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Vibration Analysis of Individual Leaf of Leaf Spring for Two Different Materials Having Different End Conditions

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Abstract: This paper deals with the vibration analysis of Leaf Spring in which the comparison is done for two different materials and different end conditions. The materials used are both isotropic in nature. The material properties are mentioned in the paper during analysis. The spring is analysed with different L/R ratios using higher order shear deformation theory. The end conditions used for analysis are simply supported, clamped clamped and one end fixed other end free condition. The dependence of support conditions and radius of curvature on the frequency response is highlighted. The convergence study has been done along with comparison to validate the present formulation. The result and the analysis of the frequency of vibration can be used to optimize the frequency of leaf spring to have better ride.

It was observed during analysis that as radius of curvature of leaf spring is decreased keeping length constant the value of frequency of vibration increases. It was also observed that as the boundary conditions are changed from clamped- clamped to simply supported the frequency also decreases for same Length/Radius of curvature ratio.

Keywords: Frequency of vibration, Radius of curvature, End conditions, Isotropic beam, Length/Radius of curvature ratio,

I. INTRODUCTION

Spring is very important element of an automobile as it absorbs the vehicle vibration and give the rider a comfortable ride. Leaf spring is one of those springs which are mainly used for heavy vehicles.

The advantage of using a leaf spring is that it can be guided along a definite path as it deflects to acts as structural member in addition to energy absorbing device. Lots of efforts have been made by different people to reduce the vibration of leaf spring so that the ride could be made more comfortable. In this paper the analysis of master leaf was modeled as a curved beam and vibration was studied for different materials and for different L/R ratios with different boundary conditions.

Mahmood M. et. al. [1] did the analysis of composite leaf spring using ANSYS V5.4 software and tried to optimize the weight of the spring. Abdul Rahim Abu Talib et.al [2] tried to analyse the elliptic spring made for both light and heavy trucks .They tried to optimize the spring parameters for different ellipticity ratios.Vinkel Arora et al [3] did the fatigue life assessment of leaf spring of 65Si7 using analytical and graphical methods.They also did the life assessment using SAE design manual approach and it was compared with experimental results. Mesut Simsek et al [4] studied the vibration of straight beam within the frame work of third order shear deformation theory. They calculated the six dimensionless frequency parameters of different beams having different h/L ratios for different boundary conditions.

Ankit Gupta & Mohammad Talha [5] introduced the geometrically nonlinear vibrations response of FGM plates. They proposed displacement based new hyperbolic higher-order shear and normal deformation theory (HHSNDT). The performance of a curved beam with coupled polynomial distributions was investigated by P. Raveendranath et al [6]. The result shows excellent convergence of natural frequencies even for very thin deep arches and higher vibrational modes.

M. Kawakami [7] presented an approximate method to study the analysis for both the in-plane and out-of-plane free vibration of horizontally curved beams with arbitrary shapes and variable cross-sections.

S.M. Ibrahim et al [8] investigated large amplitude periodic forced vibration of curved beams under periodic excitation using a three-noded beam element.

Mohammad Amir and Mohammad Talha [9] analyzed the thermo-elastic vibration of shear deformable functionally graded material (FGM) curved beams with micro structural defects by the finite element method.

The material properties of FGM beams are allowed to vary continuously in the thickness direction by a simple power-law distribution in terms of the volume fractions of the constituents. Mohammad Amir and Mohammad Talha [10] studied the imperfection sensitivity in the vibration behavior of functionally graded arches with micro structural defects (porosity).
Vaibhav Ghodge [11] did the modal analysis of a cantilever beam and simply supported beam using ANSYS for Aluminium alloy, Gray cast iron, Structural steel and Copper alloy.
Yiming Fu [12] studied nonlinear free vibration and dynamic stability for the piezoelectric functionally graded beams, subjected to one-dimensional steady heat conduction in the thickness direction. J. Yuan and S. M. Dickinson [13] used Rayleigh-Ritz method for the solution of the free vibration problem of systems comprised of straight and/or curved beam components. Eigen value problems of Timoshenko and shear-deformable curved beams were analyzed by S.Y. Yang &H.C. Sin [14] using the elements with six degrees of freedom. The results of the Eigen-analysis show that the curvature-based beam elements are free of locking and are efficient. Mehdi Hajianmaleki & Mohammad S. Qatu [15] on the basis of exhaustive study on the vibration analysis of composite thin and thick beams explained a simple classic and shear deformation model that can be used for beams with any laminate. P. Chidambaram and A.W. Leissa [16] did a study on the Vibrations of planar curved beams, rings, and arches and special attention was given to the effects of initial static loading, nonlinear vibrations and the application of finite element techniques.

II. MATHEMATICAL FORMULATION

A. Displacement Field

The displacement field is expressed in terms of axial and transverse displacements of the mid-plane, along with the curvilinear coordinates x and z direction. The displacement field is based on traction-free conditions at the inner and outer surface of the curved beam[14] is written as,

$$u(x,z,t) = u_0(x,t) + z\emptyset_x(x,t) - \frac{4z^3}{h^2}\left[\emptyset_x(x,t) + \frac{\partial w_0}{\partial x}(x,t)\right]$$
$$w(x,z,t) = w_0(x,t)$$

$let\ \frac{\partial w_0}{\partial x} = \theta_x$

$$u(x,z,t) = u_0(x,t) + z\emptyset_x(x,t) - \frac{4z^3}{3*h^2}[\emptyset_x(x,t) + \theta_x(x,t)]$$
$$u(x,z,t) = u_0(x,t) + \emptyset_x\left(z - \frac{4z^3}{3h^2}\right) - \frac{4z^3}{3h^2}\theta_x(x,t)$$
$$w(x,z,t) = w_0(x,t)$$

Where, u and w represents the displacements of a point along the (x, z) co-ordinates. u_0, $w_{0,}\emptyset_x$, θ_x are four unknown displacement functions of mid-plane.

$$\begin{bmatrix} u(x,z,t) \\ w(x,z,t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \left(z - \frac{4z^3}{3h^2}\right) & -\frac{4z^3}{3h^2} \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_0 \\ w_0 \\ \emptyset_x \\ \theta_x \end{bmatrix}$$

The basic field variables can be symbolized mathematically as, $\{\boldsymbol{\lambda o}\} = \{u_0, w_{0,}\emptyset_x, \theta_x\}^T$ $\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \left(z - \frac{4z^3}{3h^2}\right) & -\frac{4z^3}{3h^2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$

$$[\boldsymbol{\lambda} o]$$

The displacement field in the compressed form can be written as

$$[\mathbf{U}] = [\check{N}]\,[\boldsymbol{\lambda} o]$$

Where, $[\mathbf{U}] = \begin{bmatrix} u \\ w \end{bmatrix}$ ***and*** $[\check{N}] = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \left(z - \frac{4z^3}{3h^2}\right) & -\frac{4z^3}{3h^2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$

B. Strain-Displacement Relations

$$\varepsilon_{xx} = 1/(1+\frac{z}{R})(\frac{\partial u}{\partial x}+\frac{w}{R})$$

Using linear strain displacement relations, the strain vectors may be defined as

$$\varepsilon_{xx} = 1/(1+\frac{z}{R})\{\frac{\partial u_0}{\partial x}+ z\frac{\partial \emptyset_x}{\partial x}-\frac{4z^3}{h^2}\left[\frac{\partial \emptyset_x}{\partial x}+\frac{\partial \theta_x}{\partial x}\right]+\frac{w_0}{R}\}$$

$$\Upsilon_{xz} = \frac{\partial u}{\partial z}+1/(1+\frac{z}{R})(\frac{\partial w}{\partial x}-\frac{u}{R})$$

$$= (\frac{\partial u_0}{\partial z}+\emptyset_x-\frac{4z^2}{h^2}[\emptyset_x+\theta_x])+1/(1+\frac{z}{R})\{\frac{\partial w_0}{\partial x}-\frac{u_0}{R}-\frac{z}{R}(\emptyset_x)-\frac{4z^3}{3h^2}[\emptyset_x+\theta_x]\}$$

As u_0 *is a function of* x *and* t *only . ie* $\frac{\partial u_0}{\partial z} = 0.$

$$\Upsilon_{xz} = 1/(1+\frac{z}{R})\{(1+\frac{z}{R})(\emptyset_x-\frac{4z^2}{h^2}[\emptyset_x+\theta_x])+\frac{\partial w_0}{\partial x}-\frac{u_0}{R}-\frac{z}{R}(\emptyset_x)-\frac{4z^3}{3*R*h^2}[\emptyset_x+\theta_x]$$

$$\Upsilon_{xz} = 1/(1+\frac{z}{R})\{(\emptyset_x+\frac{\partial w_0}{\partial x}-\frac{u_0}{R})-\frac{4z^2}{h^2}[\emptyset_x+\theta_x]-\frac{8z^3}{3*R*h^2}[\emptyset_x+\theta_x]\}$$

$$\begin{bmatrix}\varepsilon_{xx}\\ \Upsilon_{xz}\end{bmatrix} = 1/(1+\frac{z}{R})\left(\begin{bmatrix}\frac{\partial u_0}{\partial x}+\frac{w_0}{R}\\ \emptyset_x+\frac{\partial w_0}{\partial x}-\frac{u_0}{R}\end{bmatrix}+z\begin{bmatrix}\frac{\partial \emptyset_x}{\partial x}\\ 0\end{bmatrix}+z^2\begin{bmatrix}0\\ -\frac{4}{h^2}[\emptyset_x+\theta_x]\end{bmatrix}+z^3\begin{bmatrix}-\frac{4}{3h^2}\left(\frac{\partial \emptyset_x}{\partial x}+\frac{\partial \theta_x}{\partial x}\right)\\ -\frac{8}{3R*h^2}[\emptyset_x+\theta_x]\end{bmatrix}\right)$$

$$\begin{bmatrix}\varepsilon_{xx}\\ \Upsilon_{xz}\end{bmatrix} = (1/(1+\frac{z}{R}))(\begin{bmatrix}\epsilon^{\circ}{}_1\\ \epsilon^{\circ}{}_2\end{bmatrix}+z\begin{bmatrix}k_1^1\\ 0\end{bmatrix}+z^2\begin{bmatrix}0\\ k_2^2\end{bmatrix}+z^3\begin{bmatrix}k_1^3\\ k_2^3\end{bmatrix})$$

$$\begin{bmatrix}\varepsilon_{xx}\\ \Upsilon_{xz}\end{bmatrix} = 1/(1+\frac{z}{R})\begin{bmatrix}\mathbf{1} & \mathbf{0} & \boldsymbol{z} & \mathbf{0} & \mathbf{0} & z^3 & \mathbf{0}\\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \boldsymbol{z} & z^2 & \mathbf{0} & z^3\end{bmatrix}\begin{bmatrix}\epsilon^{\circ}{}_1\\ \epsilon^{\circ}{}_2\\ k_1^1\\ k_2^2\\ k_1^3\\ k_2^3\end{bmatrix}$$

$$\textbf{[T]}= 1/(1+\frac{z}{R})\begin{bmatrix}\mathbf{1} & \mathbf{0} & \boldsymbol{z} & \mathbf{0} & \mathbf{0} & z^3 & \mathbf{0}\\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \boldsymbol{z} & z^2 & \mathbf{0} & z^3\end{bmatrix}$$

where, $\epsilon^{\circ}{}_1 = \frac{\partial u_0}{\partial x}+w_0/R$

$\epsilon^{\circ}{}_2 = \emptyset_x+\frac{\partial w_0}{\partial x}-\frac{u_0}{R}$

$k_1^1 = \frac{\partial \emptyset_x}{\partial x}$

$k_2^2 = -\frac{4}{h^2}[\emptyset_x+\theta_x]$

$k_1^3 = -\frac{4}{3h^2}\left(\frac{\partial \emptyset_x}{\partial x}+\frac{\partial \theta_x}{\partial x}\right)$

$k_2^3 = -\frac{8}{3R*h^2}[\emptyset_x+\theta_x]$

$$\begin{bmatrix}\epsilon^{\circ}{}_1\\ \epsilon^{\circ}{}_2\\ k_1^1\\ k_2^2\\ k_1^3\\ k_2^3\end{bmatrix} = \begin{bmatrix}\frac{\partial}{\partial x} & \frac{\mathbf{1}}{\boldsymbol{R}} & \mathbf{0} & \mathbf{0}\\ -\frac{\mathbf{1}}{\boldsymbol{R}} & \frac{\partial}{\partial x} & \mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \frac{\partial}{\partial x} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & -\frac{4}{h^2} & -\frac{4}{h^2}\\ \mathbf{0} & \mathbf{0} & \frac{-4}{3h^2} & \frac{-4}{3h^2}\\ \mathbf{0} & \mathbf{0} & \frac{-8}{3*R*h^2} & \frac{-8}{3*R*h^2}\end{bmatrix}\begin{bmatrix}u_0\\ w_0\\ \emptyset_x\\ \theta_x\end{bmatrix}$$

$$\{\epsilon^{\circ}\} = [\boldsymbol{B}][\boldsymbol{\lambda} o]$$

Where, B= $\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\mathbf{1}}{\boldsymbol{R}} & \mathbf{0} & \mathbf{0} \\ -\frac{\mathbf{1}}{\boldsymbol{R}} & \frac{\partial}{\partial x} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{\partial}{\partial x} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\frac{4}{h^2} & -\frac{4}{h^2} \\ \mathbf{0} & \mathbf{0} & \frac{-4}{3h^2} & \frac{-4}{3h^2} \\ \mathbf{0} & \mathbf{0} & \frac{-8}{3*R*h^2} & \frac{-8}{3*R*h^2} \end{bmatrix}$ ***and*** $\boldsymbol{\lambda} o = \begin{bmatrix} u_0 \\ w_0 \\ \emptyset_x \\ \theta_x \end{bmatrix}$

The linear constitutive relations of an isotropic beam is given as:

$$\begin{Bmatrix} \boldsymbol{\sigma}_{xx} \\ \boldsymbol{\tau}_{xz} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{Q}_{11} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{Q}_{44} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_{xx} \\ \boldsymbol{\gamma}_{xz} \end{Bmatrix}$$

Where Q_{11}=E/(1-mu^2)and Q_{44} =E/(2*(1+mu)) .

$$\begin{Bmatrix} \boldsymbol{\sigma}_{xx} \\ \boldsymbol{\tau}_{xz} \end{Bmatrix} = \boldsymbol{QT\varepsilon}^{\circ}$$

III. FINITE ELEMENT FORMULATION

A. *Formulation*

A two noded element with four degrees of freedom per node is being utilized to discretize the beam geometry. The generalized displacement vector of the model is expressed by

$$\begin{bmatrix} u_0 \\ w_0 \\ \emptyset_x \\ \theta_x \end{bmatrix} = \begin{bmatrix} N_1 & N_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_1 & N_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_1 & N_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & N_1 & N_2 \end{bmatrix} \begin{bmatrix} u_0^1 \\ u_0^2 \\ w_0^1 \\ w_0^2 \\ \phi_x^1 \\ \phi_x^2 \\ \theta_x^1 \\ \theta_x^2 \end{bmatrix}$$

The shape functions N_1 and N_2 are as follows

$N_1 =$ 1- x / Le

$N_2 = x / Le$

B. *Strain Energy of the Curved Beam*

$$\Pi^e = \frac{1}{2} \int \varepsilon_i^T \sigma_i dv$$

$$= 1/2 \int \varepsilon^{\circ T} T^T QT\varepsilon^{\circ} dv$$

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C. Kinetic Energy of Beam

$$\Delta = 1/2\int \rho U^T U dV$$

$$\Delta = 1/2\int \rho\, \lambda_0^T \check{N}^T \check{N} \lambda_0 dV$$

D. Governing Equation

Free vibration analysis of curved beam has been obtained using the variational principle, which is the abstraction of the principle of virtual displacement.

$$\frac{\partial E}{\partial x} = 0 = \int \varepsilon^T \sigma dv + \int \rho U^T\, UdV$$

$$= b\int_0^L \int_{-h/2}^{h/2} \varepsilon^{\circ T} T^T Q T \varepsilon^{\circ} dz\, dx + b\int_0^L \int_{-h/2}^{h/2} \rho \lambda_0^T \check{N}^T \check{N} \lambda_0 dz\, dx$$

$$= b\int_0^L \varepsilon^{\circ T}[D]\varepsilon^{\circ}\, dx + b\int_0^L \lambda_0^T[m]\lambda_0 dx$$

Where, D= $\int_{-h/2}^{h/2} T^T Q T dz$ and m= $\int_{-h/2}^{h/2} \check{N}^T \check{N} dz$

$$= b\int_0^L \lambda_0^T B^T\, DB\lambda_0 dx + b\int_0^L \lambda_0^T[m]\lambda_0 dx \;\; X$$

$$= K\,U + M\ddot{U}$$

$$KU = \lambda\, MU, \text{ where } \lambda = \omega^2$$

Where λ *is the eigen value and* U *is the global displacement vector*.

IV. RESULT AND DISCUSSION

The frequency of vibration of curved isotropic beam was found for two different support conditions using MATLAB program. One is clamped- clamped and the other is simply supported. The convergence table is also included in the paper. The specification of the beam used was as follows:

The frequency of vibration of curved isotropic beam was found for two different support conditions using MATLAB program. One is clamped- clamped and the other is simply supported. The convergence table is also included in the paper. The specification of the beam used was as follows:

A. Material Properties (M1)

Young's Modulus E = 71.72 GPa, Density ρ = 2800 kg/m3, Poisson's ratio = 0.33

B. Material Properties (M2)

Manganese Silicon Steel, Young's Modulus E = 210 GPa ,
Density ρ = 7860 kg/m^3 , Poisson's ratio = 0.3 and Yield stress = 1680 N/mm^2 .

1) Case1.Depth h=0.002m,
width b=0.02m;
Length a=0.5 m,
Radius of curvature R=2, 4, 8m;
Support condition: Clamped-Clamped
Material M1

2) Case2.Depth h=0.002m,
Width b=0.02m;
Length a=0.5 m,
Radius of curvature R=2, 4,8m;
Support condition: Simply –Supported
Material M1

3) Case3.Depth h=0.002m,
Width b=0.02m;
Length a=0.5 m,
Radius of curvature R=2, 4,8m;
Support condition: One end fixed and other end free
Material M1

4) Case4: Depth h=0.002m,
Width b=0.02m;
Length a=0.5 m,
Radius of curvature R=2, 4,8m;
Support condition: One end fixed and other end free
Material M1 & M2

a) Case 1

Table 1. Convergence Table for Clamped-Clamped Condition

No. Of Elements	L/R=0.25	L/R=0.125	L/R=.0625
50	2.34E+03	2.34E+03	1015
100	1.99E+03	1.34E+03	737
200	1582	1108	648
400	1394	1067	622
600	1354	1056	617
800	1339	1052	616
900	1335	1049	615

Table2: Validation table for Clamped- Clamped condition of Leaf spring

Validation Table for Clamped -Clamped condition				
Boundary-Condition	L/R ratio	Calculated value	Reference Value (S.M Ibrahim et.al)	Variation %
Clamped-Clamped	0.25	1335	1247	7.056937
Clamped-Clamped	0.125	1049	988	6.174089
Clamped-Clamped	0.0625	615	579	6.217617

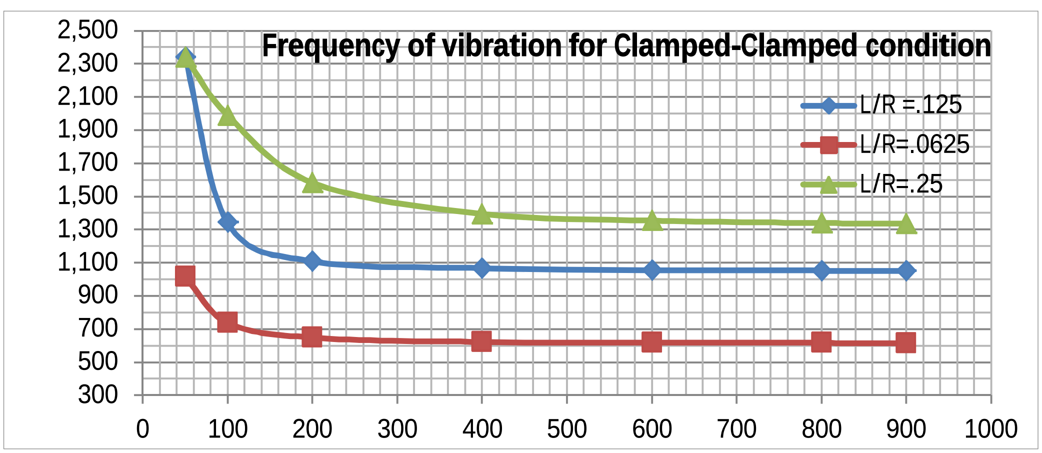

Fig.1. (Convergence graph) Vibration for Clamped-Camped Leaf Spring (X axis = no of elements, Y= frequency of vibration)

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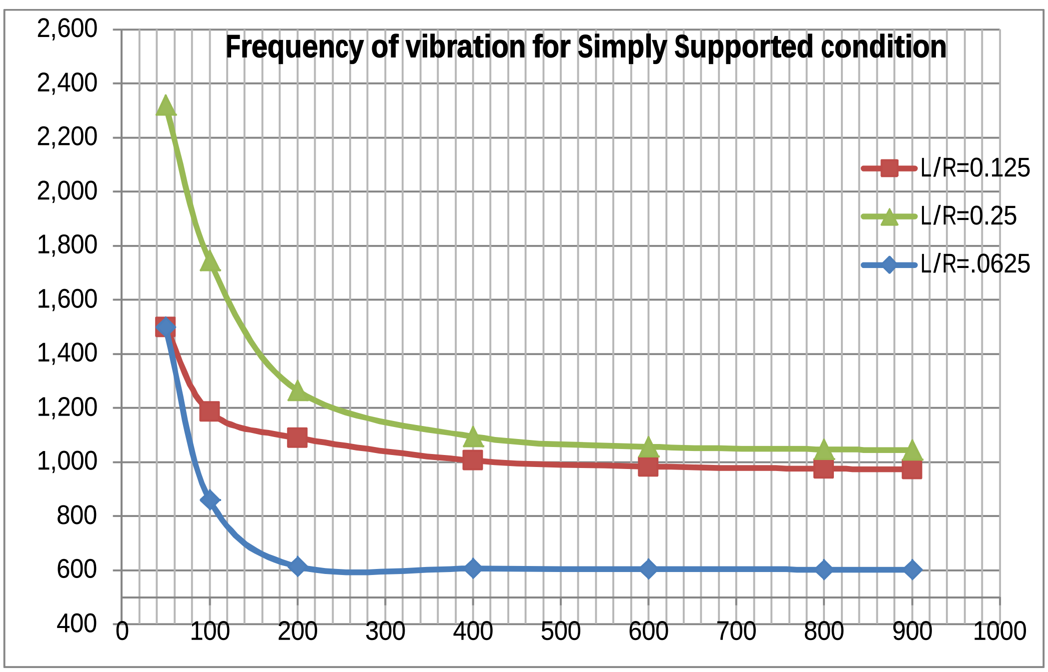
b) Case2

Table 3. Convergence Table for Simply-Supported Condition

No. Of Elements	L/R=0.25	L/R=0.125	L/R=.0625
50	2.32E+03	1.50E+03	1498
100	1745	1185	860
200	1264	1089	613
400	1094	1005	605.2
600	1056	983	603
800	1047	975	602.4
900	1043	973	602

Table 4. Validation Table for Simply Supported condition

Validation Table for Simply Supported condition				
Boundary Condition	L/R ratio	Calculated value	Reference Value (S.M Ibrahim et.al)	Variation %
Simply Supported	0.25	1043	973	7.194245
Simply Supported	0.125	973	910	6.923077
Simply Supported	0.0625	602	567	6.17284



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c) Case 3

Table 5. Convergence Table for one end fixed and one end free Condition

No. Of Elements	L/R=0.25	L/R=0.125	L/R=0.062
100	479	479.7	479.8
200	335.2	336.3	336
400	288.3	289.6	290
600	278.7	280.1	280.5
800	275.3	276.7	277.1
900	274	275.8	276.2

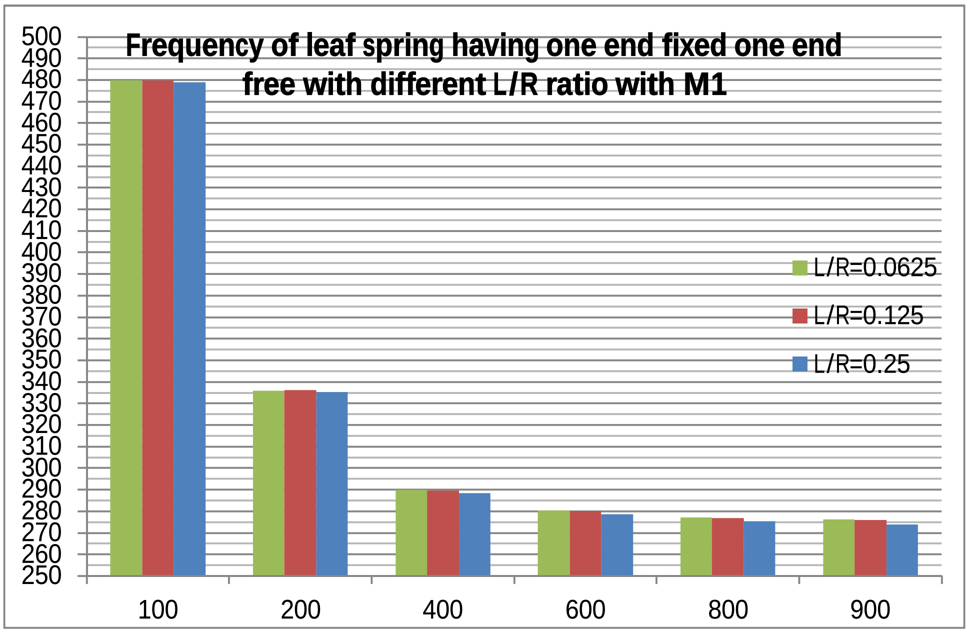

Fig.3. Column chart for the vibration of Leaf spring having one end fixed and other end free for different L/R ratios (X axis = no of elements, Y= frequency of vibration)

d) Case 4

Table 6. Convergence Table for one end fixed and one end free Condition for Manganese silicon steel used for leaf spring

No. Of Elements	L/R=0.25	L/R=0.125	L/R=0.0625
50	861	861.2	861.2
100	491.3	492	492.2
200	341.4	342.5	342
400	292	293.5	293.8
600	282.1	283.5	283.8
800	278.4	279.9	280.3
900	277.5	278.9	279.3

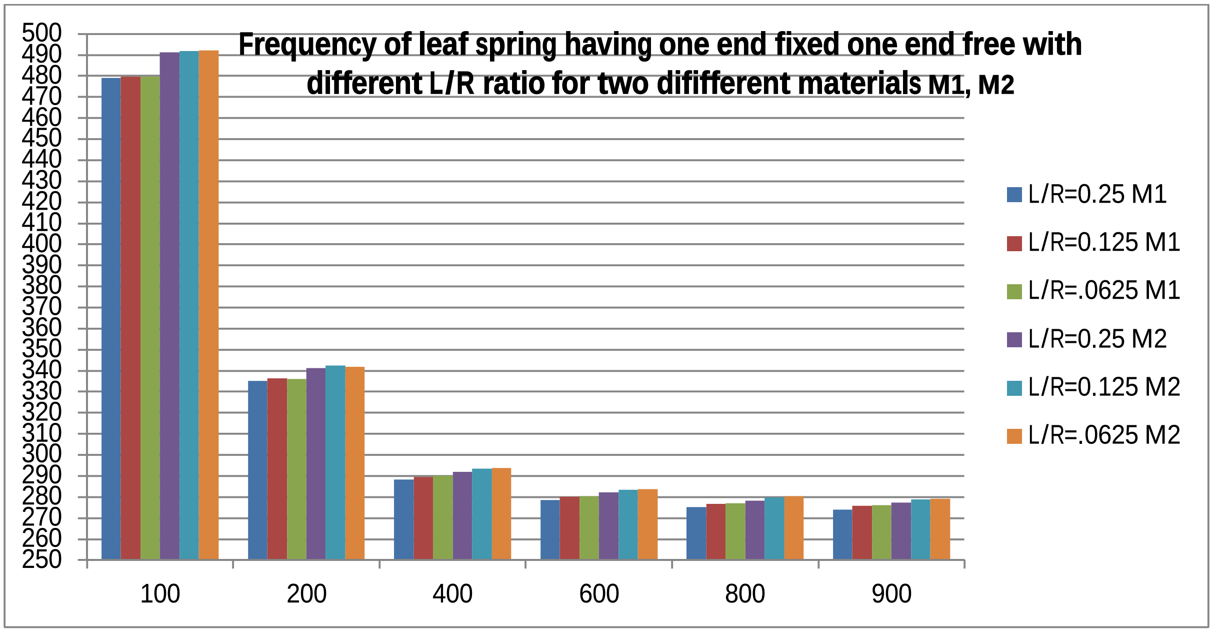

Fig.4. Column chart for the vibration of Leaf spring having one end fixed and other end free for different L/R ratios and for material M1 and M2.
(X axis = no of elements, Y= frequency of vibration)

V. CONCLUSION

The vibration analysis of the leaf spring was done as it is very important aspect of spring design. MATLAB was used for the analysis. The effect of end condition on the vibration of leaf spring was studied for L/ R =.0625, 0.125 and 0.25. The result was also validated with the S.M Ibrahim et.al paper and was found to have around 6 to 7 percent variation which is quite acceptable. During the analysis it was found that as the L/R ratio is increased the value of frequency of vibration in both the clamped-clamped condition and simply supported condition increases but for clamped-clamped condition the frequency increased much rapidly compared to simply supported condition. As the leaf spring boundary condition is more similar to one end fixed and other end free, we tried to analyse the situation for different L/R ratios and also for a material suitable for leaf spring was taken. It was observed that the frequency of vibration marginally increases if the value of R is increased for both the materials i.e. Material 1 and Material 2 when cantilever condition was considered.

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