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Double Partitioned Ranked Set Sampling: An Efficient Estimation Technique

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Abstract: Following Ranked Set Sampling (RSS) due to McIntyre (1952), Takahasi and Wakimoto (1968), Dell and Clutter (1972) and the Median Ranked Set Sampling (MRSS) method by Muttlak(1997), a new sampling strategy has been proposed. While the newly proposed sampling design is called Double Partitioned Ranked Set Sampling(DPRSS), the estimator based thereon, besides being unbiased for the population mean, is found to be more efficient than the corresponding estimators in simple random sampling, ranked set sampling and median ranked set sampling. The theoretical findings have been supported by suitable numerical illustration.
Keywords: Simple Random Sampling, Ranked Set Sampling, Median Ranked Set Sampling, Double Ranked Set Sampling, Double Partitioned Ranked Set Sampling.

I. INTRODUCTION

McIntyre (1952) introduced a technique of sampling called Ranked Set Sampling (RSS) for estimating the mean of a finite population. This is possible where the sampling units in a survey can be more easily ranked than quantified. The estimator thus obtained comes out to be unbiased for population mean with a variance less than that of usual sample mean based on Simple Random Sampling of the same size. Muttalak (1997) proposed an estimator using Median Ranked Set Sampling(MRSS) with a view to increasing efficiency of the estimator and reducing errors in ranking. Muttalak(2003) proposed Quartile Ranked Set Sampling (QRSS) for estimating population mean is also applicable for reducing error as compared to RSS. Al-Saleh and Al-Kadiri (2000) suggested Double Ranked Set Sampling (DRSS) for estimating the population mean. According to them the ranking at second stage is easier than first stage.

II. SAMPLING METHODS

A. Ranked Set Sampling (RSS)

RSS procedure involves selection of m random samples with m units in each sample. The m units in each sample are ranked with respect to a variable of interest without actually measuring them. Then the smallest rank is measured from the first sample, the second smallest rank from second sample and the procedure is continued till the unit with highest rank is measured from the m^{th}sample.

Theorder of observations from the lowest to the highest in the m samples can be presented as

$$\begin{array}{lll} x_{(11)} & x_{(12)} & \ldots\ldots\ldots\ldots\ldots\ x_{(1m)} \\ x_{(21)} & x_{(22)} & \ldots\ldots\ldots\ldots\ldots\ x_{(2m)} \\ x_{(m1)} & x_{(m2)} & \ldots\ldots\ldots\ldots\ldots\ x_{(mm)} \end{array}$$

The observations $x_{(11)}, x_{(22)}, \ldots\ldots\ldots\ldots, x_{(mm)}$ are then accurately measured to form RSS data. If m is small, then the cycle may be repeated for r times so as to obtain a combinedsampleofsizemr.

B. Median Ranked Set Sampling (MRSS)

MRSS procedure involves selection of m random samples each of size m units from population and ranked them within each sample. If sample size m is odd, then select lowest ranks from each of the first (m-1)/2samples, the median from (m+1)/2th sample and the highest ranks from each of the last (m-1)/2 sample. If sample size m is even, then select lowest rank from each ofthe first m/2 samples and highest rank from each of the lastm/2 samples. If m is small, then the cycle may be repeated for r times to have a combined sample of sizemr. The ranked units are then quantified.

C. Partitioned Ranked Set Sampling (PRSS)

According to PRSS procedure,select m^2 units from the population and then divide them into m sets each of having size m.If sample size is odd, then select $(p(m + 1))^{th}$ rank from first (m-1/2) sets and $(q(m + 1))^{th}$ rank from last (m-1/2) sets, with median from (m+1/2) set.If sample size is even, then select (p(m + 1))th rank from first (m/2) sets and (q(m+1))th rank from last (m/2) sets, where 0≤ p ≤1 and q = (1- p) then p+q=1.If m is small, then the cycle may be repeated for r times to have a combined sample of size mr.

D. Double Ranked Set Sampling (DRSS)

According to DRSS, select m^3 elements from a population and divide these units randomly into m^2sets eachof size m units.Apply the RSS procedure on each set to obtain m ranked set samples each of size m. Then repeat the RSS procedure again on m ranked set sample to have DRSS of size m. Then repeat the cycle for r times as per requirement.Motivated by the above works, we have proposed a new RSS technique, called Double Partitioned Ranked Set Sampling(DPRSS)along with an unbiased estimator for the population mean.The estimator thus proposed fares better than its competing estimators based on SRS, RSS and MRSS. It may be pointed out here that the proposed sampling technique can be viewed as a generalisation of Double Quartile Ranked Set Sampling (DQRSS).

E. Double Partitioned Ranked Set Sampling (DPRSS)

DPRSS technique comprises the following steps:

1) Select m^3 elements from the target population and divide these elements randomly into m^2 sets each of size m.
2) If sample size m is even,select the (p(m + 1))th rank from each set out of first $m^2/2$ samples and from the second $m^2/2$ samples the (q (m + 1))th rank from each set is selected.
3) If sample size m is odd, select from the first (m(m-1)/2) samples, the (p(m + 1))th rank from each set, the median from next m samples, and from last (m(m-1)/2) samples, select (q (m + 1))th rank from each set.
4) Here, p & q stand for pth and qthpartitioned observations, such that p + q = 1, for example p = 25% and q = 75% of the observations given. This can be done after arranging the series either inascending or in descending order visually.
5) Applying PRSS procedure on m sets obtained in above step, gives us DPRSS procedure sample of size m.
6) The whole cycle may be repeated r times to obtain a sample size of mr from DPRSS.
7) From above we have to examine mr samples out of m^3r population size using DPRSS.

Here, we have to remember that, the ranking should be done by visual inspection or by any economical procedure and actual quantification is done at final stage.

To understand the above procedure, let us consider the following two example.

F. Example-1(when sample size is odd)

For odd sample size, we have to apply DPRSSO method which may be described as follows.

Let m=5, then we have to select random sample of 25 sets, each should contain 5 units. Let $X^{(n)}_{j(i;m)}$ be the ith value (i=1,2,...,5) out of the jthset (1,2,.......25) at the nthstage.

After ranking, the units within each subset may be taken as

$$X_1^{(0)} = \{X^{(0)}_{1(1;5)}, X^{(0)}_{1(2;5)} \ldots\ldots\ldots\ldots X^{(0)}_{1(5;5)}\},$$

$$X_2^{(0)} = \{X^{(0)}_{2(1;5)}, X^{(0)}_{2(2;5)} \ldots\ldots\ldots\ldots X^{(0)}_{2(5;5)}\},$$

$$X_{25}^{(0)} = \{X^{(0)}_{25(1;5)}, X^{(0)}_{25(2;5)} \ldots\ldots\ldots\ldots X^{(0)}_{25(5;5)}\}$$

Now applying PRSSO method on each 25 sets, The first partitioned value p(m+1)th (for p=25%) = 25% (5+1)th= 1.5th observation, which indicates the first or lowest observation, i.e., we have to assume p(m+1)thrank from each of first m(m-)/2=10sets.Similarly, the last partitioned value q(m+1)th (for q=75%)= 75%(5+1)th=4.5th observation indicates the fifth observation or largest rank from each of last 10 sets and median of each 5 sets containing 5 units will give middle 5 observations for next stage.

Using the above procedure, we arrive at

$$X_{1\,(1;5)}^{(1)} = \min\left(X_1^{(0)}\right),$$

$$X_{10\,(1;5)}^{(1)} = \min\left(X_{10}^{(0)}\right),$$

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$$X_{11}{}^{(1)}_{(M;5)} = \text{median}\left(X_{11}{}^{(0)}\right),$$

$$X_{12}{}^{(1)}_{(M;5)} = \text{median}\left(X_{12}{}^{(0)}\right),$$

.

.

$$X_{15}{}^{(1)}_{(M;5)} = \text{median}\left(X_{15}{}^{(0)}\right),$$

$$X_{16}{}^{(1)}_{(5;5)} = \max\left(X_{16}{}^{(0)}\right),$$

$$X_{17}{}^{(1)}_{(5;5)} = \max\left(X_{17}{}^{(0)}\right),$$

..and $X_{25}{}^{(1)}_{(5;5)} = \max\left(X_{25}{}^{(0)}\right)$

The above observations t can be reorganised in the following 5 sets

$$X_1{}^{(1)} = \left\{X_{1(1;5)}^{(1)}, X_{2(1;5)}^{(1)} \ldots\ldots, X_{5(1;5)}^{(1)}\right\},$$

$$X_2{}^{(1)} = \left\{X_{6(1;5)}^{(1)}, X_{7(1;5)}^{(1)} \ldots\ldots, X_{10(1;5)}^{(1)}\right\},$$

$$X_3{}^{(1)} = \left\{X_{11(M;5)}^{(1)}, X_{12(M;5)}^{(1)} \ldots, X_{15(M;5)}^{(1)}\right\},$$

$$X_4{}^{(1)} = \left\{X_{16(5;5)}^{(1)}, X_{17(5;5)}^{(1)} \ldots\ldots, X_{20(5;5)}^{(1)}\right\},$$

$$X_5{}^{(1)} = \left\{X_{21(5;5)}^{(1)}, X_{22(5;5)}^{(1)} \ldots\ldots X_{25(5;5)}^{(1)}\right\}$$

Now, applying the same procedure once again to the above data, we get DPRSSO technique which will have p(m+1) th rank from (m-1)/2= 2 sets and choose q(m+1)th = highest rank from last 2 set and the median from middle set. Then DPRSSO partitioned sample is

$$X_1{}^{(2)}_{(1;5)} = \min\left(X_1{}^{(1)}\right),$$

$$X_2{}^{(2)}_{(1;5)} = \min\left(X_2{}^{(1)}\right),$$

$$X_3{}^{(2)}_{(M;5)} = \text{median}\left(X_3{}^{(1)}\right),$$

$$X_4{}^{(2)}_{(5;5)} = \max\left(X_4{}^{(1)}\right)$$

and $X_5{}^{(2)}_{(5;5)} = \max\left(X_5{}^{(1)}\right)$

The sample observations thus obtained constitute a random sample, i.e., the observations are the realisation of 5 i.i.d. random variables. These 5 observation are to be measured.

Let m=6, Hence we have to select 6^3=216 units in 36 sets, each have 6 units. Let us assume that, $X^{(n)}_{j(i;m)}$ be the ith observation(i=1,2,....,6) out of the jthset (j=1,2,.......36) at the nthstage.

After arranging, the units within each sets, we have

$$X_1{}^{(0)} = \left\{X_{1(1;6)}^{(0)}, X_{1(2;6)}^{(0)} \ldots\ldots X_{1(6;6)}^{(0)}\right\},$$

$$X_2{}^{(0)} = \left\{X_{2(1;6)}^{(0)}, X_{2(2;6)}^{(0)} \ldots\ldots X_{2(6;6)}^{(0)}\right\},$$

$$X_{36}{}^{(0)} = \left\{X_{36(1;6)}^{(0)}, X_{36(2;6)}^{(0)} \ldots\ldots X_{36(6;6)}^{(0)}\right\}.$$

Now, applying of PRSSE method on each of 36 sets, The first partitioned values $(p(m+1))^{th}$observation,i.e., the lowest rank from each offirst $m^2/2=18$ sets. The last partitioned value $q(m+1)^{th}$rank,i.e,largest rank from each of last $m^2/2=18$ observations.

Using the above procedure, we have

$$X_{1(1;5)}^{(1)} = \min\left(X_1^{(0)}\right),$$

$$X_{2(1;5)}^{(1)} = \min\left(X_2^{(0)}\right),$$

.

$$X_{18(1;6)}^{(1)} = \min\left(X_{18}^{(0)}\right),$$

$$X_{19(6;6)}^{(1)} = \max\left(X_{19}^{(0)}\right),$$

$$X_{20(6;6)}^{(1)} = \max\left(X_{20}^{(0)}\right),$$

and $X_{36(6;6)}^{(1)} = \max\left(X_{36}^{(0)}\right)$

The obtained values can be rearranged in the following 5 sets

$$X_1^{(1)} = \left\{X_{1(1;6)}^{(1)}, X_{2(1;6)}^{(1)} \ldots\ldots X_{6(1;6)}^{(1)}\right\},$$

$$X_2^{(1)} = \left\{X_{7(1;6)}^{(1)}, X_{8(1;6)}^{(1)} \ldots\ldots X_{12(1;6)}^{(1)}\right\},$$

$$X_3^{(1)} = \left\{X_{13(1;6)}^{(1)}, X_{14(1;6)}^{(1)} \ldots\ldots X_{18(1;6)}^{(1)}\right\},$$

$$X_4^{(1)} = \left\{X_{19(6;6)}^{(1)}, X_{20(6;6)}^{(1)} \ldots\ldots X_{24(6;6)}^{(1)}\right\},$$

$$X_5^{(1)} = \left\{X_{25(6;6)}^{(1)}, X_{26(6;6)}^{(1)} \ldots\ldots X_{30(6;6)}^{(1)}\right\},$$

$$X_6^{(1)} = \left\{X_{31(6;6)}^{(1)}, X_{32(6;6)}^{(1)} \ldots\ldots X_{36(6;6)}^{(1)}\right\}$$

Now , applying the same procedure once again to the above data, we have $p(m+1)^{th}$,i.e., smallest rank out of first m/2= 3 sets and $q(m+1)^{th}$, i.e., highest observations from last 3 sets. Then

$$X_{1(1;6)}^{(2)} = \min\left(X_1^{(1)}\right),$$

$$X_{2(1;6)}^{(2)} = \min\left(X_2^{(1)}\right),$$

$$X_{3(1;6)}^{(2)} = \min\left(X_3^{(1)}\right),$$

$$X_{4(6;6)}^{(2)} = \max\left(X_4^{(1)}\right),$$

$$X_{5(6;6)}^{(2)} = \max\left(X_5^{(1)}\right),$$

$$X_{6(6;6)}^{(2)} = \max\left(X_6^{(1)}\right)$$

The sample observations thus obtained constitute a random sample, i.e., the observations are the realisation of 6 i.i.d. random variables. These 5 observation are to be measured.

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III. GENERAL SET UP AND SOME BASIC RESULTS:

Let $X_{11}, X_{12}, \ldots\ldots X_{1m};$

$X_{21}, X_{22}, \ldots\ldots X_{2m};$

$X_{m^2 1}, X_{m^2 2}\ldots\ldots X_{m^2 m};$

be m^2 independent random sets of size m.

Let us assume that, each variable X_{ij} has common distribution functioncdf F(x) with probability density function pdf f(x) having mean μ and variance σ^2 respectively.Let $X_{i(1)}, X_{i(2)}\ldots X_{i(m)}$, where ($i = 1, 2\ldots m^2$) be the ordered statistics of the i^{th} sample $X_{i1}, X_{i2}, \ldots X_{im}$($i = 1.2 \ldots m^2$)

The SRS estimator of the population mean from a sample size m is given by,

$$\bar{X}_{SRS} = \frac{1}{m}\sum_{i=1}^{m} X_i \text{, with variance } \sigma^2/m. \tag{3.1}$$

The estimator of the population mean for RSS of size m(McIntyre (1952)) is given by,

$$\bar{X}_{RSS} = \frac{1}{m}\sum_{i=1}^{m} X_{i(i;m)}$$

and $$Var(\bar{X}_{RSS}) = \frac{1}{m^2}\sum_{i=1}^{m} \text{var}(X_{i(i;m)})$$

$$= \frac{\sigma^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}(\mu_{i(i;m)} - \mu)^2 \tag{3.2}$$

since $\sum_{i=1}^{m}(\mu_{i(i;m)} - \mu)^2 \geq 0$, $\bar{X}_{RSS}$ is more efficient than $\bar{X}_{SRS}$ based on same number of measured observations.

The DRSS estimator of population mean from a sample of size m(Al-Saleh and Al-Omari (2002)) is given by

$$\bar{X}^{(2)}_{DRSS} = \frac{1}{m}\sum_{i=1}^{m} X_i^{(2)}$$

and $$Var(\bar{X}^{(2)}_{DRSS}) = \frac{1}{m^2}\sum_{i=1}^{m} \text{var}(X^{(2)}_{i(i;m)}) = \frac{1}{m}[\sigma^2 - \frac{1}{m}\sum_{i=1}^{m}(\mu_i^{(2)} - \mu)^2] \tag{3.3}$$

where μ and σ^2 are the mean and the variance of the population respectively.

It is interest to have attention that theDRSS method is suggested by Al-Saleh and Al-Omari (2002)constitude by apply the usual RSS method on m^2sets each of size m, which is difference from our work based on DPRSS technique where we apply PRSS method on m^2 sets each of size m.

To estimate the population mean using DPRSS method,

Suppose,atK^{th} cycle, for (K = 1, 2 .... r),

A. For even sample size, let $X^{(2)}{}_{i\,(p(m+1)k)}$ be the first partitioned values for the i sets(i = 1, 2 .....,l ; l = m/2) and $X^{(2)}{}_{j\,(q\,(m+1)\,k)}$ be the last partitioned value for the jsets (j = l + 1, ...m). Then the partitioned sample,

$[X^{(2)}_{1(p(m+1))k,}, X^{(2)}_{2(p(m+1))k,}, \ldots.., X^{(2)}_{\frac{m}{2}(p(m+1))k,}][X^{(2)}_{\frac{m}{2}+1(q(m+1))k,}, X^{(2)}_{\frac{m}{2}+2(q(m+1))k,}, \ldots.., X^{(2)}_{m(q(m+1))k,}]$units are i.i.d., however, all units are mutually independent but not identically distributed. These measured units are DPRSSE(Double Partitioned Ranked Set Sampling even Size). (3.4)

B. If the sample size is odd. let $X^{(2)}{}_{i\,(p(m+1)k)}$ be the first partitioned values of the i sets(i = 1, 2.....h; h=(m-1)/2),wit

$X^{(2)}{}_{m\left(\frac{m+1}{2}\right)k}$ is the median and $X^{(2)}{}_{j(q(m+1)k)}$ be the last partitioned values for the jsets (j = h + 2, ...m). Then the partitioned samples are

$[X^{(2)}_{1(p(m+1))k}, X^{(2)}_{2(p(m+1))k}, \ldots, X^{(2)}_{h(p(m+1))k}], [X^{(2)}_{(h+1)(m+1)k}]$, $[X^{(2)}_{(h+2)(q(m+1))k}, X^{(2)}_{(h+3)(q(m+1))k}, \ldots, X^{(2)}_{m(q(m+1))k}]$ units are i.i.d., however, all units are mutually independent but not identically distributed. These measured units are DPRSSO(Double Partitioned Ranked Set Sampling odd Size). (3.5)

The estimators of the population mean using DPRSS for sample size even and odd respectively are given by,

$$\bar{X}^{(2)}_{DPRSSE} = \frac{1}{m}\left(\sum_{i=1}^{l} X^{(2)}_{i(p(m+1);m)} + \sum_{j=l+1}^{m} X^{(2)}_{j(q(m+1);m)}\right), \text{ where } l=m/2 \tag{3.6}$$

$$\bar{X}^{(2)}_{DPRSSO} = \frac{1}{m}\left(\sum_{i=1}^{h} X^{(2)}_{i(p(m+1);m)} + \sum_{j=h+2}^{m} X^{(2)}_{j(q(m+1);m)} + X^{(2)}_{((h+1);m)}\right), \text{ where } h=(m-1)/2 \tag{3.7}$$

The variance of $\bar{X}^{(2)}_{DPRSSE}$ and $\bar{X}^{(2)}_{DPRSSO}$ respectively are given by,

$$\begin{aligned} Var(\bar{X}^{(2)}_{DPRSSE}) &= \frac{1}{m^2}\left(\sum_{i=1}^{l} \operatorname{var}(\bar{X}^{(2)}_{i(p(m+1);m)}) + \sum_{j=l+1}^{m} \operatorname{var}(\bar{X}^{(2)}_{j(q(m+1);m)})\right) \\ &= \frac{1}{m^2}\left(\frac{m}{2}\operatorname{var}^{(2)}_{(p(m+1);m)} + \frac{m}{2}\operatorname{var}^{(2)}_{(q(m+1);m)}\right) \\ &= \frac{1}{2m}\left(\operatorname{var}^{(2)}_{(p(m+1);m)} + \operatorname{var}^{(2)}_{(q(m+1);m)}\right) \end{aligned} \tag{3.8}$$

$$\begin{aligned} Var(\bar{X}^{(2)}_{DPRSSO}) &= \frac{1}{m^2}\left(\sum_{i=1}^{h} \operatorname{var}(\bar{X}^{(2)}_{i(p(m+1);m)}) + \sum_{j=h+1}^{m} \operatorname{var}(\bar{X}^{(2)}_{j(q(m+1);m)}) + \operatorname{var}^{(2)}_{((h+1);m)}\right) \\ &= \frac{1}{m^2}\left(\frac{m-1}{2}.\operatorname{var}^{(2)}_{(p(m+1);m)} + \frac{m-1}{2}.\operatorname{var}^{(2)}_{(q(m+1);m)}\right) + \frac{1}{m^2}\operatorname{var}(X^{(2)}_{(h+1);m}) \\ &= \frac{m-1}{2m^2}\left(\operatorname{var}^{(2)}_{(p(m+1);m)} + \operatorname{var}^{(2)}_{(q(m+1);m)}\right) + \frac{1}{m^2}\operatorname{var}(X^{(2)}_{(h+1);m}) \end{aligned} \tag{3.9}$$

The properties of DPRSS estimators are

If the parent distribution is symmetric about mean μ, then

The DPRSS estimator is unbiased about population mean.

$$Var(\bar{X}^{(2)}_{DPRSS}) < Var(\bar{X}_{RSS}) < Var(\bar{X}_{SRS})$$

If the underlying distribution is asymmetric about mean μ, then it is found that

$MSE(\bar{X}^{(2)}_{DPRSS}) < \operatorname{var}(\bar{X}_{RSS}) < \operatorname{var}(\bar{X}_{SRS})$, where, MSE is the mean square error and

$$MSE(\bar{X}^{(2)}_{DPRSS}) = \operatorname{var}(\bar{X}^{(2)}_{DPRSS}) + (bias(\bar{X}^{(2)}_{DPRSS}))^2$$

IV. COMPARISION OF ESTIMATORS

We can compare the three estimators for μ based on RSS, MRSS and DPRSS procedures. For this purpose, we define the following Relative Precisions (RP).

A. For rss

$$RP_1 = \frac{\operatorname{Var}(\bar{\mu})}{\operatorname{Var}(\hat{\mu})}, \text{ if } \hat{\mu} \text{ is an unbiased estimator}$$

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$$= \frac{\mathrm{Var}(\bar{\mu})}{\mathrm{MSE}(\hat{\mu})}, \text{ if } \hat{\mu} \text{ is a biased estimator}$$

B. For mrss

$$\mathrm{R\,P}_2 = \frac{\mathrm{Var}(\bar{\mu})}{\mathrm{Var}(\mu^{(1)})}, \text{ if } \mu^{(1)} \text{ is a an unbiased estimator}$$

$$= \frac{\mathrm{Var}(\bar{\mu})}{\mathrm{MSE}(\mu^{(1)})}, \text{ if } \mu^{(1)} \text{ is a biased estimator}$$

C. For dprss

$$\mathrm{R\,P}_3 = \frac{\mathrm{Var}(\bar{\mu})}{\mathrm{Var}(\mu^{(2)})}, \text{ if } \mu^{(2)} \text{ is an unbiased estimator}$$

$$= \frac{\mathrm{Var}(\bar{\mu})}{\mathrm{MSE}(\mu^{(2)})}, \text{ if } \mu^{(2)} \text{ is a biased estimator}$$

$$\text{as } \mathrm{MSE}(\mu^{(2)}) = \mathrm{Var}(\mu^{(2)}) + (\mathrm{bias})^2$$

As we know from above results, there is no biased in population mean in case of symmetric distributions, we have to examine the PR for symmetric and asymmetric distribution. Table-1 shows the PR for 10 symmetric and asymmetric distributions for m=6, 7, 11, 12 for each simulation 50,000 iterations are performed for p=25%.

Table-1: PR efficiency for RSS, MRSS and DPRSS of 25% w.r.t. SRS with sample size 6,7, 11 and 12

Distribution	m	RSS		MRSS		DPRSS	
			Bias		Bias		bias
Uniform(0,1)	6	3.400	0.000	3.114	0.000	14.966	0.000
	7	3.815	0.000	3.706	0.000	21.332	0.000
	11	6.213	0.000	5.617	0.000	45.425	0.000
	12	6.500	0.000	6.649	0.000	64.737	0.000
Uniform(0,2)	6	3.400	0.000	3.132	0.000	15.167	0.000
	7	3.815	0.000	3.671	0.000	22.021	0.000
	11	6.6213	0.000	5.632	0.000	45.213	0.000
	12	6.503	0.000	6.651	0.000	65.135	0.000
Normal(0,1)	6	3.191	0.000	3.593	0.000	10.609	0.000
	7	3.585	0.000	3.927	0.000	17.809	0.000
	11	5.112	0.000	5.980	0.000	31.127	0.000
	12	5.237	0.000	6.127	0.000	36.426	0.000
Normal(1,2)	6	3.110	0.000	3.445	0.000	10.952	0.000
	7	3.535	0.000	4.251	0.000	13.359	0.000
	11	5.195	0.000	6.240	0.000	35.046	0.000
	12	5.652	0.000	6.412	0.000	36.958	0.000
Logistic(-1,1)	6	2.668	0.000	3.592	0.000	11.207	0.000
	7	3.243	0.000	4.112	0.000	12.428	0.000
	11	4.599	0.000	6.755	0.000	34.804	0.000
	12	4.911	0.000	6.728	0.000	34.315	0.000
Exponential(1)	6	2.135	0.000	2.995	0.219	9.294	0.007
	7	2.564	0.000	3.213	0.049	8.219	0.015
	11	3.671	0.000	3.542	0.105	28.555	0001
	12	3.922	0.000	4.693	0.061	8.303	0.083

Exponential(2)	6	2.207	0.000	3.122	0.168	8.372	0.015
	7	2.476	0.000	2.751	0.013	8.598	0.029
	11	3.659	0.000	3.521	0.053	28.406	0.000
	12	3.962	0.000	4.735	0.031	8.409	0.042
Gamma(1,2)	6	2.218	0.000	3.022	0.183	9.395	0.033
	7	2.537	0.000	3.111	0.012	8.135	0.178
	11	3.638	0.000	3.539	0.314	28.510	0.02
	12	3.990	0.000	4.711	0.184	8.350	0.250
Gamma(1,3)	6	2.416	0.000	3.025	0.0279	9.572	0.148
	7	2.669	0.000	3.282	0.023	8.496	0.047
	11	3.728	0.000	3.594	0.210	28.877	0.001
	12	3.918	0.000	4.697	0.123	8.372	0.167
Weibull(1,3)	6	2.459	0.000	3.029	0.274	9.660	0.047
	7	2.755	0.000	3.334	0.227	8.503	0.178
	11	3.699	0.000	3.576	0.313	28.675	0.002
	12	3.960	0.000	4.751	0.158	8.480	0.295

From above, we get the following information

A gain in efficiency attainted using DPRSS for estimation population mean for symmetric distribution. As example for N(1,2) with m=12, the relative efficiency of the DPRSS is 36.958 comparing it, with RSS and MRSS 5.652 and 6.412.

For asymmetric asymmetric distribution, gain in efficiency is attainted with smaller bias using DPRSS. for example, for Weibull with m=12, the relative efficiency of DPRSS is 8.480 with bias 0.249 for estimating population mean having parameter1 and 3, comparing with RSS and MRSS is 3.960 and 4.751 with bias 0.185.

V. RELATIVE SAVING

From relative precision, we have

$$RP_2 = \frac{Var(SRS)}{Var(DPRSS)} = \frac{Var(\hat{\mu})}{Var(\hat{\mu}^{(2)})} = \frac{\sigma^2/m}{\sigma^2/m - \frac{1}{m^2}\sum_{i-1}^{m}(\mu_{(i)}{}^{(2)} - \mu_{(i)}^{(1)})^2 - \frac{1}{m^2}\sum_{i-1}^{m}(\mu_{(i)}^{(1)} - \mu)^2}$$

$$= \frac{1}{1 - \frac{1}{\sigma^2}\left[\frac{1}{m}\sum_{i=1}^{m}(\mu_{(i)}^{(2)} - \mu_{(i)}^{(1)})^2 + \frac{1}{m}\sum_{i=1}^{m}(\mu_{(i)}^{(1)} - \mu)^2\right]} = \frac{1}{1 - RS^*}$$

Where, $RS^* = \frac{1}{m\sigma^2}\left[\sum_{i=1}^{m}(\mu_{(i)}^{(2)} - \mu_{(i)}^{(1)})^2 + \sum_{i=1}^{m}(\mu_{(i)}^{(1)} - \mu)^2\right]$

$= \frac{Var(\hat{\mu}) - Var(\hat{\mu}^{(2)})}{Var(\hat{\mu})}$ is called relative saving(RS) for DPRSS. (5.1)

Similarly ,we can have RS for RSS

$$RS = \frac{1}{m\sigma^2}\left[\sum_{i=1}^{m}(\mu_{(i)}^{(1)} - \mu)^2\right] \quad (5.2)$$

Hence, comparing RS^* and RS, we have

Relative Saving for DPRSS > Relative Saving for RSS

VI. APPLICATION TO REAL DATA SET

For the performance of mean estimation using a collection of real data set, which consists of the olive yield of each of 64 trees(for more details see Al-Saleh and Al-Omari (2002)). In this study, balanced ranked set sampling is considered. All the sampling done without replacement using the statistical programming 'R'. we obtained the mean and variance of sample mean using SRS, RSS, DPRSS technique using sample size m=3,4,5. We compare the averages of 70,000 sample estimate.

Let,$t_i$be the olive yield of the ith tree i-1,2,...,64. The mean μ, and the variance σ^2 of the population, respectively, are ,

$$\mu = \frac{1}{64}\sum_{i=1}^{64} t_i = 9.766 kg/tree$$

$$\sigma^2 = \frac{1}{64}\sum_{i=1}^{64}(t_i-\mu)^2 = 26.114 kg^2/tree$$

The skewness of the population is 0.475, indicates positively skewed ,i.e., asymmetrical distribution. Hence, we have to find out $MSE(\bar{X}^{(2)}_{DPRSS})$ and efficiency values of $\bar{X}^{(2)}_{RSS}$ and $\bar{X}^{(2)}_{DPRSS}$ relative to $\bar{X}^{(2)}_{SRS}$ for m=3, 4, 5.

TABLE 2: The efficiency values of RSS and DPRSS relative to SRS with sample size m=3,4,5

Sample size	methods	mean	Variance		efficicency
m=3	SRS	9.787	8.344		-
	RSS	9.784	4.294		1.954
	DPRSS	10.185	MSE	BIAS	4.741
			1.760	0.407	
m=4	SRS	9.784	6.159		-
	RSS	9.775	2.564		2.383
	DPRSS	10.899	MSE	BIAS	2.960
			2.070	1.271	
m=5	SRS	9.777	4.843		-
	RSS	9.775	1.696		2.870
	DPRSS	9.852	MSE	BIAS	8.061
			0.598	0.111	

On the basis of above table, the DPRSS mean at any stage is close to the population mean 9.766, and there is a bias along with MSE as it is a asymmetrical distribution. Hence, DPRSS is much more efficient than SRS, RSS.

VII. SAMPLING WITH ERROR IN RANKING

In RSS, sampling mean is unbiased estimator of population mean without any proper information that, whether it is perfect or imperfect. Hence, it has a smaller variance as compared with SRS having same sample size. So Muttalak(2003) showed that QRSS with error in ranking is unbiased estimator of population mean with assumption that population is symmetric about its mean. Hence applying the above with DPRSS method in ranking with error may be defined as follows,

Let $Y^{(2)}_{i(p(m+1);m)}$ and $Y^{(2)}_{i(q(m+1);m)}$ be the first and lastjudgement double partitioned value ofi^{th} sample (i=1,2,.....,m) having errors in ranking.Then using DPRSS technique, the estimator of population mean with error in ranking canbe represented as

$$\left\{ \hat{Y}^{(2)}_{DPRSSE_e} = \frac{1}{mr}\sum_{k=1}^{r}\left(\sum_{i=1}^{l} X^{(2)}_{i(p(m+1))k} + \sum_{i=l+1}^{m} X^{(2)}_{i(q(m+1))k}\right), \qquad l = m/2 \right.$$

$$\hat{Y}^{(2)}_{DPRSS_e} = \qquad \hat{Y}^{(2)}_{DPRSSO_e} = \frac{1}{mr}\sum_{k=1}^{r}((\sum_{i=1}^{h} X^{(2)}_{i(p(m+1))k} + \sum_{i=h+2}^{m} X^{(2)}_{i(q(m+1))k}) + X^{(2)}_{(h+1)[(m+1)/2]k}), \qquad h=(m-1)/2$$

The estimator of population mean μ in ranking with error having following properties,

$\hat{Y}^{(2)}_{DPRSS_e}$ with ranking in error is unbiased estimator of population mean with assumption that population is symmetric about its mean.

Var($\hat{Y}^{(2)}_{DPRSS_e}$) <Var (SRS) for symmetric distribution

and for asymmetric distribution about its mean, MSE($\hat{Y}^{(2)}_{DPRSS_e}$) <Var(SRS) for ranking in error.

The above properties can be proved based on Takahasi and Wakimottto(1968), Dell andClutter (1972), Muttalak (2003) and Al-saleh and Al-kadiri(2000).

VIII. CONCLUSION

In this article, it is observed that, the estimator of proposed Double Partitioned Ranked Set Sampling (DPRSS)is unbiased for population mean and is more efficient than SRS,RSS in case of Symmetrical distribution. From NPR analysis, it isfound that there is greater efficiency with smaller bias in case of estimating of population mean using DPRSS method for asymmetrical distribution. Again, using relative saving method, DPRSS has Greater RS as Compared with RSS.

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APPENDIX

A. *Corollary-1:*

Let X_{ij}be the values assumed by the r.v. X, having probability density function $f_{(X)}(x)$ and cdf $F_{(X)}(x)$ with mean and variance μ and σ^2 respectively. A sample of size m was selected and ranked. Let $X^{(1)}_{s,m}$ be the s^{th} smallest rank of the sample, where s=1,2,...,m. Then mean of $X^{(1)}_{s,m}$will be $F^{-1}[\alpha(s)]$and variance will be ${\sigma^{(1)}_{s,m}}^2$.

Proof:

Let X_{ij}be a random variable having mean μ and variance σ^2 respectively and random sample of size m was selected and ranked.

Let,$x_{S:m}$ =Sth smallest value of the sample where S = 1, ...m,

Then pdf and cdf of $X_{S:m}$ are

$$f_{s,m}(x) = \frac{1}{B(s, m-s+1)} F^{s-1}(x)(1-F(x))^{m-s} f(x)$$

$$F_{s:m}(x) = FB\ (F(x);\ s, m-s+1) \text{respectively.}$$

Where FB (F (x) ; S, m – s + 1) follows a beta distribution function with parameters (S, m – S + 1)

Let $\mu^{(1)}_{s:m}$ = mean of $X^{(1)}_{s:m}$

and ${\sigma^{(1)}}^2_{s:m}$ = variance of $X^{(1)}_{s:m}$ respectively,

such that, Using Taylor series as given in David &Nagarajah (2003),

$$E\left(X^{(1)}{}_{s;m}\right)=\mu^{(1)}{}_{s;m}=\int x.f_{s:m}(x)dx \ ; \ F^{-1}{}_{s;m}(P_s)$$

and, $F_{s:m}(x)=FB(F(x);\, s, m-s+1)=P_s$

$$\Rightarrow \mu^{(1)}{}_{s;m}=F^{-1}{}_{s;m}(P_s)$$

$$=F^{-1}[\alpha(s)]$$

where $\alpha(s)=PB(P_s;s.(m-s+1))$ is an partitioned function for beta distribution with p_s=s/m+1

Similarly,

$$F_{m-s+1;m}(x)=FB(F(x):m-s+1\,.s)=q_s$$

$$\Rightarrow \mu^{(1)}{}_{m-s+1;m}=F^{-1}{}_{m-s+1:m}(q_s)$$

$$=F^{-1}[PB(q_s;m-s+1,s)]$$

$$=F^{-1}[1-\alpha(s)]$$

where, $PB(q_s;m-s+1,s)=PB(1-P_s;m-s+1,s)$

$$= 1- PB(P_s :s, m-s+1)$$

$$=1-\alpha(s)$$

and $p_s + q_s = 1$

If f (x) follows symmetrical distribution for any $0\le\alpha(s)\le 1$

Then $\Rightarrow q_s-\mu=\mu-P_s$

$$\Rightarrow F^{-1}[1-\alpha(s)]-\mu=\mu-F^{-1}[\alpha(s)]$$

$$\Rightarrow F^{-1}[1-\alpha(s)]+F^{-1}[\alpha(s)]=2\mu$$

$$\Rightarrow \mu^{(1)}{}_{s;m}+\mu^{(1)}{}_{m-s+1;m}=2\mu$$

The variance of $X_{s\,;\,m}$ is given by

$$=\sigma^{(1)2}{}_{s;m}=\int\left(x-\mu^{(1)}{}_{s;m}\right)^2 f_{s;m}(x)\,dx$$

$$=\int(x-\mu)^2 f_{s;m}(x)\,dx-\left(\mu^{(1)}{}_{s;m}-\mu\right)^2$$

$$\Rightarrow \sigma^{(1)2}{}_{s;m}+\left(\mu^{(1)}{}_{s;m}-\mu\right)^2=\int(x-\mu)^2 f_{s;m}(x)dx$$

$$=\int(x-\mu)^2\frac{1}{B(s;m-s+1)}F^{s-1}(x)\left((1-F(x))^{m-s}\right)f(x)dx$$

$$<\int (x-\mu)^2 f(x)\,dx=\sigma^2$$

$$\Rightarrow \sigma^{(1)2}{}_{s;m}+\left(\mu^{(1)}{}_{s:m}-\mu\right)^2<\sigma^2$$

as $\dfrac{F^{s-1}(x)\,[1-f(x)]^{m-s}}{B\ (s,m-s+1)}<1$

The variance of $X_{s:m}$ may also represented as

$$=\sigma^{(1)2}{}_{s;m}=\int\frac{\left(F^{-1}(u)-F^{-1}[\alpha(s)]\right)^2}{B(s,m-s+1)}u^{s-1}(1-u)^{m-s}.du$$

For symmetrical f (x),

$$=\sigma^{(1)2}{}_{s;m} = \int \frac{\left(F^{-1}(u)-F^{-1}[\alpha(s)]\right)^2}{B(s, m-s+1)} u^{s-1}(1-u)^{m-s}.du$$

$$= \int \frac{\left(F^{-1}(1-u)-F^{-1}[1-\alpha(s)]\right)^2}{B(m-s+1, s)} u^{m-s}(1-u)^{s-1}.du$$

$$=\sigma^{(1)2}{}_{m-s+1;m}$$

B. Corollary -2

Let $X^{(2)}{}_{s;m}$ be the sth smallest value of a random sample of size m. The sample was selected from a population having probability density function $f^{(1)}{}_{s,m}(x)$ and cdf $F^{(1)}{}_{s,m}(x)$ with mean and variance $\mu^{(1)}_{s,m}$ and $\sigma^{(1)2}_{s,m}$ respectively. After ranking a size of m sample was selected and let $x^{(2)}{}_{s,m}$ be the sth smallest rank of the sample, where s=1,2,...,m. Then mean of$X^{(2)}{}_{s,m}$will be $F^{-1}[\alpha.\alpha(s)]$and variance will be $\sigma^{(2)2}_{s,m}$.

Proof:

Let $X^{(2)}{}_{ij}$be a random variable from population having mean μ and variance σ^2 respectively
When a random sample from population of size m was selected and ranked.
Let, $X^{(2)}{}_{s:m}$ =Sth smallest value of the sample where S = 1, ...m,
Then pdf of population is

$$f^{(1)}{}_{s;m}(x)=\frac{1}{B(s;m-s+1)}F_{s,m}{}^{s-1}(x)\left(1-F_{s,m}(x)\right)^{m-s}f_{s,m}(x)$$

where, the mean and variance of $x^{(1)}_{ij}$ are $\mu^{(1)}$ and $\sigma^{(1)2}$ respectively.

and also let $x^{(2)}{}_{m-s+1;m}$ be the (m – s + 1)th smallest value

$$\text{Then}\left.\begin{array}{l} E\left(x^{(2)}{}_{s;m}\right)=\mu^{(2)}{}_{s;m} \\ E\left(x^{(2)}{}_{m-s+1;m}\right)=\mu^{(2)}{}_{m-s+1;m} \\ V\left(x^{(2)}{}_{s;m}\right)=\sigma^{(2)^2}{}_{s;m} \\ V\left(x^{(2)}{}_{m-s+1;m}\right)=\sigma^{(2)^2}{}_{m-s+1;m} \end{array}\right]$$

Then,

$$\mu^{(2)}{}_{s:m} = F^{-1}{}_{s;m}[\alpha(s)]$$
$$= F^{-1}{}_{s,m}[\alpha.\ \alpha(s)]=P^*{}_s$$

again, $\mu^{(2)}{}_{m-s+1;m} = F^{-1}{}_{s;m}[1-\alpha(s)]$

$$=F^{-1}_{s,m}[1-\alpha.\alpha(s)]$$
$$=q^*{}_s$$

For symmetric distribution for $0 \leq \alpha \leq 1$

$$\Rightarrow q^*{}_s - \mu = \mu - P^*_s$$
$$\Rightarrow F^{-1}[1-\alpha.\alpha(s)] - \mu = \mu - F^{-1}[\alpha.\alpha(s)]$$
$$\Rightarrow F^{-1}[1-\alpha.\alpha(s)] + F^{-1}[\alpha.\alpha(s)] = 2\mu$$
$$\Rightarrow \mu^{(2)}{}_{m-s+1;m} + \mu^{(2)}{}_{s;m} = 2\mu$$

The variance of $X^{(2)}_{s:m}$ will be

$$\sigma^{(2)2}{}_{s;m} = \int \left(x - \mu^{(2)}{}_{s;m}\right)^2 f_{s;m}(x)dx$$

$$= \int \left(x - \mu^{(1)}{}_{s;m}\right)^2 f_{s;m}(x)dx - \left(\mu^{(2)}{}_{s;m} - \mu^{(1)}{}_{s;m}\right)^2$$

$$\Rightarrow \sigma^{(2)^2}{}_{s;m} + \left(\mu^{(2)}{}_{s;m} - \mu^{(1)}{}_{s;m}\right)^2 = \sigma^{(1)2}{}_{s;m}$$

$$\text{as} \int \left(x - \mu^{(1)}{}_{s;m}\right)^2 f_{s;m}(x)dx = \sigma^{(2)^2}{}_{s;m}$$

$$\Rightarrow \sigma^{(2)2}_{s,m} < \sigma^{(1)2}_{s,m}$$

Again

$$\Rightarrow \sigma^{(2)^2}{}_{s;m} + \left(\mu^{(2)}{}_{s;m} - \mu^{(1)}{}_{s;m}\right)^2 + \left(\mu^{(1)}{}_{s;m} - \mu\right)^2 = \sigma^{(1)2}{}_{s;m} + \left(\mu^{(1)}{}_{s;m} - \mu\right)^2 = \sigma^2_{s,m}$$

$$\Rightarrow \operatorname{var}(DPRSS) < \operatorname{var}(RSS) < \operatorname{var}(SRS)$$

The variance of $X^{(2)}_{s:m}$ may also represented as

$$= \sigma^{(2)2}{}_{s;m} = \int \frac{\left(F^{(1)\text{-}1}_{s,m}(u) - F^{(1)\text{-}1}_{s,m}\left[\alpha(s)\right]\right)^2}{B(s, m-s+1)} u^{s-1}(1-u)^{m\text{-}s}.du$$

For symmetrical f (x),

$$= \sigma^{(2)2}{}_{s;m} = \int \frac{\left(F^{(1)\text{-}1}_{s,m}(u) - F^{(1)\text{-}1}_{s,m}\left[\alpha(s)\right]\right)^2}{B(s, m-s+1)} u^{s-1}(1-u)^{m\text{-}s}.du$$

$$= \int \frac{\left(F^{(1)-1}_{s,m}(1\text{-}u) - F^{-1}_{s,m}\left[\alpha.\alpha(s)\right]\right)^2}{B(m-s+1,\ s)} u^{m-s}(1-u)^{s\text{-}1}.du$$

$$= \int \frac{\left(F^{(1)\text{-}1}_{s,m}(1\text{-}u) - F^{-1}\left[1\text{-}\alpha.\alpha(s)\right]\right)^2}{B(m-s+1,\ s)} u^{m-s}(1-u)^{s\text{-}1}.du$$

$$= \int \frac{\left(F^{(1)\text{-}1}_{s,m}(1\text{-}u) - F^{(1)\text{-}1}_{s,m}\left[1\text{-}\alpha(s)\right]\right)^2}{B(m-s+1,\ s)} u^{m-s}(1-u)^{s\text{-}1}.du$$

$$= \sigma^{(2)2}{}_{m-s+1;m}$$

C. Corollary-3

1. $\hat{\mu}^{(2)}_{DPRSS}$ is an unbiased estimator of the population mean, for givenassumption that population is symmetric about its mean.

<u>Proof</u> :

For k^{th} cycle and i^{th} sample,

D. If m is even,

$$\left[x^{(2)}{}_{1(p(m+1))k}\ .\ x^{(2)}{}_{2(p(m+1))k} \ldots\ldots x^{(2)}_{\frac{m}{2}}(p(m+1))k\right]\left[x^{(2)}_{\frac{m}{2}+1}(q(m+1)k,\ x^{(2)}_{\frac{m}{2}+2}(q(m+1)k \ldots\ldots\right.$$

$x^{(2)}_{m}(q(m+1))k\Big]$ is the sample of size DPRSSE.

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$$\Rightarrow \mu^{(2}{}_{DPRSSE} = \frac{1}{m}\left(\sum_{i=1}^{\ell} X^{(2)}{}_{i(s;m)} + \sum_{j=\ell+1}^{m} X^{(2)}{}_{j(m-s+1;m)}\right)$$

$$\Rightarrow E\left(\mu^{(2)}{}_{DPRSSE}\right) = \frac{1}{m}\left(\sum_{i=1}^{\ell} E\left(X^{(2)}{}_{i(s;m)}\right) + \sum_{j=\ell+1}^{m} E\left(X^{(2)}{}_{j(m-s+1;m)}\right)\right)$$

$$= \frac{1}{m}\left(\frac{m}{2}\,.\,\mu^{(2)}(s;m) + \frac{m}{2}\,\mu^{(2)}(m-s+1;m)\right)$$

$$= \frac{1}{m}\times\frac{m}{2}\left(\mu^{(2)}{}_{s;m} + \mu^{(2)}(m-s+1;m)\right)$$

$$= \frac{1}{2}\times 2\mu$$

$$= \mu$$

E. If m is odd,$[X^{(2)}{}_{1\,(p\,(m+1)\,k)}, X^{(2)}{}_{2\,(p\,(m+1)\,k)} \ldots\ldots X^{(2)}{}_{\frac{m-1}{2}\,(p\,(m+1)\,k)}],[X^{(2)}{}_{\frac{m-1}{2}+1\,(\,(m+1)/2)\,k}],[X^{(2)}{}_{\frac{m-1}{2}+2\,(q\,(m+1)\,k}$,........,
$X^{(2)}{}_{\frac{m-1}{2}+3\,(q\,(m+1)\,k)} \ldots\ldots X^{(2)}{}_{m\,(q\,(m+1)\,k)}$]is the sample of size DPRSSO.

$$\Rightarrow \hat{\mu}^{(2)}{}_{DPRSSO} = \frac{1}{m}\left(\sum_{i=1}^{h} X^{(2)}{}_{i(s;m)} + \sum_{j=h+2}^{m} X^{(2)}{}_{j(m-s+1;m)} + X_{(\frac{m-1}{2}+1)(\frac{m+1}{2}}\right)$$

$$\Rightarrow E\left(\hat{\mu}^{(2)}{}_{DPRSSO}\right) = \frac{1}{m}\left(\sum_{i=1}^{n} E\left(X^{(2)}{}_{i(s;m)}\right) + \sum_{j=h+2}^{m} E\left(X^{(2)}{}_{j(m-s+1;m)}\right) + E\left(X^{(2)}{}_{\frac{m+1}{2};m}\right)\right)$$

$$= \frac{1}{m}\left[\frac{m-1}{2}\left(\mu^{(2)}{}_{s;m} + \mu^{(2)}{}_{m-r+1;m}\right) + \mu\right] = \mu$$

Hence, $\hat{\mu}^{(2)}{}_{DPRSS}$ is an unbiased estimator of the population mean.

F. *Corollary-4*

$\mathrm{Var}\left(\overline{X}_{DPRSS}\right)$ is less than each of $\mathrm{Var}\left(\overline{X}_{SRS}\right)$ and $\mathrm{Var}\left(\overline{X}_{RSS}\right)$.

<u>Proof</u> :

For m is even,

Then Variance will be

$$\Rightarrow \mathrm{var}\left(\hat{\mu}_{DPRSSE}\right) = \frac{1}{m^2}\left(\sum_{i=1}^{m/2} \mathrm{var}\left(X^{(2)}{}_{i(s;m)}\right)\right) + \frac{1}{m^2}\sum_{j=\frac{m}{2}+1}^{m} \mathrm{var}\left(X^{(2)}{}_{i\,(m-s+1;m)}\right)$$

$$= \frac{1}{2m}\left(\sigma^{(2)^2}{}_{s;m} + \sigma^{(2)^2}{}_{m-s+1;m}\right) = \frac{\sigma_{s,m}^{(2)2}}{m}$$

$$\Rightarrow \mathrm{var}(DPRSSE) < \mathrm{var}(RSS) < \mathrm{var}(SRS)$$

For m is odd,

Then Variance can be defined as

$$\Rightarrow \operatorname{var}(\hat{\mu}_{DPRSSO}) = \frac{1}{m}\left(\sum_{i=1}^{m-1/2} \operatorname{var}\left(X^{(2)}{}_{i(s;m}\right) + \operatorname{var}\left(X^{(2)}{}_{\frac{m+1}{2}\,.\,m}\right) + \sum_{i\,\frac{m+1}{2}}^{m} \operatorname{var}\left(X^{(2)}{}_{j(m-s+1;m)}\right)\right)$$

$$= \frac{1}{m^2}\left(\frac{m-1}{2}\left(\sigma^{(2)^2}{}_{s;m} + \sigma^{(2)^2} m-s+1;m\right) + \sigma^{(2)^2}{}_{\frac{m+1}{2};m}\right)$$

$$= \frac{1}{m^2}\left(\frac{m-1}{2}(2\sigma_{s,m}^{(2)2}) + \sigma_{s,m}^{(2)2}\right)$$

$$= \frac{1}{m}\sigma_{s,m}^{(2)2}$$

$$\Rightarrow \operatorname{var}(DPRSSO) < \operatorname{var}(RSS) < \operatorname{var}(SRS)$$