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Some Properties of External E-Openand E-Closed Sets in Topological Spaces

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Abstract: In this work, some conditions for e-disconnectedness of a topological space in terms of maximal and minimal e-open sets and also some similarresults in terms of maximal and minimal e-closed sets along with interrelationships between themare investigated. Generally, we find that if a space has a setwhich is both maximal and minimal e-open, then either this set is the onlynontrivial e-open set in the space or the space is e-disconnected. We alsoobtain a result concerning a minimal e-open set on a subspace.
Keywords and Phrases: Maximal e-open set, minimal e-open set, maximal e-closed set, minimal e-closed set, e-disconnected.

I. INTRODUCTION AND PRELIMINERIES

E. Ekici [4] introduced e-open (resp. e-closed) sets in general topology. Nakaoka and Oda [1] introduced and studied the concept of minimalopen sets in a topological space. Dualizing the concept of minimal open sets, Nakaoka and Oda [2] introduced and studied the idea of maximal open sets (Definition 2.2). Thereafter, as consequences of maximal and minimal open sets, Nakaoka and Oda [3] introduced and studied notions of maximal and minimal closed sets. Nakaoka and Oda [3], also obtained some interrelationsamong four concepts: maximal open sets, minimal open sets, maximal closedsets, minimal closed sets. In this paper, along with some other properties, westudy a topological space in which a set may have double nature such as a set which is both maximal e-open and minimal e-open e.g. Theorem 2.9 and we show that the space may be e-disconnected.

For a subset A of a topological space (X,τ), e-Cl(A) denotes the e-closure of A with respect to the topological space (X,τ). Sometimes the topological space(X,τ) is simply denoted by X. By a proper open set of a topological space X,we mean an open set G $\neq\varphi$,X and by a proper closed set, we mean a closed set E $\neq\varphi$,X. For a topological space (X,τ) and A $\subset$X, we write $(A,\tau_{/A})$ todenote the subspace on A of (X,τ).

II. MAXIMAL AND MINIMAL E-OPEN SETS

In this paper, we obtain some results on e-disconnectedness of a topological space and hence we recall the following definition of e-disconnectedness and some more results.

A. Definition 2.1. A topological space X is e-disconnected if there exist disjoint nonemptye-open sets G and H such that G $\cup$H = X.

B. Definition 2.2. A proper nonempty open subset U ofX is said to be a maximal e-open set if any e-open set which contains U is X orU.

*C. Theorem 2.3.*In any topological space (X,τ), if A be a maximal e-open set and B be ae-open set of (X,τ). Then either A $\cup$B = X or B $\subset$A.Proof.LetA be a maximal e-open set and B be ae-open setof(X,τ). If A$\cup$B = X, then we are done. But if A$\cup$B $\neq$X, then we have to prove that B $\subset$A.NowA $\cup$ B $\neq$Xmeans B $\subset$A $\cup$ B and A $\subset$A$\cup$ B. Therefore, we have, A $\subset$A$\cup$ B and A is maximal e-open, then by definition,A$\cup$ B = X or A $\cup$ B = A but A $\cup$ B $\neq$X, then A $\cup$ B = A which implies B $\subset$A.

Definition 2.4. A nonempty open subset U of X issaid to be a minimal e-open set if any e-open set which is contained in U is U orφ.

Theorem 2.5. If U is a minimal open set and W isan open set, then either U $\cap$W =φor U $\subset$W.

Proof**:** Easy and follows from definition.

Theorem 2.6**.**In any topological space (X,τ),

Let A be a maximal e-open set of (X,τ) and x an element of X\A. Then for any e-open set B containing x,X\A $\subset$B.

Let A be a maximal e-open setof (X,τ). Then, either of the following (i) and (ii) holds.

For each x$\in$X\A and each e-open set B containing x, B = X.

*1)*There exists ae-open set B such that X\A $\subset$ B and B $\subset$ X.

*2)*Let A be a maximal e-open setof (X,τ). Then, either of the following (i) and (ii) holds.

*3)*For each x$\in$ X\A and each e-open set B containing x, we have X\A $\subset$ B.

4)There exists ae-open set B such that $X\backslash A = B \neq X$.

Proof.(1) Since $x \in X\backslash A$, we have $B \not\subset A$ for any e-open set B containing x.Then,$A \cup B = X$ by Theorem 2.3. Therefore,$X\backslash A \subset B$.
(2) If (i) does not hold, then there exists an element x of X\A and ae-opensetB containing x such that $B \subset X$.By (1), we have,$X\backslash A \subset B$.(3) If (ii) does not hold, then, by (1), we have $X\backslash A \subset B$ for each $x \in X\backslash A$and each e-open set B containing x. Hence, we have $X\backslash A \subset B$.

D. Theorem 2.7. If G is a maximal e-open set and H is a minimal e-open set of atopological space X, then either $H \subset G$ or the space is disconnected.

Proof. Using the e-maximality of G by Theorem 2.3, we get either $G \cup H = X$ or$H \subset G$. Using the e-minimality of H by Theorem 2.5, we get $G \cap H = \varphi$or $H \subset G$.The case $G \cup H = X, H \subset G$ implies $G = X$ and the case $H \subset G, G \cap H = \ldots$implies $H = \varphi$. So only feasible cases are $G \cup H = X, G \cap H = \varphi$and $H \subset G$.If $G \cup H = X, G \cap H = \varphi$, then the space is e-disconnected.

Remark 2.8. $G \cup H = X, G \cap H = \varphi$imply $G = X - H$. In Theorem 2.7, if$H \not\subset G$, then G and H both are also e-closed. Theorem 2.7 may be stated asfollows: If G is a maximal e-open set and H is a minimal e-open set of a topologicalspace X, then either $H \subset G$ or $G = X - H$.

Theorem 2.9. If a topological space X has a set which is both maximal andminimale-open sets, then either this set is the only nontrivial e-open set in the spaceor the space is e-disconnected.

Proof. Let G be both maximal and minimal e-open, and H be any e-open set.Then we get $G \subset G \cup H$. By the e-maximality of G, we have the following twocases.

Case I: $G = G \cup H$. Then we get $H \subset G$. Since G is minimale-open set, we have, $H = \varphi$or $H = G$.

Case II: $G \cup H = X$. Considering G as a minimal e-open set, we get byTheorem 2.5, $G \cap H = \varphi$or $G \subset H$. Since G is maximale-open, $G \subset H$ implies$G = H$ or $H = X$.

Considering all cases, we get $G = H$ or $G \cup H = X$ and $G \cap H = \varphi$. If$G \cup H = X$ and $G \cap H = \varphi$, then the space is e-disconnected.

Remark 2.10:It is trivial that if a topological space X has only one proper e-open set, thenthat set is both maximal and minimal e-open. If there are only two proper e-opensets in a space and the e-open sets are disjoint, then both are maximal andminimale-open sets. If G and H are only two proper e-open sets in a topological space suchthat $G \subset H$, then G is a minimal e-open set and H is a maximal e-open set in thespace. However, there may not exist a set which is both maximal and minimal e-open in ae-disconnected space which can be seen fromExample 2.13.

Corollary 2.11. If G is both maximal and minimal e-open, and E is ae-closed setin a topological space X, then either $G = X - E$ or $G = E$.

Proof. Given G is both maximal and minimal e-open, and E is ae-closed set.So $X - E$ is an e-open set. Proceeding like the proof of Theorem 2.9, we get, $G = X - E$ or $G \cup (X - E) = X$ and $G \cap (X - E) = \varphi$. $G \cup (X - E) = X$ and$G \cap (X - E) = \varphi$imply $G = E$.

Corollary 2.12. If G is both maximal and minimal e-open in a topological spaceX, then either G is the only proper e-open set in the space or proper e-open setsof the space are G and $X - G$ only.

Proof. Let H be any proper e-open set of the space. Proceeding like the proofof Theorem 2.8, we get $G = H$ or $G \cup H = X$ and $G \cap H = \varphi$, $G \cup H = X$ and$G \cap H = \varphi$imply $H = X - G$.

Example 2.13.Let us consider the topological space (X,τ) such that $X = \{a, b, c, d\}$and
$\tau = \{X,\varphi,\{a\},\{c\},\{a,c\},\{a, b\}, \{a, b,c\}, \{a,c,d\}\}$.

We find that$\delta O(X,\tau) = \{X,\varphi,\{c,d\},\{a, b\}\}$; $e\text{-}O(X,\tau) = \{X,\varphi,\{a\},\{c\},\{c,d\},\{a,c\}, \{b,c\},\{a, b\}, \{a,b,c\}, \{a,c,d\},\{a,b,d\},\{b,c,d\}\}$.

Here, the topological space (X,τ) is e-disconnected since there exists two e-open sets {a,b} and {c,d} such that $X=\{a,b\}\cup\{c,d\}$with $\{a,b\}\cap\{c,d\}= \varphi$. But the space has no such e-open set which is both e-maximal open and e-minimal open set.

Theorem 2.14. If A and B are two different e-maximal open sets in a topologicalspace X with $A \cap B$ is a e-minimal closed set, then X is e-disconnected.

Proof. Since A and B are e-maximal, we have $A \cup B = X$. We put $G = A – A \cap B, H = B$ or $G = A, H = B – A \cap B$. We observe that G and H are disjoint e-open setswith $G \cup H = X$. So, X is e- disconnected.

Theorem 2.15. If U is a maximal e-open set, then eithere-Cl(U) = X or e-Cl(U) = U.

E. Proof

Since, U is a maximal e-open set, then by Theorem 2.7, following two cases occur:

F. Case I

For each $x \in X \backslash U$ and each e-open set W containing x, we have, $X\backslash U \subset W$. Let x be any element of $X\backslash U$ and W be any e-open set containing x. Since $X\backslash U \neq W$, we have $W \cap U \neq \varphi$ for any e-open setW containing x.Thus, $X\backslash U \subset$ e-Cl(U). Since $X = X \cup (X\backslash U) \subset U \cup$e-Cl(U)=e-Cl(U)$\subset X$, we have e-Cl(U) = X.

G. Case II

There is an e-open set W such that $X\backslash U=W\neq X$, Since $X\backslash U=W$ is an e-open set, U is an e-closed set. Therefore, U= e-Cl(U).
Theorem 2.16. If there existsae-maximal open set which is not e-dense in atopological space, then the top. space is e-disconnected.
Proof. Suppose that U be ae-maximal open set which is not e-dense in X. By Theorem2.15, U = e-Cl(U). If we take, G = U and H = X-e-Cl(U), then (G,H) is ae-separation for X. Thus, the top.space is e-disconnected.

II. MAXIMAL AND MINIMAL CLOSED SETS

In this section, similarresults in terms of maximal and minimal e-closed sets along with interrelationships between themare investigated.

A. Definition 3.1. A proper nonempty e-closed subset Fof X is said to be a maximal e-closed set if any e-closed set which contains F is Xor F.

B. Theorem 3.2. If E is a maximal closed set and F isany closed set, then either $E \cup F = X$ or $F \subset E$.

C. Definition 3.3.A proper nonempty closed subset Eof X is said to be a minimal e-closed set if any e-closed set which is contained inE is E or φ.

D. Theorem 3.4.In any topological space (X,τ), if F be a minimal e-closed set and G be ae-closed set of (X,τ). Then either $F \cap G = \varphi$ or $F \subset G$..LetF be a minimal e-closed set and G be ae-closed setof(X,τ). If $F \cap G = \varphi$, then there is nothing to prove. But if $F \cap G \neq \varphi$, then we have to prove that $F \subset G$.Now if $F \cap G \neq \varphi$, then $F \cap G \subset F$ and $F \cap G \subset G$. Since $F \cap G \subset F$ and given that F is minimal e-closed, then by definition $F \cap G = F$ or $F \cap G = \varphi$. But $F \cap G \neq \varphi$,then$F \cap G = F$ which shows that$F \subset G$.

By Theorem 2.5 and Theorem 3.4, we note that if G is both minimal e-openand minimal e-closed, and E is e-clopen, then either $G \subset E$ or $G \cap E = \varphi$.Analogous to Theorem 2.7, Theorem 2.9,Corollary 2.11 and Corollary 2.12,we have Theorem 3.5, Theorem 3.6, Corollary 3.7 and Corollary 3.8 respectively. Since proofs of theorems and corollaries are similar to the proof of corresponding theorems and corollaries already establish, we omit proofs.

Theorem 3.5. If E is a maximal e-closed set and F is a minimal e-closed set ofa topological space X, then either $F \subset E$ or $E \cup F = X$, $E \cap F = \varphi$.

Theorem 3.6. If a topological space X has a set E which is both maximal andminimale-closed, then either of the following is true.

1) E is the only proper e-closed set in the space

2) If there exists another proper e-closed set F, then $E\cup F = X$ and $E\cap F = \varphi$.

Corollary 3.7. If E is both maximal and minimal e-closed, and G is an e-openset in a topological space X, then either $E = X - G$ or $E = G$.

E. Corollary 3.8. If E is both maximal and minimal e-closed in a topological spaceX, then either E is the only proper e-closed set in the space or proper e-closed setsof the space are E and $X - E$ only.

F. Remark 3.9:It is trivial that if a topological space X has only one proper e-closed set,then that set is both maximal and minimal e-closed. If there are only two propere-closed sets in a space and the e-closed sets are disjoint, then both are maximaland minimal e-closed. If E and F are only two proper e-closed sets in a topologicalspace such that $F \subset E$, then F is a minimal e-closed and E is a maximal e-closedset in the space.

G. Example 3.10.In the topological space ofExample 2.13, there existdisjoint proper e-closed sets {a,b} and {c,d} such that X={a,b}∪{c,d}. But there is no proper e-closed set in the space which is both maximal and minimal e-closed set. Thus,we conclude that there may exist closed sets E, F in X such that $E \cup F = X$and $E \cap F = \varphi$but there may not exists a set which is both maximal andminimal closed.

H. Theorem 3.11. If G is both maximal open and minimal closed, H is open andE is closed, then either of the following is true.

1) $H \subset G \subset E$.

2) $H \subset G$ and $G \cap E = \varphi$.

3) $G \cup H = X$ and $G \subset E$.

4) $G \cup H = X, G \cap E = \varphi$.

Proof. Considering G as a maximal open set, By Theorem 2.3 we get $H \subset G$or $G \cup H = X$. Considering G as a minimal closed set, by Theorem 3.4, $G \subset E$or $G \cap E = \varphi$. $H \subset G$ and $G \subset E$ imply $H \subset G \subset E$. The remaining probablecombinations are $H \subset G$, $G \cap E = \varphi$; $G \cup H = X$, $G \subset E$ and $G \cup H = X, G \cap E = \varphi$.

H. Corollary 3.12. If G is both maximal open and minimal e-closed, then G and$X - G$ are only proper e-clopen sets in the space.

Proof. Let E be e-clopen in X. Putting H = E in Theorem 3.11, we get G = Eor $G = X - E$.

I. Theorem 3.13. If G is both maximal e-open and maximal e-closed, and E ise-clopen, then either $E \subset G$ or $G \cup E = X$.

Proof.Similar to the proof of Theorem 3.11. Corresponding to Theorem 3.11 and Corollary 3.12, we have Theorem 3.14and Corollary 3.15 respectively. The proofs of the theorem and the corollaryare omitted as they are similar to proofs already established.

*J.*Theorem 3.14. If G is both minimal e-open and maximal e-closed, H is open andE is e-closed, then either of the following is true.

1) $E \subset G \subset H$.

2) $G \subset H, G \cup E = X$.

3) $G \cap H = \varphi, E \subset G$.

4) $G \cup E = X$, $G \cap H = \varphi$.

*K.*Corollary 3.15. If G is both minimal e-open and maximal e-closed, then G and$X - G$ are only proper e-clopen sets in the space.

*L.*Theorem 3.16. Let A,G be open sets in X such that $A \cap G \neq \varphi$, Then, $A \cap G$ is a minimal open set in (A, τ_A) if G is a minimal open set in (X,τ).

Proof. If $A \cap G$ is not a minimal e-open set in (A,τ_A), there exists an e-open set$U \neq \varphi$in (A,τ_A) such that $U \not\subseteq A \cap G$. Since G is a minimal open set in Xand $A \cap G \neq \varphi$, we have by Theorem 2.5, $G \subset A$ which implies $A \cap G = G$. Abeing open in X, U is also open in X. So we get a set U open in X such that$\varphi \neq U \not\subseteq G$ which is a contradiction to our assumption that G is a minimal open set in X.

*M.*Theorem 3.17.Let U be a maximal e-open set. Then e-Int(X\U) =X-U or e-Int(X\U) = φ.

Proof.By Theorem 2.6, we have following cases (1) e-Int(X\U) = φ or (2) e-Int(X\U) = X\U.

*N.*Theorem 3.18.Let U be a maximal e-open set and S a nonempty subset of X\U. Then e-Cl(S)= X\U.

Proof.Since$\varphi \neq S \subset X\backslash U$, we have $W \cap S \neq \varphi$ for any element x of X\U and any e-open set W of x by Theorem 2.15. Then $X\backslash U \subset$e-Cl(U). Since X\U is a e-closed set and $S \subset X\backslash U$, we see that e-Cl(S) $\subset$e-Cl(X\U) = X\U. Therefore X\U= e-Cl(S).

*O.*Corollary 3.19.Let U be a maximal e-open set and M a subset of X with $U \subset M$. Then e-Cl(M) =X.

Proof.Since$U \subset M \subset X$, there exists a nonempty subset S of X\U such that $M = U \cup S$. Hence we have e-Cl(M)= e-Cl($U \cup S$)= e-Cl(U) $\cup$e-Cl(S) $\supset$(X\U) $\cup$U= X by Theorem 3.18. Therefore e-Cl(M) =X.

P. Theorem3.20.Let U be a maximal e-open set and assume that the subset X\U has two element at least. Then e-Cl(X\{a}) =X for any element of X\U.

Proof.Since$U \subset X\backslash\{a\}$ by our assumption, we have the result by Corollary 3.19.

Q. Theorem3.21.Let U be a maximal e-open set, and N be a proper subset of X with $U \subset N$. Then, e-Int(N)= U.

Proof.IfN = U, then e-Int(N)= e-Int(U)=U. Otherwise, $N \neq U$, and hence $U \subset N$. It follows that $U \subset$e-Int(N). Since U is a maximal e-open set, we also have,e-Int(N) $\subset$U. Therefore,e-Int(N) = U.

III. CONCLUSION

In this work, the concept of maximal e-open sets, minimal e-closed sets, e-semi maximal open and e-semi minimal closed sets which are fundamental results for further research on topological spaces are introduced and aimed in investigating the properties of these new notions of open sets with example, counter examples and some of their fundamental results are also established. Hope that the findings in this paper will help researcher enhance and promote the further study on topological spaces to carry out a general framework for their applications in separation axioms, connectedness, compactness etc. and also in practical life.

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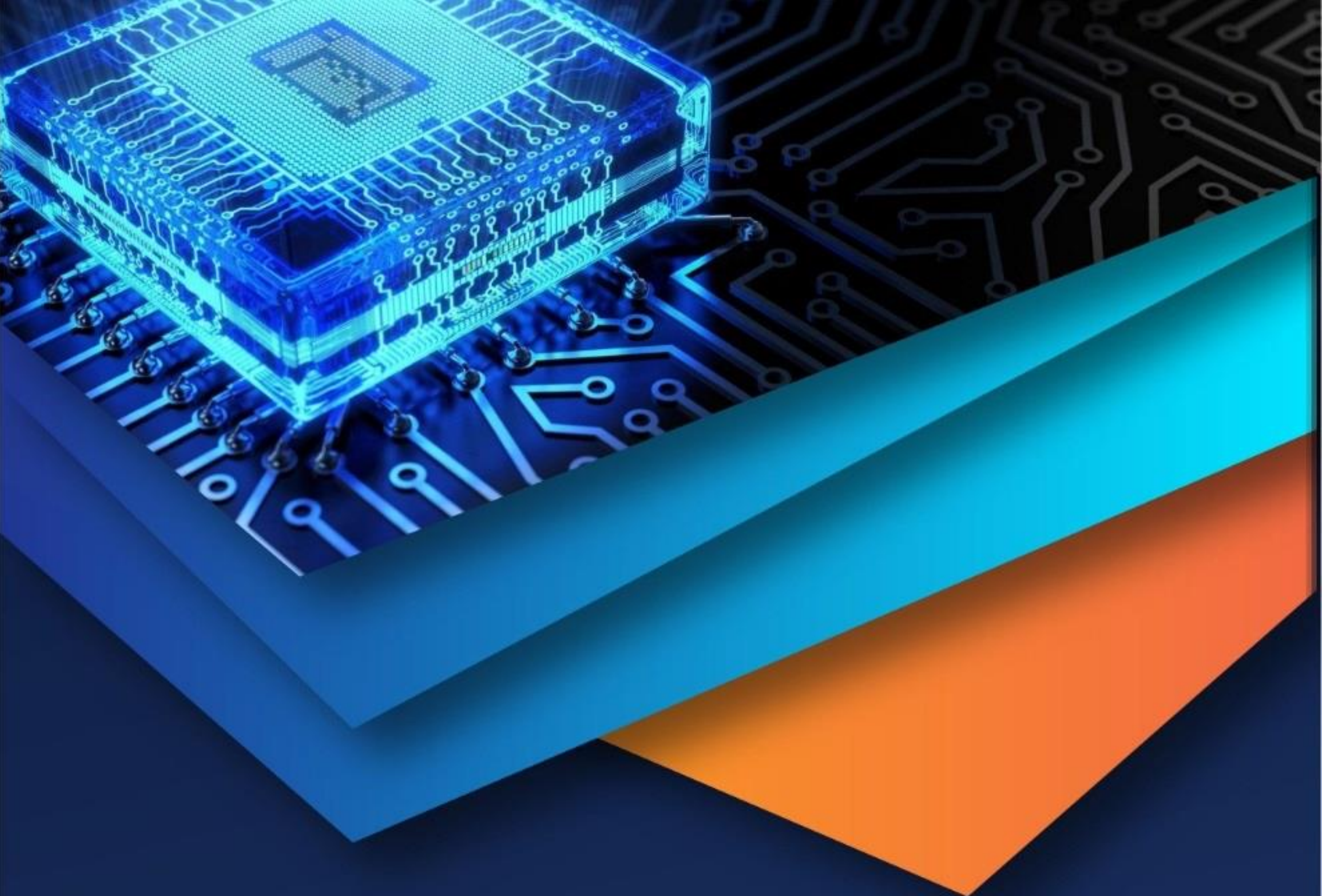

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