iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 **Issue:** X **Month of publication:** October 2017

DOI: http://doi.org/10.22214/ijraset.2017.10148

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue X, October 2017- Available at www.ijraset.com

Cubic Harmonious Labeling of Path Related Graphs

Mini. S.Thomas[1] , Mathew Varkey T.K [2]
[1]Asst.Prof, Department of Mathematics, ILM Engineering College, Eranakulam, India
[2]Asst.Prof, Department of Mathematics, T.K.M College of Engineering, Kollam, Kerala, India

Abstract: ***A (n,m) graph G =(V,E) is said to be Cubic Harmonious Graph(CHG) if there exists an injective function $f:V(G)\rightarrow\{1,2,3,\ldots\ldots\ldots,m^3+1\}$ such that the induced mapping $f^*_{chg} : E(G)\rightarrow \{1^3,2^3,3^3,\ldots\ldots\ldots,m^3\}$ defined by $f^*_{chg}(uv) = (f(u)+f(v)) \bmod (m^3+1)$ is a bijection. The vertex labels are distinct and edge labels are also distinct as well as cubic. In this paper, focus will be given on the result "cubic harmonious labeling of caterpillar graph and (P_n,S_m) graph***
Keywords: Caterpillar graph,Cubic harmonious graph, Cubic harmonious labeling, Harmonious graph, Path graph.

I. INTRODUCTION

Consider a graph $G=(V(G),E(G))$ with m edges. A function f defined by f: $V(G)\rightarrow\{1,2,3,\ldots,m-1\}$, is called a harmonious labeling of G if it is an injective and it induces a bijective function f^* defined by $f^*(e) = (f(u)+f(v)) \pmod m$. where $e = uv$ for $u,v \in v(G)$, $e\in E((G)$. By taking the edge labels of a sequentially labeled graph with m edges modulo m, we obviously obtain a harmoniously labeled graph. More than 150 papers have been published on harmonious labeling and comprehensive information can be found in [3].Labeling of special type of Square Harmonious Labeling is examined in [9].Cubic Graceful Labeling is introduced in [5]. Cubic Harmonious Labeling is defined in [6].

In this paper , we study the existence of cubic harmonious labeling for graphs obtained by caterpillar and (P_n,S_m)

A. *Definition 1.1*

The path on n vertices is denoted by P_n.

B. *Definition1.2*

A complete bipartite graph $K_{1,n}$ is called a ***star***and it has (n+1) vertices and n edges

C. *Definition1.3*

The *graph* (P_n,S_m) is obtained from n copies of the star graph S_m and the path P_n: $\{u_1,u_2\ldots.u_n\}$ by joining u_j with the vertex of the j^{th} copy of S_m by means of and edge for $1\leq j\leq n$.

D. *Definition1.4*

The ***caterpillar*** $S(X_1, X_2\ldots X_m)$ is obtained from the path $P_n$by joining X_i vertices to each of the i^{th} vertex of P_n $(1\leq i\leq n)$

II.MAIN RESULTS

A. *Theorem 2. 1*

The graph (P_n, S_m) is cubic harmonious graph.

1) Proof :

Let $\{u_1,u_2,\ldots.u_n\}$ be the vertices of the path P_n and $\{v_{0j},v_{ij},v_{2j},\ldots\ldots,v_{mj}\}$ be the vertices of j^{th} copy P_m for $1\leq j\leq n$.

The graph (P_n,S_m) is obtained from n copies of the star graph S_m and the path P_n: $\{u_1,u_2\ldots.u_n\}$ by joining u_j with the vertex of the j^{th} copy of S_m by means of and edge for $1\leq j\leq n$.

Then

$$V((P_n,S_m)) = \begin{cases} u_i; & 1\leq i\leq n \\ v_{0i}; & 1\leq i\leq n \end{cases}$$

$v_{ij};\quad 1\le i\le m,\quad 1\le j\le n$

and

$$E((P_n,S_m)) = \begin{cases} u_i u_{i+1}; & 1\le i\le n-1 \\ u_i v_{0i}; & 1\le i\le n \\ v_{oj} v_{ij}; & 1\le i\le m,\ 1\le j\le n \end{cases}$$

Define an injection $f: V((P_n,S_m)) \to \{1,2,\dots\dots,(mn+2n-1)^3+1\}$ by

$f(u_i) = (mn+2n+1-i)^3 + (mn+2n-1)^3 + 1 - f(u_{i-1})$; $\quad 2\le i\le n$

$f(V_{oj}) = (mn+j)^3 + (mn+2n-1)^3 + 1 - f(u_j)$; $\quad 1\le j\le n$

$f(v_{ij}) = (mn-ni+j)^3 + (mn+2n-1)^3 + 1 - f(V_{oj})$; $\quad 1\le i\le m,\ 1\le j\le n$

The induced edge mapping are

$f^*(u_iu_{i+1}) = (mn+2n+1-i)^3$; $\quad 1\le i\le n-1$

$f^*(u_iv_{0i}) = (mn+i)^3$; $\quad 1\le i\le n$

$f^*(v_{oj}v_{ij}) = (mn-ni+j)^3$; $\quad 1\le i\le m,\ 1\le j\le n$

The vertex label are in the set $\{1,2,\dots\dots\dots,(mn+2n-1)^3+1\}$ Then the edge labels are distinct and cubic. ie. $\{1^3, 2^3, \dots\dots\dots\dots\dots, (mn+2n-1)^3\}$. Hence the graph (P_n, S_m) is cubic harmonious.

B. Theorem 2.2

The caterpillar $S(X_1, X_2 \dots X_m)$ is cubic harmonious graph for all n>1.

1) *Proof :* Let G be the caterpillar S ($x_1, x_2 \dots x_m$) is obtained from the path P_n by joining X_i vertices to each of the i^{th} vertex of P_n $(1\le i\le n)$

Let $V(G) = \{v_i;\ 1\le i\le n\}\ U\{v_{ij};\ 1\le i\le n-1,\ 1\le j\le m\}$

and

$E(G) = \{v_iv_{i+1};\ 1\le i\le n-1\}\ U\{v_iv_{ij};\ 1\le i\le n-1,\ 1\le j\le m\}$

Then $|V(G)| = m+n$ and $|E(G)| = m+n-1$

Define an injection $f: V(G) \to \{1,2,\dots\dots,(m+n-1)^3+1\}$ by

$f(v_1) = (m+n-1)^3 + 1;$

$f(v_i) = (m+n+1-i)^3 + (m+n-1)^3 + 1 - f(v_{i-1});$ $\quad 2\le i\le n$

$f(v_{ij}) = (m+1-j)^3 + (m+n-1)^3 + 1 - f(v_i);$ $\quad 1\le i\le n-1, 1\le j\le m;$

The induced edge mapping are

$f^*(v_i v_{i+1}) = (m+n-1)^3$; $\quad 1\le i\le n-1$

$f^*(v_iv_{ij}) = (m+1-j)^3$; $\quad 1\le j\le m$

The vertex label are in the set $\{1,2,\dots\dots,(m+n-1)^3+1\}$. Then the edge labels are distinct and cubic. The edge sets are $\{1^3,2^3,3^3,\dots\dots,(m+n-1)^3\}$. Hence the caterpillar is cubic harmonious graph.

III.CONCLUSION

The harmonious labeling is one of the most important labeling techniques. As all the graphs are not harmonious, it is very interesting to investigate graphs or graph families which admit harmonious labeling.We have reported the cubic harmonious labeling of different graphs.

REFERENCES

[1] N. Adalin Beatress, P.B.Sarasija, Square Harmonious Graphs, International Jounal of Advanced Research in Science, Engineering and Technology. Vol.3, Issue 2 , February 2016
[2] Frank Harary, Graph Theory, Narosa Publishing House, New Delhi, 2001
[3] J.A . Gallian, A dynamic survey of graph labeling, The Electronic journal of Combinatrorics,(2016
[4] R.L.Graham, N.J.A.Sloane, On additive bases and harmonious graphs ,SIAM.Algebr.Disc.Meth.,Vol 1, No 4(1980),382-40

[5] Mini.S.Thomas and Mathew Varkey T.K, Cubic Graceful Labeling , Global Journal of Pure And Applied Mathematics, Volume 13,Number 9, pp 5225-5234, Research India Publications June (2017)

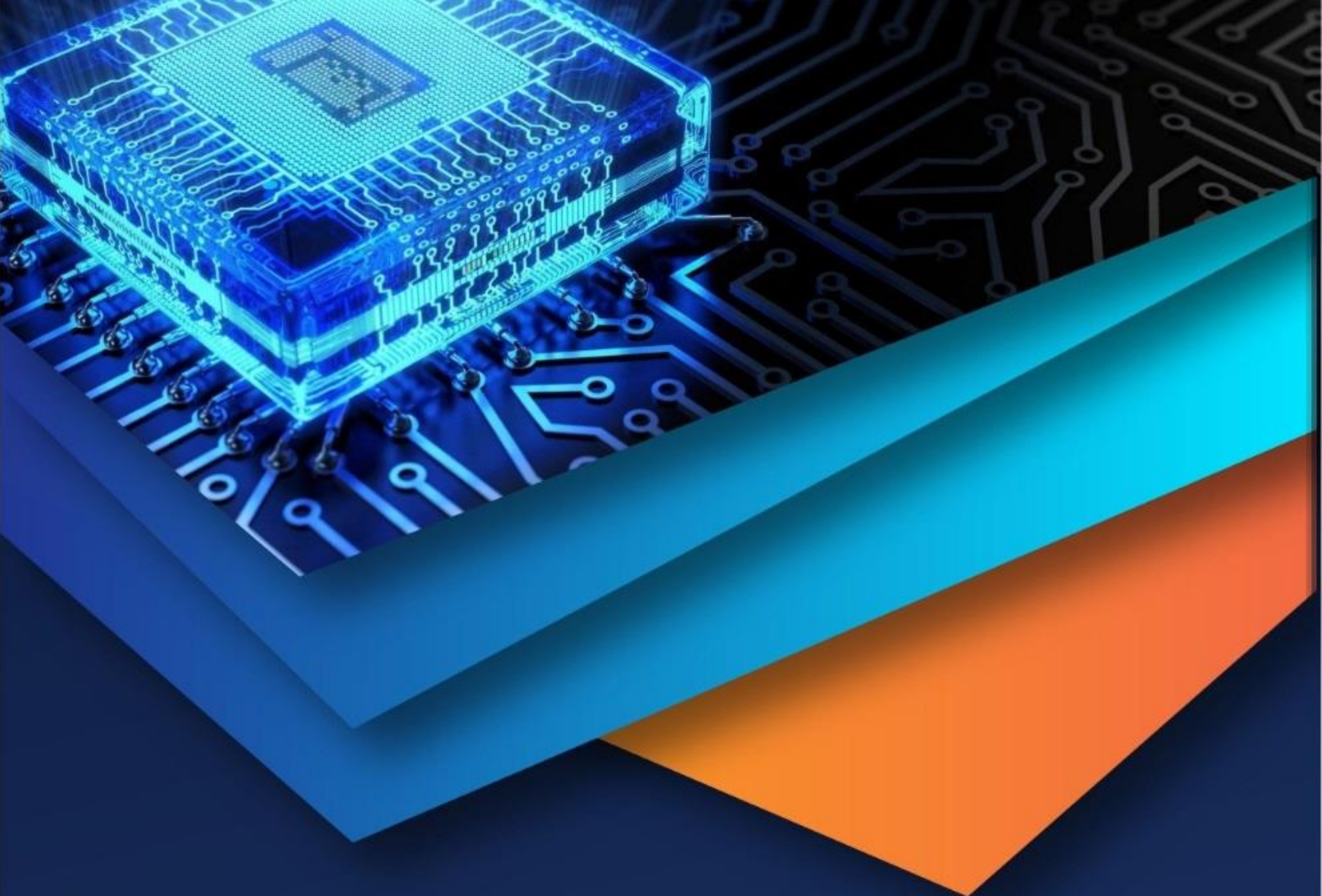
[6] Mini.S.Thomas and Mathew Varkey T.K, Cubic Harmonious Labeling . International Journal of Engineering Development of Research (Accepted) (2017)

[7] P.B.Sarasija and N.Adalin Beatress, Even - Odd harmonious graphs, international journal of Mathematics and Soft Computing Vol.5, No1,23-29, (2015).

[8] S.C.Shee, "On harmonious and related graphs" Ars Combinatoria, vol 23, pp 237-247,(1987).

[9] T.Tharmaraj and P.B.Sarasija, Square graceful graphs, international journal of Mathematics and Soft Computing Vol.4, No 1,129-137, (2014)

[10] T.Tharmaraj and P.B.Sarasija, Some Square graceful graphs, international journal of Mathematics and Soft Computing Vol.5 No.1,119-127 . (2014).



10.22214/IJRASET

INDEX COPERNICUS
45.98

IMPACT FACTOR:
7.129

TOGETHER WE REACH THE GOAL

IMPACT FACTOR:
7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY