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Special Dio3-Tuples for Pronic Number –I

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Abstract: We search for three distinct polynomials with integer coefficients such that the product of any two members of the set added with their sum and increased by a non-zero integer (or polynomial with integer coefficients) is a perfect square.

Keywords: NDio 3-tuples, Pronic numbers, Polynomials.

Notation: Pro_n = Pronic number of rank n .

I. INTRODUCTION

Many mathematicians considered the problem of the existence of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n [1] and also for any linear polynomials n . Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-14].

In this communication, we present a few special dio 3-tuples for Pronic numbers of different ranks with their corresponding properties.

II. BASIC DEFINITION

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be a special dio 3-tuple with property $D(n)$ if $a_i * a_j + (a_i + a_j) + n$ is a perfect square for all $1 \leq i < j \leq 3$, where n may be non-zero integer or polynomial with integer coefficients.

A. Method of Analysis

1) *Case 1:* Construction of Dio 3-tuples for Pronic number of rank $n - 1$ and n .

Let $a = Pro_{n-1}$, $b = Pro_n$ be Pronic number of rank $n - 1$ and n respectively such that $ab + (a + b) + n^2 + 1$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (a + c) + n^2 + 1 = \beta^2 \tag{1}$$

$$bc + (b + c) + n^2 + 1 = \gamma^2 \tag{2}$$

On solving equations (1) and (2), we get

$$(a - b) + (n^2 + 1)(b - a) = (b + 1)\beta^2 - (a + 1)\gamma^2 \tag{3}$$

Assume $\beta = x + (a + 1)y$ and $\gamma = x + (b + 1)y$ and it reduces to

$$x^2 = (a + 1)(b + 1)y^2 + n^2 \tag{4}$$

The initial solution of the equation (4) is given by

$$x_0 = n^2 + 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - n + 2$

On substituting the values of a and β in equation (1), we get

$$c = 4n^2 + 3 = Pro_{2n-2} + 6n + 1$$

Hence, The triple $(Pro_{n-1}, Pro_n, Pro_{2n-2} + 6n + 1)$ is a Dio 3-tuple with property $D(n^2 + 1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 1

n	(a, b, c)	$D(n)$
1	(0,2,7)	2
2	(2,6,19)	5
3	(6,12,39)	10
4	(12,20,67)	17
5	(20,30,103)	26

We present below the Dio 3-tuple for Pronic number of the rank mentioned above with suitable properties.

Table 2

a	b	c	$D(n)$
Pro_{n-1}	Pro_n	$Pro_{2n-2} + 6n + 3$	$D(3n^2 + 4)$
Pro_{n-1}	Pro_n	$Pro_{2n-2} + 6n + 5$	$D(5n^2 + 9)$
Pro_{n-1}	Pro_n	$Pro_{2n-2} + 6n + 7$	$D(7n^2 + 16)$
Pro_{n-1}	Pro_n	$Pro_{2n-2} + 6n + 9$	$D(9n^2 + 25)$
Pro_{n-1}	Pro_n	$Pro_{2n-2} + 6n + 11$	$D(11n^2 + 36)$

In general, it is noted that the triple $(Pro_{n-1}, Pro_n, Pro_{2n-2} + 6n + 2k + 1)$ is a Dio 3-tuple with the property $D((2k + 1)n^2 + t^2)$, where $k = 2, 3, 4, \dots$ and $t = 1, 2, \dots$

2) Case 2: Construction of Dio 3-tuples for Pronic number of rank $n - 2$ and n .

Let $a = Pro_{n-2}$, $b = Pro_n$ be Pronic number of rank $n - 2$ and n respectively such that $ab + (a + b) + 2n^2 - 2n - 1$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (a + c) + 2n^2 - 2n - 1 = \beta^2 \tag{5}$$

$$bc + (b + c) + 2n^2 - 2n - 1 = \gamma^2 \tag{6}$$

On solving equations (5) and (6), we get

$$(a - b) + (2n^2 - 2n - 1)(b - a) = (b + 1)\beta^2 - (a + 1)\gamma^2 \tag{7}$$

Assume $\beta = x + (a + 1)y$ and $\gamma = x + (b + 1)y$ and it reduces to

$$x^2 = (a + 1)(b + 1)y^2 + 2n^2 - 2n - 2 \tag{8}$$

The initial solution of the equation (8) is given by

$$x_0 = n^2 - n + 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - 4n + 4$

On substituting the values of a and β in equation (5), we get

$$c = 4n^2 - 4n + 5 = Pro_{2n-2} + 2n + 3$$

Hence, The triple $(Pro_{n-2}, Pro_n, Pro_{2n-2} + 2n + 3)$ is a Dio 3-tuple with property $D(2n^2 - 2n - 1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 3

n	(a, b, c)	$D(n)$
1	(0,2,5)	-1
2	(0,6,13)	3
3	(2,12,29)	11
4	(6,20,53)	23
5	(12,30,85)	39

We present below, a few Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 4

a	b	c	$D(n)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 5$	$D(4n^2 - 4n + 2)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 7$	$D(6n^2 - 6n + 7)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 9$	$D(8n^2 - 8n + 14)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 11$	$D(10n^2 - 10n + 23)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 13$	$D(12n^2 - 12n + 34)$

3) Case 3: Construction of Dio 3-tuples for Pronic number of rank $n - 2$ and $n - 1$.

Let $a = \text{Pro}_{n-2}$, $b = \text{Pro}_{n-1}$ be Pronic number of rank $n - 2$ and $n - 1$ respectively such that

$ab + (a + b) + (-n^2 + 2n - 1)$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (a + c) + (-n^2 + 2n - 1) = \beta^2 \tag{9}$$

$$bc + (b + c) + (-n^2 + 2n - 1) = \gamma^2 \tag{10}$$

On solving equations (9) and (10), we get

$$(a - b) + (-n^2 + 2n - 1)(b - a) = (b + 1)\beta^2 - (a + 1)\gamma^2 \tag{11}$$

Assume $\beta = x + (a + 1)y$ and $\gamma = x + (b + 1)y$ and it reduces to

$$x^2 = (a + 1)(b + 1)y^2 + (-n^2 + 2n - 2) \tag{12}$$

The initial solution of the equation (12) is given by

$$x_0 = n^2 - 2n + 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - 5n + 4$

On substituting the values of a and β in equation (9), we get

$$c = 4n^2 - 8n + 5 = \text{Pro}_{2n-2} - 2n + 3$$

Hence, The triple $(\text{Pro}_{n-2}, \text{Pro}_{n-1}, \text{Pro}_{2n-2} - 2n + 3)$ is a Dio 3-tuple with property $D(-n^2 + 2n - 1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 5

n	(a, b, c)	$D(n)$
1	(0,0,1)	0
2	(0,2,5)	-1
3	(2,6,17)	-4
4	(6,12,37)	-9
5	(12,20,65)	-16

We present below, a few Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 6

a	b	c	$D(n)$
Pro_{n-2}	Pro_{n-1}	$Pro_{2n-2} - 2n + 5$	$D(n^2 - 2n + 2)$
Pro_{n-2}	Pro_{n-1}	$Pro_{2n-2} - 2n + 7$	$D(3n^2 - 6n + 7)$
Pro_{n-2}	Pro_{n-1}	$Pro_{2n-2} - 2n + 9$	$D(5n^2 - 10n + 14)$
Pro_{n-2}	Pro_{n-1}	$Pro_{2n-2} - 2n + 11$	$D(7n^2 - 14n + 23)$
Pro_{n-2}	Pro_{n-1}	$Pro_{2n-2} - 2n + 13$	$D(9n^2 - 18n + 34)$

III. CONCLUSION

In this paper we have presented a few examples of constructing a special Dio 3-tuples for Pronic number of different ranks with suitable properties. To conclude one may search for Dio 3-tuples for higher order Pronic number with their corresponding suitable properties.

REFERENCES

- [1] Balkar A, Duvenport H, "The equations $3x^2 - 2 = y^2$ and $8x^2 - 7 = z^2$ ", Quart.J.Math.Oxford Ser, 1969, 20(2), 129-137.
- [2] Jones B.E, "A second variation on a problem of Diophantus and Davenport", Fibonacci Quart, 1977, 15, 323-330.
- [3] Brown E, "Sets in which $xy + k$ is always a perfect square", Math.Comp, 1985, 45, 613-620.
- [4] Beardon A.F, Deshpande M.N, "Diophantine Triples", The Mathematical Gazette, 2002 86, 258-260
- [5] Deshpande M.N, "Families of Diophantine triplets", Bulletin of the Marathwada Mathematical Society, 2003, 4, 19-21.
- [6] Fujita Y, "The extendability of Diophantine pairs $\{k - 1, k + 1\}$ ", Journal of Number Theory, 2008,128, 322-353.
- [7] Gopalan M.A and Pandichelvi V, "On the extendability of the Diophantine triple involving Jacobsthal numbers $(J_{2n-1}, J_{2n+1} - 3, 2J_{2n} + J_{2n-1} + J_{2n+1} - 3)$ ", International Journal of Mathematics & Applications, 2009, 2(1), 1-3.
- [8] Srividhya G, "Diophantine Quadruples for Fibonacci numbers with property D(1)", Indian Journal of Mathematics and Mathematical Science, 2009, 5(2), 57-59.
- [9] Gopalan M.A, Srividhya G, "Two special Diophantine Triples", Diophantus J.Math, 2012, 1(1), 23-27.
- [10] Gopalan M.A, Srividhya G, "Diophantine Quadruple for Fibonacci and Lucas numbers with property D(4)", Diophantus J.Math, 2012, 1(1), 15-18.
- [11] Andrej Dujella, Zagreb, Croatia, "The Problem of Diophantus and Davenport for Gaussian Integers", Glas.Mat.Ser.III, 1997, 32, 1-10.
- [12] Gopalan M.A, Geetha K, Manju Somanath, "On Special Diophantine Triples", Archimedes Journal of Mathematics, 2014, 4(1), 37-43.
- [13] Gopalan M.A, Geetha V, Vidhyalakshmi S, "Dio 3-tuples for Special Numbers-I", The Bulletin of Society for Mathematical Services and Standards, 2014, 10, 1-6.
- [14] Gopalan M.A, Geetha K, Manju Somanath, "Special Dio 3-tuples", The Bulletin of Society for Mathematical Services and Standards, 2014, 10, 22-25.



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