



# **iJRASET**

International Journal For Research in  
Applied Science and Engineering Technology



---

# **INTERNATIONAL JOURNAL FOR RESEARCH**

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume: 11    Issue: XI    Month of publication: November 2023**

**DOI: <https://doi.org/10.22214/ijraset.2023.56572>**

**[www.ijraset.com](http://www.ijraset.com)**

**Call:  08813907089**

**E-mail ID: [ijraset@gmail.com](mailto:ijraset@gmail.com)**

# Type-II Censored analysis based on Exponentiated Inverted Weibull distribution

V. Sastry Ch

Coordinator, Department of Statistics, KRU Dr. MRAR College of PG Studies, Nuzvid, A.P, India

**Abstract:** In this paper, find the parameter estimation for the Exponentiated Inverted Weibull (EIW) distribution based on type II censored data. Here detailed discussion on loglikelihood functions for complete and censoring models of different parameters in Exponentiated Inverted Weibull (EIW) distribution. To estimate the parameters, calculate the Mean Square Error, Total Deviation and data tested with Kolmogorov and Smirnov method.

**Keywords:** Censoring, Exponential, Inverted Weibull, Weibull, Simulation, Kolmogorov-Smirnov Test, MSE, Total Deviation.

## I. INTRODUCTION

Exponentiated Inverted Weibull distribution is a particular case of Generalized Exponentiated Weibull distribution. If a random variable  $X$  has Exponentiated Weibull distribution to  $\frac{1}{x}$  said to have Generalized Exponentiated Weibull distribution. It is used in reliability Engineering, system reliability, life-testing, survival analysis, data compression and many other fields. The two shape parameters Exponentiated Inverted Weibull distribution and their properties are in A.Flaih et.al (2012). Seunghyung et.al., (2017) proposed families of Inverted Exponentiated Weibull distribution. In Inverted Exponentiated Weibull distribution substitute the shape parameter value is ONE get the Exponentiated Inverted Weibull distribution.

## II. EXPONENTIATED INVERTED WEIBULL DISTRIBUTION (EIWD)

Exponentiated Inverted Weibull Distribution (EIWD) cumulative distribution function is

$$F(x) = \left( e^{-x^{-\alpha}} \right)^\lambda \tag{1}$$

Probability density function of EIWD is

$$f(x) = \lambda \alpha x^{-(\alpha+1)} \left( e^{-x^{-\alpha}} \right)^\lambda \tag{2}$$

The distribution function of  $x$  takes the presented in equation (1) where  $\alpha$  and  $\lambda$  both are shape parameters and  $0 < x < \infty$  and  $\alpha > 0, \lambda > 0$ . The probability density function of EIWD in (2).

Here if  $\lambda = 1$ , this EIWD becomes the standard Inverted Weibull distribution

## III. CHARACTERISTICS OF EXPONENTIATED INVERTED WEIBULL DISTRIBUTION (EIWD):

In this section presented the properties for Exponentiated Inverted Weibull (EIW) Distribution.

The Reliability function  $R(x) = 1 - \left( e^{-x^{-\alpha}} \right)^\lambda \quad x, \alpha, \lambda > 0 \tag{3}$

The hazard function 
$$h(x) = \frac{\alpha \lambda x^{-(\alpha+1)} \left( e^{-x^{-\alpha}} \right)^\lambda}{1 - \left( e^{-x^{-\alpha}} \right)^\lambda} \tag{4}$$

Inverse transformation the method for random variable  $x_l = \left( -\frac{1}{\lambda} \ln l \right)^{-\frac{1}{\alpha}} \tag{5}$

The Median  $\left( l = \frac{1}{2} \right) \quad x_l = \left( \frac{\lambda}{\ln 2} \right)^{-\frac{1}{2}} \tag{6}$

**A. Method of Moments**

The  $k^{th}$  moments of the EIW distribution is given as follows:

$$E(x^k) = \int_0^\infty x^k f(x) dx = \int_0^\infty x^k \alpha \lambda x^{-(\alpha+1)} (e^{-x^{-\alpha}})^\lambda dx$$

$$= \lambda \frac{k}{\alpha} \left(1 - \frac{k}{\alpha}\right); \alpha > k \tag{7}$$

Let  $k=1$  in Eq (7), we obtain the Mean as:

$$E(x^k) = \lambda \frac{1}{\alpha} \left(1 - \frac{1}{\alpha}\right); \lambda > 1 \tag{8}$$

**IV. COMPUTATIONAL RESULTS**

*Mean Square Error (MSE)*

In order to understand the accuracy of the Exponentiated Inverted Weibull distribution, most of the studies address the question as a statistical error and calculate the Mean Square Error (MSE) based on the measured data.

$$MSE = \frac{\sum_{i=1}^n (\hat{F}(x_i) - F(x_i))^2}{n} \tag{9}$$

where  $\hat{F}(x_i)$  is the measured value and  $F(x_i)$  is the Exponentiated Inverted Weibull distribution parameters based calculated value.

*Total Deviation (TD)*

The objective of our experiments is to compare the method of MLE parameters value. We have generated random samples with known parameters. For each sample, we have varied the size from 5 to 100. To be able to compare, we calculated the Total Deviation (TD) for each method as follows:

$$TD = \left| \frac{\hat{\alpha} - \alpha}{\alpha} \right| + \left| \frac{\hat{\lambda} - \lambda}{\lambda} \right| \tag{10}$$

where  $\alpha$  and  $\lambda$  are the known parameters, and  $\hat{\alpha}$  and  $\hat{\lambda}$  are the estimated parameters by any method.

*Kolmogorov-Smirnov Test (KS Test)*

It is based on a comparison between the Empirical Distribution Function (ECDF) of Exponentiated Inverted Weibull distribution pdf values and given  $n$  order data points.

$$D_n = \text{Sup}_{1 \leq i \leq n} \left| \hat{F}(x_i) - F(x_i) \right| \tag{11}$$

The Statistics  $D_n$  converges to zero almost surely  $n \rightarrow \infty$

**V. DISTRIBUTION OF ORDER STATISTICS**

In this section discuss the order statistics value. Let  $x_1, x_2, \dots, x_n$  be the  $n$  iid random variables from the population have pdf  $f(X)$  and CDF  $F(X_i)$ .

$X_{1:n} \leq X_{2:n} \leq \dots \leq X_{n:n}$  Ordered random variables the pdf of  $r$ th order statistics  $X_{r:n}$  is

$$f_{r:n}(x) = \frac{n!}{(n-r)!(r-1)!} [F(x)]^{r-1} [1-F(x)]^{n-r} f(x) \tag{12}$$

*The Log-Likelihood function on Complete Data*

If  $X_1, X_2, \dots, X_n$  is ordered random sample from Exponentiated Inverted Weibull distribution given by pdf  $f(X)$ , then the Log-Likelihood function (LL) becomes:

$$L(\alpha, \lambda) = n \log \alpha + n \log \lambda - (\lambda + 1) \sum_{i=1}^n \log x_i - \alpha \sum_{i=1}^n x_i^\lambda \tag{13}$$

There the MLE of  $\alpha$  and  $\lambda$  which maximize equation (18) must satisfy the nonlinear normal equations given by

$$\frac{\partial \text{Log}L}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^n x_i^{-\lambda} = 0 \tag{14}$$

$$\frac{\partial \text{Log}L}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^n \log x_i - \alpha \sum_{i=1}^n x_i^{-\lambda} \log x_i = 0 \tag{15}$$

*The Log-Likelihood function on Type-II Censoring Statistics Data*

Type II censored data is derived from research or observation of live test on a research object which is product, individual, system, unit or component. Unknown  $n$  is the number of research objects and which is a random sample of where and  $r$  is the sequential survival time in a study.

Likelihood function for type II censor data with parameter in general is as follows:

$$L(\lambda, \alpha; X_i) = \frac{n!}{(n-r)!} \left[ \prod_{i=1}^r f(X_i) \right] [1 - F(X_r)]^{n-r} \tag{16}$$

The MLE of the parameter  $\lambda$  and  $\alpha$  under type-II censoring can be shown to be of the form

$$\frac{\partial \text{Log}L}{\partial \lambda} = \frac{n}{\lambda} + \sum_{i=1}^r X_i^{-\alpha} - (n-r) \frac{X_r^{-\alpha} e^{-\lambda X_r^{-\alpha}}}{1 - e^{-\lambda X_r^{-\alpha}}} = 0 \tag{17}$$

$$\frac{\partial \text{Log}L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^r \log(X_i) - (r-1)\lambda \sum_{i=1}^r X_i^{-(\alpha+1)} + \lambda(n-r) \frac{X_r^{-\alpha} \log(X_r) e^{-\lambda X_r^{-\alpha}}}{1 - e^{-\lambda X_r^{-\alpha}}} = 0 \tag{18}$$

The effects of the Type-II censored sampling scheme on the estimation of the unknown parameters of  $\lambda$  and  $\alpha$  with respect to the complete sampling is compared. This comparison based on Mean Square Error (MSE), Kolmogorov-Smirnov Test (KS Test) and Total Deviation (TD) of a  $n$  experiment with items.

The R Language is environment is Perform the estimation of the parameters. In R Software version 4.1.3 the packages of miscTools and maxLik used for analytical procedures for estimating the parameter of  $\alpha$  and  $\lambda$  as Complete and Censored samples.

**VI. RESULTS**

In this section demonstrate the Complete and Censored data to estimation are too complicate to study analytically. The numerical procedure is described below:

- 1) 10000 random sample of sizes 5, 10, 15, 20, 25, 30,50, 75, and 100 are generated form Exponentiated Inverted Weibull distribution.
- 2) Sets of Parameters values are  $\alpha = 0.5, 1.5, 2.0, 2.5, \text{ and } 3.0$ ;  $\lambda = 0.5, 1.5, 2.0, 2.5, \text{ and } 3.0$
- 3) The Maximum Likelihood Estimation of Complete and Censored samples is estimate of unknown parameters are obtain.
- 4) Mean Square Error (MSE), Kolmogorov-Smirnov Test (KS Test) and Total Deviation (TD) estimates of different parameters compute.

n	r	$\alpha$	$\lambda$	$\hat{\alpha}$	$\hat{\lambda}$	MSE	KS	p-value	TD
5	3	1.5	0.5	3.9649	0.3391	0.0139	0.5217	0.2867	1.5828
5	4	1.5	0.5	3.46	0.2802	0.0176	0.6170	0.0558	2.0043
5	5	1.5	0.5	3.4001	0.1367	0.0493	0.7010	0.0057	2.6226
5	4	1.5	1.5	3.3503	0.4121	0.132	0.6630	0.0300	3.8100
5	5	1.5	1.5	3.1776	0.3060	0.2037	0.7100	0.0048	3.4800
5	5	1.5	2.0	3.1776	0.5628	0.2037	0.7100	0.0048	3.6312
5	5	1.5	2.5	3.8177	0.9530	0.252	0.6600	0.0120	2.6321
5	4	2.0	0.5	3.6515	0.3209	0.0093	0.5790	0.0874	1.1839
5	5	2.0	0.5	3.3898	0.2479	0.0199	0.6600	0.0120	1.199
5	5	2.0	1.5	4.5335	1.6490	0.0493	0.7010	0.0057	1.366
5	5	2.0	1.5	4.5335	1.6490	0.0493	0.7010	0.0057	1.366
5	5	2.0	2.0	3.4786	0.5449	0.1611	0.7030	0.0055	1.4668
5	5	2.0	2.5	4.2368	0.9029	0.2037	0.7100	0.0048	1.7572
5	4	2.5	0.5	4.5643	0.3209	0.0093	0.5790	0.0874	1.1839
5	5	2.5	0.5	4.2372	0.2479	0.0199	0.6600	0.0120	1.199
5	5	2.5	2.0	5.6668	3.1653	0.0493	0.7010	0.0057	1.8494
5	5	2.5	2.5	7.9321	14.139	0.071	0.7220	0.0038	6.8284

n	r	$\alpha$	$\lambda$	$\hat{\alpha}$	$\hat{\lambda}$	MSE	KS	p-value	TD
5	5	2.5	3.0	4.3482	1.1030	0.1611	0.7030	0.0055	1.3716
5	4	3.0	0.5	5.4772	0.3209	0.0093	0.5790	0.0874	1.1839
10	7	0.5	0.5	0.8208	0.0667	0.0965	0.7071	5e-04	1.5082
10	8	0.5	0.5	0.8918	0.0426	0.1447	0.688	3e-04	1.6985
10	9	0.5	0.5	0.9729	0.0266	0.2013	0.7109	0	1.8928
10	10	0.5	0.5	1.0583	0.0159	0.2672	0.745	0	2.0849
10	7	1.5	1.5	3.8873	0.3737	0.1337	0.6641	0.0015	2.3424
10	8	1.5	1.5	3.9104	0.2663	0.1856	0.677	4e-04	2.4294
10	9	1.5	1.5	3.7964	0.1827	0.2491	0.69	1e-04	2.4091
10	10	1.5	1.5	3.8052	0.1476	0.3071	0.741	0	2.4385
10	7	2.0	2.0	4.869	0.8993	0.1063	0.7481	2e-04	1.9848
10	8	2.0	2.0	4.7801	0.6496	0.1489	0.697	2e-04	2.0653
10	9	2.0	2.0	4.5818	0.4523	0.2022	0.7089	1e-04	2.0648
10	10	2.0	2.0	4.2334	0.2992	0.2672	0.745	0	1.9671
10	9	2.5	2.5	4.2192	0.7646	0.1169	0.7699	0	1.3818
10	10	2.5	2.5	3.8014	0.5791	0.136	0.792	0	1.2889
10	9	3.0	3.0	4.5829	1.0039	0.0922	0.7719	0	1.193
10	10	3.0	3.0	4.2709	0.7112	0.1272	0.787	0	1.1866
15	12	0.5	0.5	0.8731	0.0194	0.2432	0.75	0	1.7072
15	13	0.5	0.5	0.9574	0.0137	0.2839	0.77	0	1.8874
15	14	0.5	0.5	1.0588	0.0093	0.3256	0.784	0	2.0991
15	15	0.5	0.5	1.1776	0.0062	0.364	0.791	0	2.3427
15	12	1.5	0.5	2.1658	0.0646	0.0865	0.7847	0	1.3147
15	13	1.5	0.5	2.1965	0.0493	0.1175	0.7581	0	1.3658
15	14	1.5	0.5	2.2163	0.0377	0.154	0.7706	0	1.4021
15	15	1.5	0.5	2.2021	0.035	0.1631	0.7873	0	1.3982
15	12	2.0	0.5	2.3585	0.0915	0.0594	0.7767	0	0.9963
15	13	2.0	0.5	2.4064	0.073	0.0815	0.7921	0	1.0573
15	14	2.0	0.5	2.3948	0.0662	0.0888	0.8096	0	1.0649
15	15	2.0	0.5	2.4336	0.0582	0.1005	0.8263	0	1.1005
15	12	2.5	0.5	2.9481	0.0915	0.0594	0.7767	0	0.9963
15	13	2.5	0.5	3.008	0.073	0.0815	0.7921	0	1.0573
15	14	2.5	0.5	2.9935	0.0662	0.0888	0.8096	0	1.0649
15	15	2.5	0.5	3.042	0.0582	0.1005	0.8263	0	1.1005
15	12	3.0	0.5	3.5378	0.0915	0.0594	0.7767	0	0.9963
15	13	3.0	0.5	3.6096	0.073	0.0815	0.7921	0	1.0573
15	14	3.0	0.5	3.2946	0.0713	0.0859	0.8066	0	0.9555
15	15	3.0	0.5	3.2823	0.0647	0.093	0.8223	0	0.9646
15	12	0.5	1.5	0.9076	0.0524	0.4028	0.8057	0	1.7802
15	13	0.5	1.5	0.9013	0.0481	0.4335	0.8211	0	1.7706
15	14	0.5	1.5	0.9351	0.0389	0.4917	0.8366	0	1.8444
15	15	0.5	1.5	0.9254	0.0363	0.5216	0.8483	0	1.8267
15	12	1.5	1.5	3.9531	0.1141	0.272	0.7047	0	2.5594
15	13	1.5	1.5	4.0169	0.0975	0.3139	0.7361	0	2.613
15	14	1.5	1.5	4.0437	0.0859	0.3528	0.7616	0	2.6386
15	15	1.5	1.5	3.819	0.0805	0.3757	0.7813	0	2.4923
15	12	2.0	1.5	3.6004	0.3001	0.11	0.7317	0	1.6001
15	13	2.0	1.5	3.466	0.2233	0.1459	0.7441	0	1.5842
15	14	2.0	1.5	3.2134	0.1982	0.1556	0.7666	0	1.4746
15	15	2.0	1.5	3.0166	0.1471	0.1986	0.799	0	1.4102
15	12	2.5	1.5	4.2333	0.3486	0.0865	0.7897	0	1.4609
15	13	2.5	1.5	4.1106	0.2632	0.1171	0.7541	0	1.4688
15	14	2.5	1.5	3.9291	0.1973	0.1536	0.7666	0	1.4401
15	15	2.5	1.5	3.6702	0.1753	0.1631	0.7873	0	1.3512

n	r	$\alpha$	$\lambda$	$\hat{\alpha}$	$\hat{\lambda}$	MSE	KS	p-value	TD
15	12	3.0	1.5	4.0387	0.3653	0.0586	0.7797	0	1.1027
15	13	3.0	1.5	3.9683	0.2917	0.0807	0.7921	0	1.1283
15	14	3.0	1.5	3.7616	0.2552	0.0883	0.8096	0	1.0837
15	15	3.0	1.5	3.6504	0.2214	0.1005	0.8263	0	1.0692
15	12	0.5	2.0	1.014	0.0655	0.4464	0.7537	0	1.9953
15	13	0.5	2.0	1.0345	0.0502	0.5112	0.7671	0	2.0439
15	14	0.5	2.0	1.0527	0.0443	0.5641	0.7866	0	2.0832
15	15	0.5	2.0	1.0922	0.0363	0.6279	0.8103	0	2.1663
15	13	1.5	2.0	4.9105	0.2194	0.3399	0.7332	0	3.164
15	14	1.5	2.0	4.4384	0.1937	0.3625	0.7556	0	2.8621
15	15	1.5	2.0	2.3471	0.0787	0.4306	0.8373	0	1.5254
15	12	2.0	2.0	4.9545	0.256	0.2436	0.6947	0	2.3492
15	13	2.0	2.0	4.8941	0.2182	0.284	0.7301	0	2.3379
15	14	2.0	2.0	4.8199	0.1864	0.3255	0.762	0	2.3167
15	15	2.0	2.0	4.7102	0.1627	0.364	0.791	0	2.2738
15	12	2.5	2.0	4.4181	0.5936	0.0868	0.7917	0	1.4704
15	13	2.5	2.0	4.2407	0.4362	0.117	0.7531	0	1.4782
15	14	2.5	2.0	3.9943	0.315	0.1535	0.7656	0	1.4402
15	15	2.5	2.0	3.6702	0.2675	0.1631	0.7873	0	1.3343
15	14	3.0	2.0	4.7932	0.315	0.1535	0.7656	0	1.4402
15	15	3.0	2.0	4.4042	0.2675	0.1631	0.7873	0	1.3343
15	12	0.5	2.5	1.2582	0.0671	0.5172	0.7627	0	2.4897
15	13	0.5	2.5	1.0929	0.0484	0.5859	0.7442	0	2.1665
15	14	0.5	2.5	1.1138	0.0426	0.6463	0.7651	0	2.2106
15	15	0.5	2.5	1.1077	0.0393	0.6987	0.7817	0	2.1998
15	14	1.5	2.5	2.4852	0.1203	0.413	0.8266	0	1.6087
15	15	1.5	2.5	2.4617	0.108	0.454	0.8413	0	1.5979
15	13	2.5	2.5	4.7153	0.6032	0.1475	0.7411	0	1.6448
15	14	2.5	2.5	4.2987	0.3991	0.1902	0.764	0	1.5598
15	15	2.5	2.5	4.0934	0.313	0.2277	0.807	0	1.5122
15	12	3.0	2.5	5.4824	0.9244	0.087	0.7937	0	1.4577
15	13	3.0	2.5	5.2145	0.6591	0.117	0.7521	0	1.4745
15	14	3.0	2.5	4.8553	0.4576	0.1534	0.7646	0	1.4354
15	15	3.0	2.5	4.4042	0.3712	0.1631	0.7873	0	1.3196
15	12	0.5	3.0	1.415	0.0507	0.5861	0.7083	0	2.8131
15	13	0.5	3.0	1.3826	0.0476	0.6429	0.7312	0	2.7494
15	14	0.5	3.0	1.3893	0.043	0.7053	0.7521	0	2.7642
15	15	0.5	3.0	1.3731	0.04	0.7638	0.7687	0	2.7329
15	12	1.5	3.0	2.7999	0.1964	0.3681	0.8007	0	1.8011
15	13	1.5	3.0	2.7511	0.1752	0.409	0.8181	0	1.7757
15	14	1.5	3.0	2.7222	0.1552	0.455	0.8346	0	1.7631
15	15	1.5	3.0	2.6092	0.1414	0.4842	0.8453	0	1.6923
15	12	2.0	3.0	7.8347	1.2859	0.3036	0.7143	0	3.4887
15	13	2.0	3.0	7.2186	0.9864	0.341	0.7352	0	3.2805
15	15	2.0	3.0	3.1295	0.1484	0.4306	0.8373	0	1.5153
15	14	3.0	3.0	4.9069	0.6252	0.1533	0.7646	0	1.4273
15	15	3.0	3.0	4.4042	0.4851	0.1631	0.7873	0	1.3064
20	16	0.5	0.5	0.9601	0.0125	0.2729	0.785	0	1.8953
20	17	0.5	0.5	0.6668	0.0149	0.3238	0.8312	0	1.3039
20	18	0.5	0.5	0.7009	0.0121	0.3546	0.8444	0	1.3776
20	19	0.5	0.5	0.7391	0.0097	0.389	0.8564	0	1.4588
20	20	0.5	0.5	0.7498	0.0089	0.4012	0.864	0	1.4818
20	15	1.5	1.5	4.3146	0.1041	0.2646	0.7357	0	2.807
20	16	1.5	1.5	2.3319	0.0627	0.316	0.8195	0	1.5128

n	r	$\alpha$	$\lambda$	$\hat{\alpha}$	$\hat{\lambda}$	MSE	KS	p-value	TD
20	17	1.5	1.5	2.3654	0.0561	0.3468	0.8342	0	1.5396
20	18	1.5	1.5	2.4066	0.0497	0.3812	0.8474	0	1.5712
20	19	1.5	1.5	2.3458	0.0476	0.3934	0.8564	0	1.5321
20	20	1.5	1.5	2.2539	0.0461	0.4016	0.864	0	1.4719
20	17	2.0	2.0	3.0769	0.0924	0.3245	0.8302	0	1.4923
20	18	2.0	2.0	3.0877	0.0836	0.355	0.8434	0	1.5021
20	19	2.0	2.0	3.1058	0.0751	0.3892	0.8554	0	1.5154
20	20	2.0	2.0	2.9993	0.0711	0.4012	0.864	0	1.4641
20	16	2.5	2.5	4.9244	0.3601	0.194	0.7325	0	1.8257
20	17	2.5	2.5	4.8139	0.3049	0.2236	0.765	0	1.8036
20	18	2.5	2.5	4.6892	0.2585	0.2542	0.796	0	1.7723
20	19	2.5	2.5	4.5619	0.2237	0.2826	0.82	0	1.7353
20	20	2.5	2.5	4.3382	0.1978	0.2994	0.838	0	1.6562
20	17	3.0	3.0	5.6759	0.6175	0.1662	0.7582	0	1.6861
20	18	3.0	3.0	4.8297	0.3533	0.2061	0.789	0	1.4921
20	19	3.0	3.0	4.6618	0.294	0.2357	0.819	0	1.4559
20	20	3.0	3.0	4.1963	0.2451	0.2414	0.841	0	1.3171
30	25	0.5	0.5	0.6598	0.013	0.2917	0.865	0	1.2937
30	26	0.5	0.5	0.6699	0.012	0.3006	0.8705	0	1.3159
30	27	0.5	0.5	0.6726	0.0114	0.3063	0.876	0	1.3225
30	28	0.5	0.5	0.6859	0.0097	0.3342	0.8823	0	1.3523
30	29	0.5	0.5	0.7032	0.0088	0.3482	0.8865	0	1.3888
30	30	0.5	0.5	0.7045	0.0076	0.379	0.8657	0	1.3939
30	25	1.5	1.5	2.2812	0.0521	0.2967	0.863	0	1.4861
30	26	1.5	1.5	2.2833	0.0458	0.3247	0.8695	0	1.4917
30	27	1.5	1.5	2.2842	0.0433	0.3389	0.876	0	1.4939
30	28	1.5	1.5	2.2375	0.0373	0.3696	0.8546	0	1.4668
30	29	1.5	1.5	2.1848	0.0323	0.4013	0.8565	0	1.4350
30	30	1.5	1.5	2.2094	0.0297	0.4262	0.8637	0	1.4531
30	25	2.0	2.0	3.0776	0.0812	0.2918	0.862	0	1.4982
30	26	2.0	2.0	3.0269	0.0774	0.3006	0.8675	0	1.4748
30	27	2.0	2.0	2.9451	0.0741	0.3061	0.874	0	1.4355
30	28	2.0	2.0	2.9214	0.0655	0.3341	0.8803	0	1.428
30	29	2.0	2.0	2.9026	0.062	0.3482	0.8865	0	1.4203
30	30	2.0	2.0	2.818	0.0533	0.379	0.8657	0	1.3824
30	20	2.5	2.5	7.0629	0.5041	0.1817	0.757	0	2.6235
30	25	2.5	2.5	3.6356	0.1169	0.2708	0.859	0	1.4075
30	28	2.5	2.5	3.4294	0.0978	0.3006	0.8783	0	1.3327
30	29	2.5	2.5	3.335	0.0925	0.3067	0.8845	0	1.297
30	30	2.5	2.5	3.2689	0.0874	0.3153	0.8897	0	1.2726
30	20	3.0	3.0	7.1131	0.7412	0.1426	0.736	0	2.1240
30	25	3.0	3.0	3.9434	0.1599	0.2291	0.851	0	1.2612
30	28	3.0	3.0	3.6986	0.1234	0.2741	0.8773	0	1.1917
30	29	3.0	3.0	3.6103	0.1156	0.2819	0.8835	0	1.1649
30	30	3.0	3.0	3.4833	0.1082	0.2864	0.8887	0	1.1250
50	25	0.5	0.5	0.5703	0.0271	0.1576	0.814	0	1.0865
50	30	0.5	0.5	0.5831	0.0174	0.2113	0.8283	0	1.1315
50	35	0.5	0.5	0.6056	0.0109	0.2808	0.8584	0	1.1894
50	40	0.5	0.5	0.6577	0.0076	0.3263	0.878	0	1.3003
50	45	0.5	0.5	0.7095	0.0054	0.3716	0.8928	0	1.4083
50	48	0.5	0.5	0.7299	0.0047	0.3886	0.8992	0	1.4505
50	49	0.5	0.5	0.676	0.0046	0.4086	0.8942	0	1.3428
50	50	0.5	0.5	0.6753	0.0045	0.411	0.897	0	1.3415
50	25	1.5	1.5	2.4992	0.0824	0.1623	0.801	0	1.6112

n	r	$\alpha$	$\lambda$	$\hat{\alpha}$	$\hat{\lambda}$	MSE	KS	p-value	TD
50	30	1.5	1.5	2.4091	0.0469	0.2385	0.803	0	1.5748
50	35	1.5	1.5	2.4484	0.034	0.3003	0.8394	0	1.6096
50	40	1.5	1.5	2.4444	0.0278	0.3448	0.865	0	1.6111
50	45	1.5	1.5	2.3936	0.024	0.3809	0.8858	0	1.5798
50	48	1.5	1.5	2.1395	0.02	0.4135	0.8903	0	1.4130
50	49	1.5	1.5	2.1163	0.0196	0.4173	0.8932	0	1.3978
50	50	1.5	1.5	2.1171	0.019	0.4259	0.896	0	1.3987
50	25	2.0	2.0	3.6099	0.1313	0.1605	0.8	0	1.7393
50	30	2.0	2.0	3.4376	0.0865	0.2114	0.8193	0	1.6755
50	35	2.0	2.0	3.3058	0.0567	0.28	0.8324	0	1.6246
50	40	2.0	2.0	3.2422	0.0464	0.3257	0.862	0	1.5979
50	45	2.0	2.0	3.1544	0.0385	0.3712	0.8848	0	1.5580
50	48	2.0	2.0	3.0438	0.0354	0.3885	0.8962	0	1.5042
50	49	2.0	2.0	2.7554	0.0299	0.4085	0.8942	0	1.3627
50	50	2.0	2.0	2.7011	0.0293	0.411	0.897	0	1.3359
50	25	2.5	2.5	4.5888	0.2022	0.1486	0.796	0	1.7546
50	30	2.5	2.5	4.341	0.1612	0.1751	0.8327	0	1.6719
50	35	2.5	2.5	3.9494	0.094	0.2391	0.8283	0	1.5422
50	40	2.5	2.5	3.8501	0.0693	0.3006	0.861	0	1.5123
50	45	2.5	2.5	3.7138	0.056	0.3483	0.8858	0	1.4631
50	48	2.5	2.5	3.5929	0.0501	0.3721	0.8982	0	1.4171
50	49	2.5	2.5	3.558	0.0484	0.3792	0.9016	0	1.4038
50	50	2.5	2.5	3.5364	0.0467	0.3885	0.905	0	1.3959
50	25	3.0	3.0	5.3057	0.306	0.1242	0.783	0	1.6666
50	30	3.0	3.0	5.0038	0.2263	0.1573	0.8277	0	1.5925
50	35	3.0	3.0	4.6253	0.1776	0.1799	0.8574	0	1.4826
50	40	3.0	3.0	4.2287	0.1156	0.231	0.867	0	1.3711
50	45	3.0	3.0	3.8873	0.0755	0.2992	0.8868	0	1.2706
50	48	3.0	3.0	3.7902	0.0654	0.3296	0.9002	0	1.2416
50	49	3.0	3.0	3.7603	0.0627	0.3386	0.9036	0	1.2325
50	50	3.0	3.0	3.7044	0.0604	0.3436	0.907	0	1.2147
75	30	0.5	0.5	0.5318	0.0296	0.1286	0.7853	0	1.0043
75	40	0.5	0.5	0.5685	0.0144	0.207	0.848	0	1.1081
75	45	0.5	0.5	0.6012	0.0108	0.2381	0.8678	0	1.1808
75	50	0.5	0.5	0.5749	0.009	0.2652	0.876	0	1.1318
75	55	0.5	0.5	0.5998	0.0071	0.2932	0.8906	0	1.1853
75	60	0.5	0.5	0.6376	0.0055	0.3268	0.9007	0	1.2642
75	65	0.5	0.5	0.6663	0.0045	0.3532	0.9092	0	1.3236
75	70	0.5	0.5	0.6766	0.0036	0.3866	0.9084	0	1.3459
75	75	0.5	0.5	0.6953	0.003	0.4194	0.9173	0	1.3846
75	35	1.5	1.5	2.566	0.0492	0.1883	0.7787	0	1.6779
75	40	1.5	1.5	2.5912	0.0401	0.2189	0.811	0	1.7007
75	45	1.5	1.5	2.5768	0.0344	0.2438	0.8351	0	1.6949
75	50	1.5	1.5	2.2767	0.0273	0.2744	0.862	0	1.4996
75	55	1.5	1.5	2.317	0.0226	0.3134	0.8796	0	1.5296
75	60	1.5	1.5	2.3212	0.0196	0.3433	0.8927	0	1.5344
75	65	1.5	1.5	2.284	0.0176	0.3671	0.9042	0	1.5110
75	70	1.5	1.5	2.2092	0.0142	0.4119	0.9074	0	1.4633
75	71	1.5	1.5	2.2084	0.014	0.4171	0.9098	0	1.4629
75	72	1.5	1.5	2.2178	0.0135	0.4274	0.9112	0	1.4696
75	73	1.5	1.5	2.2234	0.0129	0.4388	0.9146	0	1.4737
75	74	1.5	1.5	2.2087	0.0127	0.4416	0.916	0	1.4640
75	75	1.5	1.5	2.2172	0.0123	0.4524	0.9183	0	1.4699
75	30	2.0	2.0	3.8014	0.1251	0.1307	0.7673	0	1.8381



<b>n</b>	<b>r</b>	<b><math>\alpha</math></b>	<b><math>\lambda</math></b>	<b><math>\hat{\alpha}</math></b>	<b><math>\hat{\lambda}</math></b>	<b>MSE</b>	<b>KS</b>	<b>p-value</b>	<b>TD</b>
75	35	2.0	2.0	3.7049	0.0815	0.1753	0.7747	0	1.8117
75	40	2.0	2.0	3.6953	0.0671	0.2064	0.808	0	1.8141
75	45	2.0	2.0	3.6635	0.0562	0.2374	0.8331	0	1.8036
75	50	2.0	2.0	3.1355	0.0427	0.2643	0.858	0	1.5464
75	55	2.0	2.0	3.0795	0.0367	0.2925	0.8766	0	1.5214
75	60	2.0	2.0	3.0841	0.0318	0.3263	0.8907	0	1.5262
75	65	2.0	2.0	3.0258	0.0281	0.3528	0.9032	0	1.4989
75	70	2.0	2.0	2.8843	0.0236	0.3864	0.9044	0	1.4304
75	71	2.0	2.0	2.8463	0.0221	0.3991	0.9088	0	1.4121
75	72	2.0	2.0	2.8315	0.0217	0.4032	0.9112	0	1.4049
75	73	2.0	2.0	2.8344	0.021	0.4135	0.9136	0	1.4067
75	74	2.0	2.0	2.7932	0.0207	0.4149	0.916	0	1.3863
75	75	2.0	2.0	2.7811	0.0203	0.4194	0.9173	0	1.3804
75	30	2.5	2.5	4.9943	0.2435	0.1094	0.798	0	1.9003
75	35	2.5	2.5	4.7028	0.1418	0.149	0.7883	0	1.8244
75	40	2.5	2.5	4.6869	0.1058	0.1898	0.804	0	1.8324
75	45	2.5	2.5	4.6227	0.0875	0.222	0.8311	0	1.8141
75	50	2.5	2.5	4.5053	0.075	0.2494	0.851	0	1.7722
75	55	2.5	2.5	3.8337	0.0542	0.2757	0.8726	0	1.5118
75	60	2.5	2.5	3.6747	0.0469	0.299	0.8897	0	1.4512
75	65	2.5	2.5	3.5358	0.0411	0.3224	0.9022	0	1.3979
75	70	2.5	2.5	3.4135	0.0361	0.3469	0.9134	0	1.3509
75	71	2.5	2.5	3.4085	0.035	0.3551	0.9158	0	1.3494
75	72	2.5	2.5	3.3878	0.0343	0.3594	0.9172	0	1.3414
75	73	2.5	2.5	3.3581	0.0335	0.3625	0.9186	0	1.3298
75	74	2.5	2.5	3.3191	0.0328	0.3646	0.921	0	1.3145
75	75	2.5	2.5	3.3102	0.032	0.3715	0.9223	0	1.3113
75	55	3.0	3.0	4.812	0.0904	0.2455	0.8678	0	1.5739
75	60	3.0	3.0	4.1778	0.0635	0.275	0.8877	0	1.3714
75	65	3.0	3.0	3.9455	0.0556	0.2888	0.9002	0	1.2966
75	70	3.0	3.0	3.8614	0.0467	0.3263	0.9134	0	1.2716
75	71	3.0	3.0	3.7969	0.0455	0.3275	0.9158	0	1.2505
75	72	3.0	3.0	3.7328	0.0443	0.3287	0.9182	0	1.2295
75	73	3.0	3.0	3.7168	0.0431	0.3351	0.9196	0	1.2246
75	74	3.0	3.0	3.6531	0.0419	0.3362	0.922	0	1.2037
75	75	3.0	3.0	3.6359	0.0408	0.3424	0.9233	0	1.1984
100	50	0.5	0.5	0.5313	0.0134	0.1924	0.856	0	1.0357
100	60	0.5	0.5	0.578	0.0086	0.2396	0.8857	0	1.1388
100	70	0.5	0.5	0.6035	0.0059	0.2851	0.8964	0	1.1951
100	80	0.5	0.5	0.6369	0.004	0.3404	0.916	0	1.2658
100	90	0.5	0.5	0.687	0.0029	0.3812	0.9248	0	1.3682
100	95	0.5	0.5	0.7021	0.0025	0.4003	0.9289	0	1.3991
100	96	0.5	0.5	0.7094	0.0024	0.4076	0.9302	0	1.4139
100	97	0.5	0.5	0.7135	0.0023	0.4105	0.9304	0	1.4224
100	98	0.5	0.5	0.7194	0.0022	0.419	0.9326	0	1.4343
100	99	0.5	0.5	0.7258	0.0022	0.4237	0.9328	0	1.4473
100	100	0.5	0.5	0.7301	0.0021	0.4266	0.933	0	1.4559
100	50	1.5	1.5	2.3612	0.0355	0.1992	0.829	0	1.5505
100	60	1.5	1.5	2.4246	0.0255	0.2516	0.8667	0	1.5995
100	70	1.5	1.5	2.3239	0.0184	0.3035	0.8861	0	1.537
100	80	1.5	1.5	2.3442	0.0143	0.3554	0.9065	0	1.5533
100	90	1.5	1.5	2.3395	0.0117	0.403	0.9208	0	1.5519
100	95	1.5	1.5	2.3313	0.0106	0.4261	0.9269	0	1.5471
100	96	1.5	1.5	2.3377	0.0104	0.4324	0.9282	0	1.5516

n	r	$\alpha$	$\lambda$	$\hat{\alpha}$	$\hat{\lambda}$	MSE	KS	p-value	TD
100	97	1.5	1.5	2.3449	0.0102	0.4391	0.9294	0	1.5565
100	98	1.5	1.5	2.3399	0.01	0.4423	0.9296	0	1.5532
100	99	1.5	1.5	2.3368	0.0099	0.4458	0.9308	0	1.5513
100	100	1.5	1.5	2.3438	0.0097	0.4529	0.932	0	1.5561
100	50	2.0	2.0	3.3817	0.0553	0.1917	0.825	0	1.6632
100	60	2.0	2.0	3.3786	0.0414	0.2388	0.8607	0	1.6686
100	70	2.0	2.0	3.2145	0.0309	0.2843	0.8801	0	1.5918
100	80	2.0	2.0	3.1089	0.0231	0.3398	0.9045	0	1.5429
100	90	2.0	2.0	3.0363	0.0193	0.381	0.9188	0	1.5085
100	95	2.0	2.0	2.9526	0.0176	0.4001	0.9269	0	1.4675
100	96	2.0	2.0	2.9545	0.0172	0.4075	0.9282	0	1.4687
100	97	2.0	2.0	2.9418	0.017	0.4104	0.9294	0	1.4624
100	98	2.0	2.0	2.9368	0.0164	0.419	0.9316	0	1.4602
100	99	2.0	2.0	2.9331	0.0161	0.4236	0.9318	0	1.4585
100	100	2.0	2.0	2.9203	0.0159	0.4266	0.933	0	1.4522
100	70	2.5	2.5	4.0062	0.0502	0.2547	0.8854	0	1.5824
100	80	2.5	2.5	3.7065	0.0358	0.3043	0.9015	0	1.4683
100	90	2.5	2.5	3.5655	0.028	0.3548	0.9198	0	1.415
100	95	2.5	2.5	3.4872	0.0254	0.3757	0.9269	0	1.3847
100	96	2.5	2.5	3.4778	0.0249	0.3802	0.9282	0	1.3812
100	97	2.5	2.5	3.4296	0.0245	0.3809	0.9304	0	1.362
100	98	2.5	2.5	3.4171	0.0241	0.3848	0.9306	0	1.3572
100	99	2.5	2.5	3.4032	0.0237	0.3884	0.9318	0	1.3518
100	100	2.5	2.5	3.3984	0.0231	0.3949	0.933	0	1.3501
100	60	3	3	4.9519	0.0905	0.2003	0.8547	0	1.6205
100	70	3.0	3.0	4.7059	0.0691	0.2391	0.8834	0	1.5456
100	80	3.0	3.0	4.3984	0.0545	0.2712	0.906	0	1.448
100	81	3.0	3.0	4.3317	0.0533	0.2718	0.9083	0	1.4261
100	90	3.0	3.0	3.9507	0.0381	0.3182	0.9198	0	1.3042
100	95	3.0	3.0	3.8491	0.0333	0.3437	0.9279	0	1.2719
100	96	3.0	3.0	3.8332	0.0322	0.3518	0.9302	0	1.267
100	97	3.0	3.0	3.8181	0.0316	0.356	0.9314	0	1.2622
100	98	3.0	3.0	3.7998	0.031	0.3597	0.9326	0	1.2563
100	99	3.0	3.0	3.789	0.0303	0.3655	0.9338	0	1.2529
100	100	3.0	3.0	3.743	0.0297	0.3665	0.935	0	1.2378

### VII. CONCLUSION

Simulation study is carried out to compare the performance of different estimate values. The performances of the parameters of the Exponentiated Inverted Weibull (EIW) Distribution a detailed study on the statistical properties is presented. The theory the MLE of Exponentiated Inverted Weibull Distribution for complete and censored data has been provided.

A simulation study is implemented for investigating the accuracy of different estimates for different sample sizes for complete and censored values into MLE method. This study some of structural properties of the Type-II Censored statistics with MLE. The MLE method is employed for estimating the model parameters different order values. I hope that the proposed model will attract wider application in areas such as engineering, survival and medical data, economics, among other.

### VIII. ACKNOWLEDGEMENTS

The author would like to thanks Flaih et. al. (2012) proposed distribution Exponentiated Inverted Weibull Distribution is more flexible distribution to facilitate better modeling of lifetime data and his study.

## REFERENCES

- [1] Abernathy, R. B. (2004). The New Weibull Handbook, 4th Edition, Dept. AT Houston, Texas 77252-2608, USA.
- [2] Aleem, M and Pasha G.R (2003). Ratio, product and single Moments of Order Statistics from Inverse Weibull Distribution. J.Stat. Vol. 10, (1) PP(1-7).
- [3] Alizadeh, M., Ghosh, I., Yousof, H. M., Rasekhi, M., and Hamedani, G. G. (2017). The generalized odd generalized exponential family of distributions: Properties, characterizations and application. Journal of Data Science, 15(3), 443-465.
- [4] B.C. Arnold, N. Balakrishnan and H.N. Nagaraja, (1992). A First Course in Order Statistics, Wiley, New York.
- [5] Balakrishnan, N., Kocherlakota, (1986). On the moments of order statistics from the doubly truncated logistic distribution, Journal of Statistical Planning and Inference, 13, pp.117-129.
- [6] Barry C. Arnold, N. Balakrishnan and H.N. Nagaraja, (1992), A First Course in Order Statistics, Wiley & Sons, Inc.,
- [7] Calabria, R., and Pulcini, G. (1990). On the maximum likelihood and least squares estimation in the inverse Weibull distribution, Statistica Applicata, 2, 53-66.
- [8] David H.A., Nagaraja H. N., (2003). Order Statistics, Third Edition, JohnWiley, New York.
- [9] de Gusmão, F. R. S., Ortega, E. M. M., and Cordeiro, G. M. (2011). The generalized inverse Weibull distribution. Statistical Papers, 52, 591-619.
- [10] Elbatal, I., and Muhammed, H. Z. (2014). Exponentiated generalized inverse Weibull distribution. Applied Mathematical Sciences, 8, 3997-4012.
- [11] Flaih, A., Elsalloukha, A., Mendi. E., and Milanova. M., (2012). The Exponentiated Inverted Weibull Distribution. Applied Mathematics & Information Sciences, 6, No. 2 pp.167-171
- [12] G. S. Mudholkar and A. D. Hutson, (1996), Exponentiated Weibull family: some properties and flood data application, Commun. Statist.-Theory Meth. 25, 3050-3083.
- [13] G.S.Mudholka, D.K.Srivastava and M. Freimer, (1995), The exponentiated Weibull family: a reanalysis of the bus-motorfailure data, Technometrics 37, 436-445.
- [14] Gupta, R. D., and Kundu, D. (1999). Generalized exponential distribution. Australian and New Zealand Journal of Statistics, 41 (2), 173-188.
- [15] Khan, M. S., and King, R. (2016). New generalized inverse Weibull distribution for lifetime modeling. Communications for Statistical Applications and Methods, 23(2), 147-161
- [16] Lieblein, J., (1955). On moments of order statistics from Weibull distribution. Annals of Mathematical Statistics, 24, pp. 330-333.
- [17] M. Pal, M. M. Ali and J. Woo, (2006), Exponentiated Weibull distribution, Statistica, 66, 2, 139-147.
- [18] M. S. Khan, G. R. Pasha and A. H. Pasha, (2008), Theoretical analysis of inverse weibull distribution, WSEAS Transactions on Mathematics, 7, 2.
- [19] M. Z. Raqab, (2002), Inferences for generalized exponential distribution based on record statistics, Journal of Statistical Planning and Inference, vol. 104, pp. 339-350.
- [20] Pararai, M., Warahena-Liyanaage, G., and Oluyede, B.O. (2014). A new class of generalized inverse Weibull distribution with applications. Journal of Applied Mathematics & Bioinformatics, 4(2), 17-35.
- [21] Seunghyung Lee, Yunhwan Noh, Younshik Chung, (2017), Inverted exponentiated weibull distribution with applications to lifetime data, Vol, 24, No. 3, pp. 227-240.



10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)