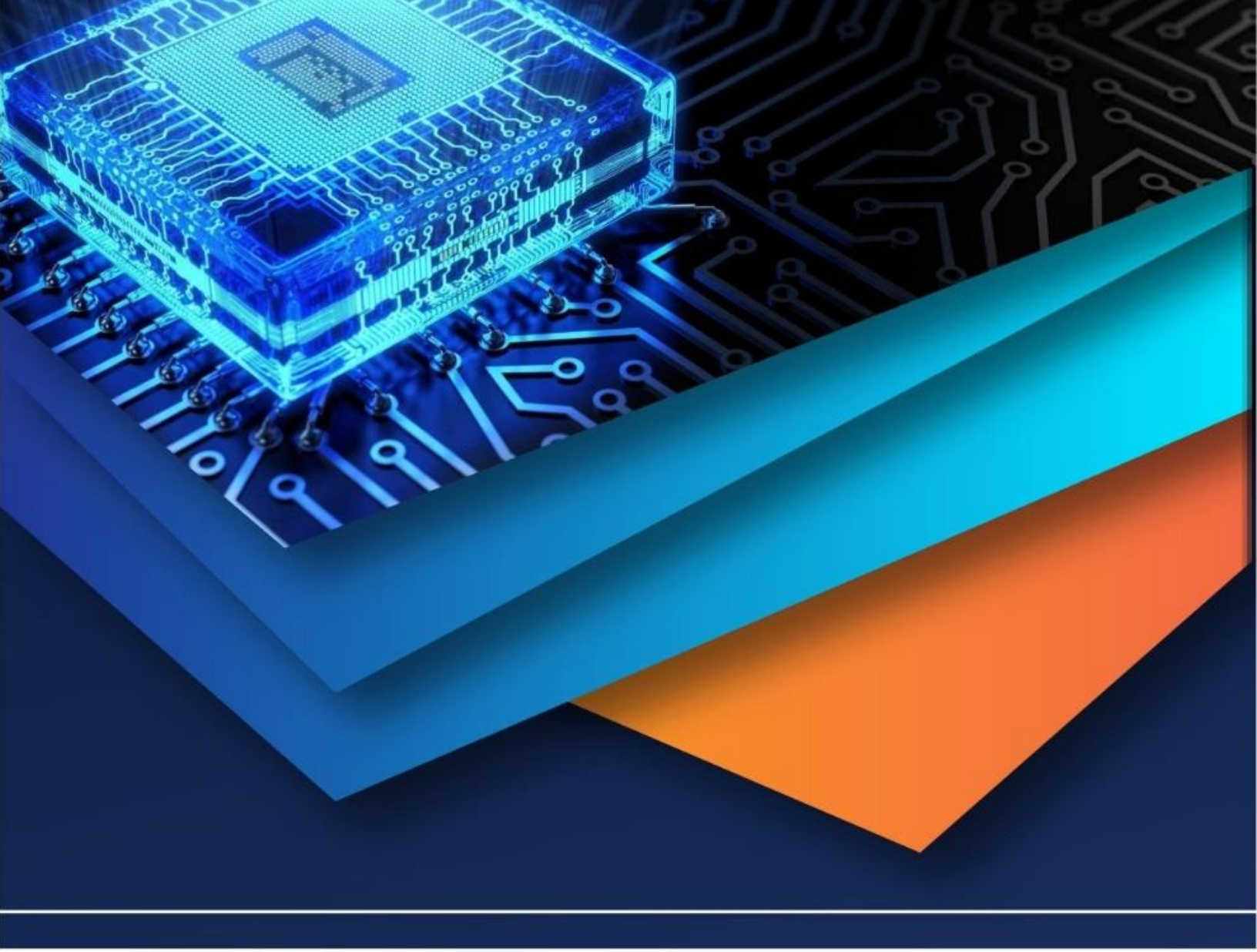

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