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# The Study on the Fundamental of Dark and Bright Solitons

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**Abstract:** *The words bright and dark are taken from the field of optics, where they are used to describe the bright spots and black shadows that appear in optical fibres. However, in the 1830s, soliton observations were made in water. Oceanographers were initially taken aback by the discovery in the 1960s and 1970s that bright solitons were present on the surface of deep ocean waters. However, numerous experiments have since been conducted to investigate and validate the phenomenon, some of which have identified bright solitons as the cause of rogue waves at sea. Both bright and dark solitons have now been observed in Bose-Einstein condensates, plasmas, fibre optics, and other environments. [30]*

**Keywords:** *Dark Soliton, Bright Soliton, Nonlinear Optics, Self Reinforcing, Nonlinear Schrodinger Equation, Dispersion, Self Phase Modulation.*

## I. INTRODUCTION

It is challenging to define soliton in a way that is widely accepted. Three features are offered to solitons by Drazin & Johnson (1989, p. 15). [1]. They have three characteristics: they have a permanent form, they are limited within a region, and they can interact with other solitons. They also avoid phase shifts but otherwise survive collisions unharmed. There are more formal formulations available, but they require a large amount of mathematics. Furthermore, some scientists designate events that do not satisfy these three conditions as soliton phenomena (for instance, the "light bullets" in nonlinear optics are sometimes referred to as solitons, despite the fact that they lose energy during interaction).[2] Wave patterns can be localised and permanent depending on how dispersion and nonlinearity combine. Envision a bright pulse passing through glass. It is possible that this pulse is made up of several different light frequencies. These various frequencies pass through glass at different speeds due to its dispersion, which changes the pulse's structure over time. There is also the nonlinear Kerr effect, in which the brightness or amplitude of the light determines a material's refractive index at a certain frequency. Because of the precise cancellation of the dispersion effect by the Kerr effect, a properly generated pulse maintains its shape over time. The pulse is a soliton as a result [3]. Solitons solve a wide range of entirely solvable models, including as the coupled nonlinear Schrödinger equation, the sine-Gordon equation, the Korteweg-de Vries equation, and several nonlinear Schrödinger equations. The soliton solutions are stable because of the integrability of the field equations, which is usually achieved using the inverse scattering transform. One active and expansive area of mathematics is the mathematical theory of these equations.[3] In some "undular" tidal bore occurrences occurring in a few rivers, like the River Severn, a train of solitons moves in tandem with a wavefront. As internal waves propagate down the oceanic pycnocline, propelled by the seafloor's topography, more solitons are created. Additional atmospheric solitons exist. Consider the morning glory cloud in the Gulf of Carpentaria as an example. It is created by massive linear roll clouds caused by pressure solitons travelling across a temperature inversion layer. Pressure solitons are employed in the recently developed, but not generally recognised, soliton model of neuroscience to explain the signal conduction within neurons [3]. Any solution of a system of partial differential equations that is persistent against decay to the "trivial solution" is referred to as a topological soliton, also known as a topological defect. The origin of soliton stability is not the integrability of the field equations, but topological constraints. Since the differential equations have to satisfy a set of boundary conditions and preserve the nontrivial homotopy group of the boundary, constraints are nearly often present. Therefore, solutions to differential equations can be grouped using homotopy classes [3].

To be more precise, soliton transmission in optical fibres enhances data transmission quality by mitigating two main forms of pulse degradation. The dispersion that occurs when pulses travel over lengthy fibre sections is one kind of deterioration. The other is the nonlinear effects resulting from signals interacting with the fibre and with each other in a power-dependent manner. In general, the two effects compound each other to worsen the condition; however, for some forms and powers of optical pulses, the effects can cancel each other out, at least to first-order approximation. These kinds of pulses are referred to as soliton pulses. One of the key benefits of solitons for long-distance, high-speed gearboxes is their inherent longevity. Despite soliton attenuation, soliton can be made intrinsically stable over long fibre lengths.

This provides a means of mitigating the degradation of signal quality caused by dispersion and nonlinear effects, which is a significant issue at 10 Gbit/s and becomes worse at higher transmission speeds. Because of these characteristics, researchers are working on soliton systems for planned 40-Gbit/s and long-haul 10-Gbit/s systems [4].

## II. LITERATURE REVIEW

After seeing a single wave on Scotland's Union Canal in 1834, John Scott Russell (1808–1882) became the first person to record the soliton phenomenon. He named the phenomenon the "Wave of Translation" when he was able to replicate the event in a wave tank. Similar waves to those Russell saw can be described by locally limited, strongly stable propagating solutions to the Korteweg–de Vries equation. Zabusky and Kruskal were the ones who first gave these solutions the name "soliton." With the suffix "on" reflecting its original usage to represent particles such as hadrons, baryons, and electrons and reflecting their observed particle-like activity, the name was intended to describe the solitary nature of the waves.[3].

A young engineer named John Scott Russell, engaged for a summer project in 1834 to look at ways to enhance the designs for barges intended to traverse canals—specifically, the Union Canal in Edinburgh, Scotland—made the first reported observation of a lone wave. The barge came to a sudden stop one August day when the tow line holding the mules to the vessel broke. Still, the mass of water ahead of the blunt prow of the barge rolled forward rapidly, forming a large isolated elevation, a smooth, rounded mound of water, and continuing down the canal without altering its shape or speed. Russell (1844). Russell rode up to the Wave of Translation and passed it after investigating this coincidental discovery further. The Wave of Translation was rolling at around eight or nine miles per hour, keeping its original measurements of roughly thirty feet by one foot to one and a half. Then, in carefully monitored laboratory trials, he employed a wave tank; he published the results in 1844 (Russell, 1844). He gave the following four instances: In single waves, he noticed hyperbolic secant structure. If the initial quantity of water is large enough, it can create two or more waves that progressively split away and become almost solitary. When separated, waves can pass one another "without change of any kind". In a shallow water channel of height  $h$ , a single wave with amplitude  $A$  travels at the speed of  $[g(A+h)]^{1/2}$ , where  $g$  is the gravitational acceleration. Greater amplitude waves travel faster than lower amplitude waves in this nonlinear phenomenon. In 1895, the Dutch physicist Diederick Korteweg and his pupil Gustav de Vries (KdV) (de Vries, 1895) (Scott, 2005) created the nonlinear partial differential equation (PDE) that bears their names to this day. Russell's experiments may be explained by the KdV equation (1), according to Korteweg and de Vries. Equation (1) demonstrates that the amplitude effect, a nonlinear component, and the dispersive term—which permits waves of various wavelengths to travel at different velocities—combine to determine the rate of change of the wave's height over time. Russell's wave was matched by a periodic solution discovered by Korteweg and de Vries, in addition to a solitary-wave solution. A trade-off between dispersion and nonlinearity led to these solutions. Until 1965, when mathematicians, physicists, and engineers researching water waves disregarded both Russell's findings and the work of Norman Zabusky and Martin Kruskal, who published their numerical solutions to the KdV equation (and coined the term "soliton") (Zabusky, 1965). On the cubic, Fermi–Pasta–Ulam (FPU) nonlinear lattice, Kruskal obtained a continuous description of the oscillations of unidirectional waves propagating, as stated in (1) (Fermi, 1955; Porter, 2009b; Weissert, 1997). Simultaneously, Morikazu Toda made history by being the first to identify a soliton in the discrete, integrable system that is now known as the Toda lattice (Toda, 1967).[5]

Films of interacting solitary waves in an FPU lattice, the KdV equation, and a modified KdV equation were produced by Gary Deem, Zabusky, and Kruskal in 1965; see the discussion in the review work (Zabusky, 1984). We illustrate the dynamics of solitons in the space-time diagram shown in Figure 1 using the KdV equation. Robert Miura discovered a precise transition between this modified KdV equation and equation (1) when he understood the significance of this discovery (Miura, 1976). Interest in the mathematical study of solitons arose after Clifford Gardner, John Greene, Martin Kruskal, and Robert Miura used the inverse scattering approach to solve the KdV equation's initial-value problem in 1967 (Miura, 1968; Gardner, 1967; Gardner, 1974). As a result, a suitable definition of integrability for continuum frameworks was created. By demonstrating the integrability of the nonlinear Schrödinger (NLS) problem and the availability of soliton solutions, Alexei Borisovich Shabat and Vladimir Zakharov expanded the inverse scattering technique in 1972. One of the various nonlinear PDEs for which Mark Ablowitz, David Kaup, Alan Newell, and Harvey Segur demonstrated that they had soliton solutions in 1973 was the sine-Gordon equation. Thanks to Albert Backlund's 19th-century studies of surfaces with constant negative Gaussian curvature, this equation was already known to be integrable. Since then, other researchers (in one and several spatial dimensions) have explored related soliton solutions and deduced alternative integrable PDEs.

The Kadomtsev–Petviashvili (KP) equation shows that a more intricate definition of a "soliton" in many spatial dimensions is necessary.



Analytical methods for studying solitary waves in nonintegrable equations usually involve the use of asymptotic analysis, variational approximations, and/or perturbative techniques (Kivshar, 1989). (Scott, 2005) The coupled mode equations of the fibre Bragg grating are a well-known illustration of a nonintegrable system that has exact solutions for solitary waves that are isolated in optics. One of the most active fields in physics and mathematics today is the study of solitons and solitary waves (Scott, 2005). It has affected a wide range of disciplines, including as experimental science and pure mathematics. This has produced significant findings in a number of fields, including integrable systems, biology, optics, supersymmetry, and nonlinear dynamics.

### III. MATHEMATICAL EQUATIONS OF SOLITON

Nonlinear Schrodinger equation (NLS)[6],[7]:

$$i \frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0 \tag{8}$$

$\beta_2$  is the GVD of the optical fiber

$\gamma$  is the nonlinear coefficient of the fiber,

$$\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}}$$

The effects of dispersion & assuming Gaussian pulse shape, [6],[7]:

$$i \frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} = 0 \tag{8a}$$

(without the nonlinear term)

$$\tau_{\text{out}} = \tau_{\text{in}} (1 + (\beta_2 \cdot L / \tau^2)^2)^{1/2} \tag{9}$$

$$\tau_{\text{out}} = \tau_{\text{in}} (1 + (L / L_D)^2)^{1/2} \tag{10}$$

Where ,  $L_D = \tau^2 / |\beta_2|$  is the dispersion length.

The effects of nonlinearity, [6],[7]:

$$i \frac{\partial A}{\partial z} + \gamma |A|^2 A = 0 \tag{8b}$$

(without the dispersion term)

The maximum nonlinear phase shift, [6],[7]:

$$\phi_{\text{max}} = \gamma P_0 L = L / L_{\text{NL}}$$

And the nonlinear length, [6],[7]:

$$L_{\text{NL}} = (\gamma P_0)^{-1}$$

For the Self-Phase Modulation,  $I(t)$  gives the intensity of an ultrashort pulse with a Gaussian form and constant phase at time  $t$  ,[6][7]:

$$I(t) = I_0 \exp\left(-\frac{t^2}{\tau^2}\right) \tag{11}$$

From the Optical Kerr Effect [6][7]:

$$n(I) = n_0 + n_2 \cdot I \tag{12}$$

This change in refractive index causes a displacement in the pulse's immediate phase [6][7]:

$$\phi(t) = \omega_0 t - kx = \omega_0 t - \frac{2\pi}{\lambda_0} \cdot n(I)L \tag{13}$$

The pulse shifts in frequency as a result of the phase shift. The frequency  $\omega(t)$  at any given instant is provided by ,[6][7]:

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_0 - \frac{2\pi L}{\lambda_0} \frac{dn(I)}{dt} \tag{14}$$

$$\omega(t) = \omega_0 + \frac{4\pi L n_2 I_0}{\lambda_0 \tau^2} \cdot t \cdot \exp\left(\frac{-t^2}{\tau^2}\right) \tag{15}$$

#### IV. THE BRIGHT SOLITON

The localised intensity peak of a bright soliton is located above a continuous wave (CW) backdrop, whereas the localised intensity dip of a dark soliton is located below a CW background. A "vector dark bright soliton" is a polarisation condition in which one polarisation is bright and the other is dark.[33]

The following is the form of Eq. (16) bright solitary solutions[27][28]:

$$u_b(\zeta) = \frac{A_b}{\sqrt{1 + N_b \cosh(\alpha_b \zeta)}} \tag{16}$$

where  $A_b$ ,  $N_b$ , and  $\alpha_b$  are real constants which represent wave parameters ( $A_b$  and  $\alpha_b$  related to the amplitude and pulse width of the bright wave profiles, respectively) to be determined by the physical coefficients of the model.

The following parameters: energy  $\xi$ ,  $N_b$ ,  $\alpha_b$ , and  $a_b$ [27][28]:

$$\begin{aligned} A_b &= \sqrt{\frac{2a_1}{a_3}} \\ \alpha_b &= \sqrt[4]{a_1} \\ N_b &= \sqrt{\frac{a_3^2 + 4a_1 a_5}{a_3^2}} \\ \xi &= \frac{a_1 a_2}{2a_3} \end{aligned} \tag{17}$$

under parametric circumstances [27][28]:

$$a_2 a_3 + 4a_1 a_4 = 0, a_1 > 0, a_3 > 0, a_3^2 + 4a_1 a_5 > 0 \tag{18}$$

### V. THE DARK SOLITON

The formation of dark solitons[31] is typified by a localised decrease in intensity in contrast to a continuous wave background that is more intense. All normal dispersion fibre lasers mode-locked using the nonlinear polarisation rotation method can create scalar dark solitons (also known as linearly polarised dark solitons), which can be rather stable. Because of the two polarisation components' cross-interaction, vector dark solitons[32] are far less stable. Consequently, it is intriguing to look at the evolution of these two polarisation components polarisation states. Specific medium conditions have been considered in the investigation of both forms of solitons, In temporal solitons,  $\beta_2 < 0$  or  $D > 0$ , anomalous dispersion, and this indicates that the self-phase modulation produces self-focusing in spatial solitons,  $n_2 > 0$ . [29]:

$$-\frac{1}{2} \frac{\partial^2 a}{\partial \tau^2} + i \frac{\partial a}{\partial \zeta} + N^2 |a|^2 a = 0. \tag{19}$$

Solitons-like solutions exist for this problem. Regarding  $N = 1$ , the first order [29]:

$$a(\tau, \zeta) = \tanh(\tau) e^{i\zeta} \tag{20}$$

For higher order solitons ( $N > 1$ ) we can use the following closed form expression [29]:

$$a(\tau, \zeta = 0) = N \tanh(\tau) \tag{21}$$

The solutions for dark solitary adopt the following form[27][28]:

$$u_d(\zeta) = \frac{A_d \sinh(\alpha_d \zeta)}{\sqrt{1 + N_d \sinh^2(\alpha_d \zeta)}} \tag{22}$$

In this case, the real constant  $N_d$  is assumed to be positive. The dark wave profiles' amplitude and pulse width are correlated with the real parameters  $A_d$  and  $\alpha_d$ , respectively[27][28].

The following parameters  $A_d$ ,  $\alpha_d$ , and energy  $\xi$  can be obtained [27][28] :

$$\begin{aligned} A_d &= \sqrt{\frac{2a_1 N_d}{a_3}} \\ \alpha_d &= \sqrt[2]{a_1} \\ \xi &= \frac{a_1}{2a_3} \end{aligned} \tag{23}$$

based on the parametric conditions [27][28]:

$$2(a_2 a_3 + 2a_1 a_4) - a_3 = 0, a_1 > 0, a_3 > 0, a_3^2 + 4a_1 a_5 > 0. \tag{24}$$

The precise single-wave dark solutions [27][28]:

$$E_d(z, t) = \left\{ \frac{2a_1 N_d}{a_3} \frac{\sinh^2[\sqrt[2]{a_1}(t + \beta z)]}{1 + N_d \sinh^2[\sqrt[2]{a_1}(t + 2\alpha_1 \omega z)]} \right\}^{\frac{1}{4}} \times e^{i(kz - \omega t)} \tag{25}$$

## VI. CONCLUSION

Solitons light pulses propagate without widening over very long distances in a non-linear dispersive medium. They have therefore attracted a lot of interest from the communications industry. Solitons operate in optical fibres because of two effects that, given the right circumstances, can cancel one other out. One of these is chromatic dispersion, which occurs when pulses with various wavelengths disperse as they pass through an optical fibre. Self-phase modulation, or SPM, is the alternative that works over a wider variety of pulse spectrum wavelengths. The pulse can maintain its form or dispersion once it reaches equilibrium in the fibre since dispersion and SPM can cancel one another out. The pulse may potentially more severely expand or compress as a result of SPM. Conversely, attenuation reduces the pulse's strength and makes it more difficult for it to maintain its shape over the fibre span. Optical amplifiers have been constructed to balance attenuation and preserve pulse shapes. [24][25]. Solitons are obtained by solving non-linear wave equations that characterise the propagation of waves in certain physical systems. Mathematical models of a variety of systems, including water waves, crystal lattice vibrations, and optical waveguides, show these waves as solutions. On a modulationally stable continuous-wave background, dark solitons exhibit local dips. Dark solitons in fibre lasers are less prone to loss and more stable in noisy environments than dazzling solitons.

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