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Dependence upon Debye-HüCkel Parameter In Course Of an Electro-Osmotic Flow Initiated By an Alternating Electric Field

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Abstract: A study of the electro-osmotic flow of a micro polar bio-fluid is made here. The flow regime is affected by an external alternating electric field. The channel walls through which the flow is taking place are in a state of periodic vibration with a frequency that differs from that of electric field applied. It is observed that a rise in Debye-Hückel parameter enhances both the velocity and micro rotation gradient.
Keywords: Electro-osmotic flow; Debye-Hückel parameter; Micropolar fluid.

I. INTRODUCTION

Bio-micro fluidic devices have drawn attention to many researchers for its gradually increasing multipurpose uses [1, 2]. Microchannels are the fundamental components to study the design aspect of micro-fluidic devices. Micro valves and micro pumps, in general, suffer from mechanical failure as one goes on shrinking the channels or in other words for micro channel. The things are even more difficult when the channel walls are in a state of periodic motion. But the flow which is driven by electro kinetic forces, has the advantages of not having any moving parts; which gives rise to a noise free controller of fluid flow in a micro channel.
A couple of studies on this important electro kinetic phenomenon are available in scientific literatures (cf. [3]-[8]). The studies with the consideration of rotation of micro particles in an oscillatory microchannel under the action of an alternating field were required to improve the design aspect of micro-fluidic devices. It is seen that the equation for conservation of angular momentum can reveal the behavior of suspended tiny particles that needs to be taken into account to have perfection.
Micro-continuum mechanics as refined by Erigena [9] with the use of Cauchy's laws of motion and the laws of conservation of mass and energy served best for the purpose above. Motivating towards that this study is made by developing a mathematical model taking account the momentum transport in case of flow of a fluid in a micro-channel under the influence of an alternating electric field. Moreover, on average, each ion in an ionized solution is more closely associated with oppositely charged ions in comparison with the ions of like charge. Debye–Hückel parameter has its importance for non-ideality of any electrolytic solution and for accurate estimate in Poisson-Boltzmann model ions in dilute solutions is an important aspect to study. Electrostatic interactions among ions cause departure from ideal solution and so here the case of a micropolar fluid model is considered with Debye-Hückel approximation and the dependence on Debye-Hückel parameter is studied which may act as a first step towards enabling applicability of micropolar fluid model in a right direction when the electro osmotic flow through a micro channel is considered.

II. THE MODEL

An incompressible, viscous and ionized micropolar fluid is considered to flow in the direction of x-axis through a microchannel that is bounded by two plates at the position, $y = \pm h$ [Fig. 1]. An alternating electric field is applied externally and the channel walls are in an oscillatory motion with a velocity $u_s e^{i\omega t}$. Let us consider that u = u (y,t) be the fluid velocity at time t and the continuity equation reduces to $\frac{\partial u}{\partial x} = 0$, the flow is considered to be symmetric about the axis y = 0 and so we had limited our study to the region $0 \leq y \leq h$ only.

III. ANALYSIS

Taking into account all the aforementioned assumptions and using the approximations of boundary layer, the equations that can be well fitted to explain the unsteady and fully developed electro-osmotic flow of the micropolar fluid are of the form:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \left(\nu + \frac{k}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho}\frac{\partial N}{\partial y} + \frac{\rho_e}{\rho}\left(E_x e^{i\omega_e t}\right), \qquad (1)$$

$$\frac{\partial N}{\partial t} = \frac{\gamma_s}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \qquad (2)$$

and

$$\frac{d^2\psi}{dy^2} = -\frac{\rho_e}{\varepsilon} \qquad (3)$$

Where, ρ_e is the net electric charge density distribution near a charged surface; N represents the microrotation, ω denotes the angular velocity of oscillation of the channel walls; t denotes the time variable and k stands for the vortex viscosity and ρ for the fluid density; E is the externally applied electric field and ε represents the dielectric constant of the medium. The electrical double layers are considered to be so thin that there is no interference between the two walls. The Debye length is assumed to be much smaller than the channel height, which is less than the width of the channel. The symbol γ_s represents the spin gradient viscosity and is given by

$$\gamma_s = \left(\mu + \frac{k}{2}\right) j = \mu\left(1 + \frac{\chi}{2}\right) j, \qquad (4)$$

In which, $\chi = \frac{k}{\mu}$ represents the material / micropolar parameter.

Making use of non-dimensional variables as,

$$x^* = \frac{x}{h};\ y^* = \frac{y}{h};\ u^* = \frac{u}{U_{HS}};\ N^* = \frac{Nh}{U_{HS}};\ \ p^* = \frac{p}{\frac{\mu U_{HS}}{h}};\ \omega^* = \frac{\omega h}{U_{HS}};\ \omega_e^* = \frac{\omega_e h}{U_{HS}};\ R = \frac{U_{HS} h}{\nu};\ \psi^* = \frac{\psi}{\zeta}; t^* = \frac{tU_{HS}}{h};$$

(5)

Further, we have use the symbol $j = h^2$ to denote a micro-inertia parameter per unit mass. We shall also use the unsteady Helmholtz-Smoluchowski electro-osmotic velocity U_{UHS} defined as

$$U_{UHS} = U_{HS} e^{i\omega t} = \frac{-\zeta\varepsilon E\, e^{i\omega_e t}}{\mu} \qquad (6)$$

in which M represents the mobility, ς the zeta potential, and $\mu = \rho\ \nu$, the dynamic viscosity. In terms of the above-written dimensionless variables, Eq. (1)-(3) may be rewritten as

$$R\frac{\partial u^*}{\partial t^*} = -\frac{\partial p^*}{\partial x^*} + (1+\chi)\frac{\partial^2 u^*}{\partial y^{*2}} + \chi\frac{\partial N^*}{\partial y^*} + \frac{d^2\psi^*}{dy^{*2}}\, e^{i\omega_e^* t}, \qquad (7)$$

$$R\frac{\partial N^*}{\partial t^*} = \left(1 + \frac{K}{2}\right)\frac{\partial^2 N^*}{\partial y^{*2}} - \chi\left(2N^* + \varepsilon\frac{\partial u^*}{\partial y^*}\right) \qquad (8)$$

and

$$\frac{d^2\psi^*}{dy^{*2}} = -\frac{2n_0 ez\, h^2}{\zeta\varepsilon}\sinh\left(\frac{ez\zeta}{K_B T}\psi^*\right) \qquad (9)$$

The electric potential energy is considered small compared to the thermal energy of the ions, so that the values of $|ez\zeta\psi^*| < |K_B T|$. Then equation (9) reduces to

$$\frac{d^2\psi^*}{dy^{*2}} = m^2\psi^*, \qquad (10)$$

$$m^2 = \frac{h^2}{\lambda^2} = \frac{2n_0 e^2 z^2\, h^2}{K_B T} \qquad (11)$$

m stands for the non-dimensional Debye-Huckel parameter and λ, for the thickness of the Debye layer. The solution of Eq. (10) subject to the boundary conditions

$$\psi^*(0)=0,\ \psi^*(\pm 1)=1 \text{ and } \left(\frac{d\psi^*}{dy^*}\right) = 0 \qquad (12)$$

$$N^*(0) = 0 \text{ and } N^*(1) = \beta\frac{\partial u^*}{\partial y^*}\Big|\, y^* = 1 \qquad (13)$$

where $\beta \in [-1, 0]$ is a constant

In the presence of an electrical double layer, $\beta < 0$ may be a reasonable consideration. Now, dropping the superscript ’ * ’ in the dimensionless variables and keeping in mind that electro-osmotic flow occurring in many situations, including physiological processes usually exhibit oscillatory characteristics, we consider

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$u = u_s e^{i\omega t};$ (14)

$N = N_s e^{i\omega t}$ (15)

ω, being the angular velocity of oscillation and using (12) and (15), from equation (7) and (8), we have

$$i\omega R u_s = B + (1+\chi)\frac{d^2u_s}{dy^2} + \chi\frac{dN_s}{dy} + m^2\frac{\cosh(my)}{\cosh(m)} \quad (16)$$

and

$$i\omega R N_s = \left(1+\frac{K}{2}\right)\frac{d^2N_s}{dy^2} - \chi\left(2N_s + \frac{du_s}{dy}\right) \quad (17)$$

Considering that the Reynolds number R for flows in a micro-channel, is very small, we develop a perturbation technique by writing

$$u_s = u_{s0} + Ru_{s1} + R^2u_{s2} + \cdots. \quad (18)$$

and

$$N_s = N_{s0} + RN_{s1} + R^2N_{s2} + \cdots. \quad (19)$$

The boundary conditions for the variables u_{s0}, u_{s1}, N_{s0} and N_{s1} associated with the fluid velocity and micro-rotation of the micro-particles can then be written in the form

$$\frac{du_{s0}}{dy} = 0; \quad \frac{du_{s1}}{dy} = 0; \quad N_{s0} = 0; \; N_{s1} = 0 \quad \text{at } y = 0 \quad (20)$$

and

$$u_{s0=u_0}; \; u_{s1} = 0; N_{s0} = \beta\frac{du_{s0}}{dy}; N_{s1} = \beta\frac{du_{s1}}{dy} \text{ at } y = 1 \quad (21)$$

Substituting the expansion of u_s and N_s from Eq. (18) and (19) into Eq. (16) and (17), equating like powers of R and ignoring quadratic and higher powers of R, we have computed the velocity and micro-rotation for the unsteady electro-osmotic flow of a micropolar fluid, by the use of the software MATHEMATICA. The numerical results are presented in the next section for an illustrative example.

IV. NUMERICAL ESTIMATES OF FLOW IN MICRO-BIO-FLUIDIC DEVICES

Attention is given, in particular, to determine the estimates for blood flow and for that the range of values of different parameter are taken in commensurate with that of blood.

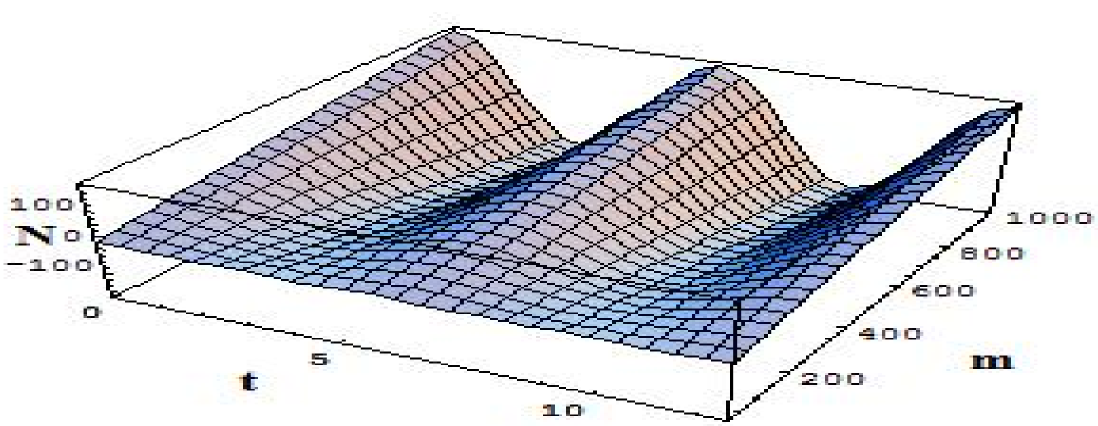

Fig. 1 Variation in micro-rotation (N) with t near the channel wall (y=0.8) for different values of m, where $B = 20{:}0;\ \chi = 4.0;\ \beta = -0.5;\ u_0 = 1.0;\ R = 0{:}002;\ \omega = 20.0;$ and $\omega_e = 25.0$

Figs. 1 and 2 show that the amplitudes of oscillation are increased with the increase in Debye-Hückel parameter m. Debye-Hückel parameter m increases with the increase in the width of the channel, it implies that any rise in height of the channel may cause an enhancement in the amplitudes of oscillation for both the velocity and microrotation.

Figs 3 and 4 reveal the enhancement of blood velocity and microrotation of erythrocytes with the increase in m. It is also noted that the velocity gradient is higher in the midpoint of the channel, whereas microrotation gradient is much more near the wall. So, the velocity gradient gradually decreases as we go from the midpoint of the channel to the boundary. The thickness of the electrical double layer is actually the Debye Length and is inversely proportional to the square root of the ion concentration. So, a lower value of the concentration of the ions gives rise to a reduction in the magnitude of the body forces causing thereby reduction in bulk flow.

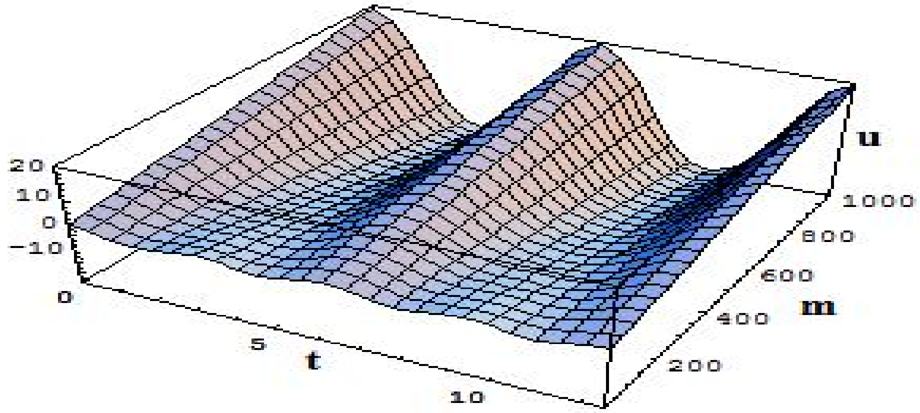

Fig. 2 Time variation of velocity at y=0.9 for different values of m, when $B = 20{:}0;\ \chi = 4.0;\ \beta = -0.5;\ u_0 = 1.0;\ R = 0.002;\ \omega = 20{:}0;$ and $\omega_e = 50.0$

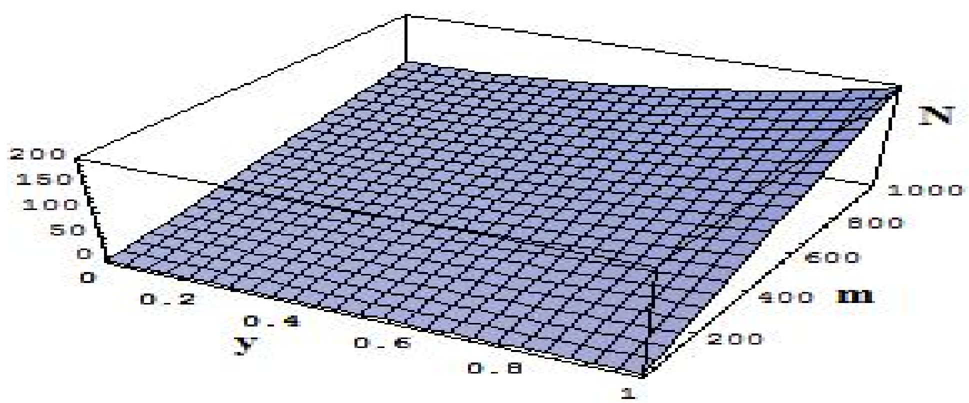

Fig. 3 Distribution of N in the case of steady flow, for different values of m, when $B = 20{:}0;\ \chi = 4.0;\ \beta = -0.5;\ u_0 = 1.0;\ R = 0.002$

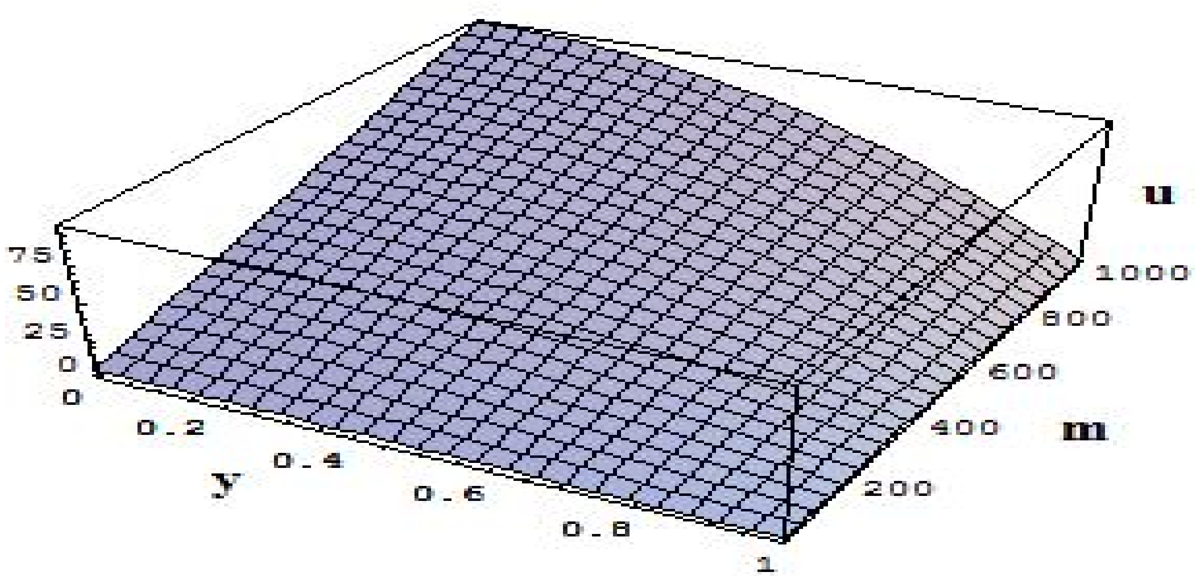

Fig. 4 Distribution of u for different values of m, when the flow is steady and $B = 20{:}0;\ \chi = 4.0;\ \beta = -0.5;\ u_0 = 1.0;\ R = 0.002.$

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V. CONCLUSION

The paper analyses a boundary value problem during unsteady electroosmotic flow of a micropolar fluid in a rectangular micro channel. Here, it is important to note that the frequency of oscillation of the channel walls is taken as different from the frequency of oscillation of the alternating electric field. The dynamics of flow is estimated using the approximation in which the non-dimensional Debye- Hückel parameter m is taken in between 10 and 1000. It is concluded from the graph that with the enhancement in channel height, both micro-rotation of microparticles and flow velocity are enhanced when the Debye layer thickness does not change.

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