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A Notes on Bailey’s Transform

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Abstract: In this paper transformation formulae for poly-basic hypergeometric functions have been established by making use of Bailey’s transform.

Keywords: Hyper geometric functions, Summations, Transformation, Polybasic, Convergence

I. INTRODUCTION

The well-known Bailey’s transformation states that, if

$$\beta_n = \sum_{r=0}^n \alpha_r U_{n-r} V_{n+r} \tag{1.1}$$

$$\text{and } \gamma_n = \sum_{r=n}^{\infty} \delta_r U_{n-r} V_{n+r} = \sum_{r=0}^{\infty} \delta_r U_r V_{r+2n} \tag{1.2}$$

where $\alpha_r, \delta_r, U_r, V_r$ are any functions of r only, and that the series for γ_n exists, then, subject to convergence,

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{1.3}$$

Bailey’s transform (1.3) has been exploited, both as a tool to find new transformations of both ordinary and basic, hypergeometric series and also to find new q -identities of Rogers-Ramanujan type. We shall make use of the following summation in our analysis.

$$\begin{aligned} & {}_3\Phi_3 \left[\begin{matrix} q^a & : & q_1^b & ; & zq^{a+1}q_1^{b+1} : q, q_1 : qq_1 : z \\ zq^{a+1} & : & zq_1^{b+1} & ; & zq^a q_1^b \end{matrix} \right]_N \\ &= \frac{(1-zq^a)(1-zq_1^b)}{(1-z)(1-zq^a q_1^b)} - \frac{[q^a; q]_{N+1} [q_1^b; q_1]_{N+1} z^{N+1}}{(1-z)(1-zq^a q_1^b) [zq^{a+1}; q]_N [zq_1^{b+1}; q_1]_N} \end{aligned} \tag{1.4}$$

As $N \rightarrow \infty$, yeilds

$$\begin{aligned} & {}_3\Phi_3 \left[\begin{matrix} q^a & : & q_1^b & ; & zq^{a+1}q_1^{b+1} : q, q_1 : qq_1 : z \\ zq^{a+1} & : & zq_1^{b+1} & ; & zq^a q_1^b \end{matrix} \right]_N \\ &= \frac{(1-zq^a)(1-zq_1^b)}{(1-z)(1-zq^a q_1^b)} \end{aligned} \tag{1.5}$$

$$\sum_{k=0}^n \frac{(1-ap^k q^k) [a; p]_k [c; q]_k c^{-k}}{(1-a) [q; q]_k [ap/c; p]_k} = \frac{[ap; p]_n [cq; q]_n}{[q; q]_n [ap/c; p]_n} \tag{1.6}$$

$$\begin{aligned} & \sum_{k=0}^n \frac{(1-ap^k q^k)(1-bp^k q^{-k}) [a, b; p]_k [c, a/bc; q]_k q^k}{(1-a)(1-b) [q, aq/b; q]_k [ap/c, bcp; p]_k} \\ &= \frac{[ap, bp; p]_n [cq, aq/bc; q]_n}{[q, aq/b; q]_n [ap/c, bcp; p]_n} \end{aligned} \tag{1.7}$$

II. NOTATIONS AND DEFINITIONS

A RATIONAL FUNCTION OF Q^N , Q BEING FIXED COMPLEX PARAMETERS CALLED THE BASE OF THE SERIES, USUALLY WITH MODULUS LESS THAN ONE. AN EXPLICIT REPRESENTATION OF SUCH SERIES IS GIVEN BY:

$${}_r\Phi_s \left[\begin{matrix} a_1, & a_2, \dots, & a_r; q; Z \\ b_1, & b_2, \dots, & b_s; q^i \end{matrix} \right]$$

$$= \sum_{n=0}^{\infty} q^{i \binom{n}{2}} \frac{[a_1, a_2, \dots, a_r; q]_n Z^n}{[b_1, b_2, \dots, b_s; q]_n} \tag{2.1}$$

where $\binom{n}{2} = n(n-1)/2$ And

$$[a_1, a_2, \dots, a_r; q]_n = [a_1, q]_n [a_2, q]_n \dots [a_r, q]_n$$

With the q-shifted factorial defined by

$$[a; q]_n = \begin{cases} 1, & \text{if } n = 0 \\ (1-a)(a-aq) \dots (1-aq^{n-1}), & \text{if } n = 1, 2, \dots \end{cases}$$

(2.2)

For the convergence of the series (2.1) we need $|q| < 1$ and $|z| < \infty$ when $i = 1, 2, \dots$ or $\max\{|q|, |z|\} < r$. When $i = 0$ provided no zeros appear in the denominator.

${}_r\Phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q; Z \\ b_1, b_2, \dots, b_s; q^i \end{matrix} \right]_N$ means that the series runs

From 0 to N only

The poly-basic hypergeometric series is defined as:

$$\begin{aligned} & \Phi \left[\begin{matrix} c_{1,1}, \dots, c_{1,r_1}, \dots, c_{m,1}, \dots, c_{m,r_m}; q_1, q_2, \dots, q_m; Z \\ d_{1,1}, \dots, d_{1,s_1}, \dots, d_{m,1}, \dots, d_{m,s_m} \end{matrix} \right] \\ &= \sum_{n=0}^{\infty} Z^n \prod_{j=1}^m \frac{[c_{j,1}, \dots, c_{j,r_j}; q_j]_n}{[d_{j,1}, \dots, d_{j,s_j}; q_j]_n} \end{aligned} \tag{2.3}$$

III. MAIN RESULTS

Taking $u_r = v_r = 1$ and

$$\begin{aligned} \delta_r &= \frac{[q^a, q]_r [q_1^b; q_1]_r [xq^{a+1}q_1^{b+1}; qq_1]_r x^r}{[xq^{a+1}; q]_r [xq_1^{b+1}; q_1]_r [xq^a q_1^b; qq_1]_r} \text{ in (1.2) we get} \\ \gamma_n &= \frac{[q^a, q]_n [q_1^b; q_1]_n [xq^{a+1}q_1^{b+1}; qq_1]_n x^n (1-xq^{a+n})(1-xq_1^{b+n})}{[xq^{a+1}; q]_n [xq_1^{b+1}; q_1]_n [xq^a q_1^b; qq_1]_n (1-x)(1-xq^{a+n}q_1^{b+n})} \\ &= \frac{(1-xq^a)(1-xq_1^b)(q^a; q)_n (q_1^b; q_1)_n x^n}{(x-1)(1-xq^a q_1^b)(xq^a; q)_n [xq_1^b; q_1]_n} \end{aligned} \tag{3.1}$$

Putting these values in (1.1) (1.2) and (1.3) we get the new form of the Bailey's transform as

If

$$\beta_n = \sum_{r=0}^n \alpha_r \tag{3.2}$$

Then

$$\begin{aligned} & \frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^a q_1^b)} \sum_{n=0}^{\infty} \frac{(q^a; q)_n [q_1^b; q_1]_n x^n}{(xq^a; q)_n [xq_1^b; q_1]_n} \alpha_n \\ &= \sum_{n=0}^{\infty} \frac{(q^a; q)_n [q_1^b; q_1]_n [xq^{a+1}q_1^{b+1}; qq_1]_n x^n}{(xq^{a+1}; q)_n [xq^a q_1^b; qq_1]_n [xq_1^{b+1}; q_1]_n} \beta_n \end{aligned} \tag{3.3}$$

We shall make use of (3.2) and (3.3) in order to establish certain new transformation formulae

(a) Choosing

$\alpha_r = z^r$ in (3.2) we get

$$\beta_n = \frac{1-z^{n+1}}{1-z}$$

Putting these values in (3.3) we get,

$$\frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^aq_1^b)} {}_3\Phi_2 \left[\begin{matrix} q^a; & q_1^b; & q, q_1; & xz \\ xq^a; & xq_1^b & & \end{matrix} \right]$$

$$= \frac{(1-xq^a)(1-xq_1^b)}{(1-z)(1-x)(1-xq^aq_1^b)} - \frac{z}{(1-z)}$$

$$\times {}_3\Phi_3 \left[\begin{matrix} q^a & ; & q_1^b & ; & xq^{a+1}; & q_1^{b+1}; & q, q_1; & qq_1; & xz \\ xq^{a+1}; & xq_1^{b+1}; & xq^aq_1^b & & & & & & \end{matrix} \right], \tag{3.4}$$

(b) Next, taking

$$\alpha_r = \frac{[q_2^\alpha; q_2]_r [q_3^\beta; q_3]_r [zq_2^{\alpha+1}q_3^{\beta+1}; q_2q_3]_r z^r}{[zq_2^{\alpha+1}; q_2]_r [zq_3^{\beta+1}; q_3]_r [zq_2^\alpha q_3^\beta; q_2q_3]_r}$$

In (3.2) and making use of (1.4) we get:

$$\beta_n = \frac{(1-zq_2^\alpha)(1-zq_3^\beta)}{(1-z)(1-zq_2^\alpha z_3^\beta)}$$

$$- \frac{[q_2^\alpha; q_2]_{n+1} [q_3^\beta; q_3]_{n+1} z^{n+1}}{(1-z)(1-zq_2^\alpha q_3^\beta) [zq_2^{\alpha+1}; q_2]_n [zq_3^{\beta+1}; q_3]_n} \tag{3.5}$$

Putting these values in (3.3) we have

$$\frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^aq_1^b)} \left[\begin{matrix} q^a; & q_1^b & ; & q_2^\alpha, q_3^\beta, zq_2^{\alpha+1}, zq_3^{\beta+1}; & q, q_1, q_2, q_3, q_2q_3; & xz \\ xq^a; & xq_1^b; & zq_2^{\alpha+1}; & zq_3^{\beta+1}; & zq_2^\alpha q_3^\beta & \end{matrix} \right]$$

$$= \frac{(1-zq_2^\alpha)(1-zq_3^\beta)(1-xq^a)(1-xq_1^b)}{(1-z)(1-x)(1-zq_2^\alpha q_3^\beta)(1-xq^aq_1^b)} - \frac{z(1-q_2^\alpha)(1-q_3^\beta)}{(1-z)(1-zq_2^\alpha q_3^\beta)}$$

$$\times {}_5\Phi_5 \left[\begin{matrix} q^a; & q_1^b; & xq^{a+1}q_1^{b+1}; & q_2^{\alpha+1}; & q_3^{\beta+1}; & q, q_1, qq_1, q_2, q_3; & xz \\ xq^{a+1}; & xq_1^{b+1}; & xq^aq_1^b; & zq_2^{\alpha+1}; & zq_3^{\beta+1} & \end{matrix} \right] \tag{3.6}$$

(c) Again, taking

$$\alpha_r = \frac{(1-\alpha P^r Q^r) [\alpha; P]_r [\beta; Q]_r \beta^{-r}}{(1-\alpha) [Q; Q]_r [\alpha P/\beta; P]_r}$$

And making use of (1.6) we get,

$$\beta_n = \frac{[\alpha P; P]_n [\beta Q; Q]_n}{[Q; Q]_n [\alpha P/\beta; P]_n}$$

Putting these values in (3.3) we obtain,

$$\frac{(1-xq^a)(1-xq_1^b)}{(1-x)(1-xq^aq_1^b)} {}_5\Phi_5 \left[\begin{matrix} q^a; & q_1^b; & \alpha PQ; & \alpha; & \beta; & q; & q_1, PQ, P, P, Q; & x/\beta \\ xq^a; & xq_1^b; & \alpha; & \alpha P/\beta; & Q; & \end{matrix} \right]$$

$$\times {}_6\Phi_5 \left[\begin{matrix} q^a; & q_1^b; & xq^{a+1}q_1^{b+1}; & \alpha P; & \beta Q; & q, q_1, qq_1, P, Q; & x \\ xq^{a+1}; & xq_1^{b+1}; & xq^aq_1^b; & \alpha P/\beta; & Q; & \end{matrix} \right]. \tag{3.7}$$

(d) Lastly, taking

$$\alpha_r = \frac{(1 - \alpha P^r Q^r)(1 - \beta P^r Q^{-r})[\alpha, \beta; P]_r [\gamma, \alpha/\beta\gamma; Q]_r Q^r}{(1 - \alpha)(1 - \beta)[Q, \alpha Q/\beta; Q]_r [\alpha P/\gamma, \beta\gamma P; P]_r}$$

In (3.2) and making use of (1.7) we get,

$$\beta_n = \frac{[\alpha P, \beta P; P]_n [\gamma Q, \alpha Q/\beta\gamma; Q]_n}{[Q, \alpha Q/\beta; Q]_n [\alpha P/\beta, \beta\gamma P; P]_n}$$

Now putting these values in (3.3) we find:

$$\frac{(1 - xq^a)(1 - xq_1^b)}{(1 - x)(1 - xq^a q_1^b)} \times {}_9\Phi_8 \left[\begin{matrix} q^a; q_1^b; \alpha; \beta; \gamma, a/\beta\gamma; \alpha PQ; \beta P/Q; q, q_1, P, Q, PQ, P/Q; xQ \end{matrix} \right]$$

$$= {}_8\Phi_7 \left[\begin{matrix} q^a; q_1^b; xq^{a+1}q_1^{b+1}; \alpha P, \beta P; \gamma Q, \alpha Q/\beta\gamma; q, q_1, q_1, P, Q; x \end{matrix} \right]$$

$$= {}_8\Phi_7 \left[\begin{matrix} xq^{a+1}; xq_1^{b+1}; xq^a q_1^b; \alpha P/\gamma, \beta\gamma P, Q, \alpha Q/\beta \end{matrix} \right] \tag{3.8}$$

A number of other interesting results can also be obtained by taking suitable values of α_r and β_n .

IV. CONCLUSIONS

Six transformation formulae for poly-basic hypergeometric functions are established by making use of Bailey's Transform.

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