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Special Dio3-Tuples for Pronic Number –I

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***Abstract:** We search for three distinct polynomials with integer coefficients such that the product of any two members of the set added with their sum and increased by a non-zero integer (or polynomial with integer coefficients) is a perfect square.*
***Keywords:** NDio 3-tuples, Pronic numbers, Polynomials.*

Notation: Pro_n = Pronic number of rank n.

I. INTRODUCTION

Many mathematicians considered the problem of the existence of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n [1] and also for any linear polynomials n. Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-14].

In this communication, we present a few special dio 3-tuples for Pronic numbers of different ranks with their corresponding properties.

II. BASIC DEFINITION

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be a special dio 3-tuple with property $D(n)$ if $a_i * a_j + (a_i + a_j) + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non-zero integer or polynomial with integer coefficients.

A. *Method of Analysis*

1) *Case 1:* Construction of Dio 3-tuples for Pronic number of rank $n-1$ and n.

Let $a = \text{Pro}_{n-1}$, $b = \text{Pro}_n$ be Pronic number of rank $n-1$ and n respectively such that $ab + (a+b) + n^2 + 1$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac + (a+c) + n^2 + 1 = \beta^2 \tag{1}$$

$$bc + (b+c) + n^2 + 1 = \gamma^2 \tag{2}$$

On solving equations (1) and (2), we get

$$(a-b) + (n^2+1)(b-a) = (b+1)\beta^2 - (a+1)\gamma^2 \tag{3}$$

Assume $\beta = x + (a+1)y$ and $\gamma = x + (b+1)y$ and it reduces to

$$x^2 = (a+1)(b+1)y^2 + n^2 \tag{4}$$

The initial solution of the equation (4) is given by

$$x_0 = n^2 + 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - n + 2$

On substituting the values of a and β in equation (1), we get

$$c = 4n^2 + 3 = \text{Pro}_{2n-2} + 6n + 1$$

Hence, The triple $(\text{Pro}_{n-1}, \text{Pro}_n, \text{Pro}_{2n-2} + 6n + 1)$ is a Dio 3-tuple with property $D(n^2+1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

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Table 1

n	(a,b,c)	$D(n)$
1	(0,2,7)	2
2	(2,6,19)	5
3	(6,12,39)	10
4	(12,20,67)	17
5	(20,30,103)	26

We present below the Dio 3-tuple for Pronic number of the rank mentioned above with suitable properties.

Table 2

a	b	c	$D(n)$
Pro_{n-1}	Pro_n	$\mathrm{Pro}_{2n-2}+6n+3$	$D(3n^2+4)$
Pro_{n-1}	Pro_n	$\mathrm{Pro}_{2n-2}+6n+5$	$D(5n^2+9)$
Pro_{n-1}	Pro_n	$\mathrm{Pro}_{2n-2}+6n+7$	$D(7n^2+16)$
Pro_{n-1}	Pro_n	$\mathrm{Pro}_{2n-2}+6n+9$	$D(9n^2+25)$
Pro_{n-1}	Pro_n	$\mathrm{Pro}_{2n-2}+6n+11$	$D(11n^2+36)$

In general, it is noted that the triple $(\mathrm{Pro}_{n-1}, \mathrm{Pro}_n, \mathrm{Pro}_{2n-2}+6n+2k+1)$ is a Dio 3-tuple with the property $D((2k+1)n^2+t^2)$, where $k=2,3,4,\ldots$ and $\mathrm{t}=1,2,\ldots$

2) Case 2: Construction of Dio 3-tuples for Pronic number of rank $n-2$ and n.

Let $a=\mathrm{Pro}_{n-2}$, $b=\mathrm{Pro}_n$ be Pronic number of rank $n-2$ and n respectively such that $ab+(a+b)+2n^2-2n-1$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac+(a+c)+2n^2-2n-1=\beta^2 \tag{5}$$

$$bc+(b+c)+2n^2-2n-1=\gamma^2 \tag{6}$$

On solving equations (5) and (6), we get

$$(a-b)+(2n^2-2n-1)(b-a)=(b+1)\beta^2-(a+1)\gamma^2 \tag{7}$$

Assume $\beta=x+(a+1)y$ and $\gamma=x+(b+1)y$ and it reduces to

$$x^2=(a+1)(b+1)y^2+2n^2-2n-2 \tag{8}$$

The initial solution of the equation (8) is given by

$$x_0=n^2-n+1,\quad y_0=1$$

Therefore, $\beta=2n^2-4n+4$

On substituting the values of a and β in equation (5), we get

$$c=4n^2-4n+5=\mathrm{Pro}_{2n-2}+2n+3$$

Hence, The triple $(\mathrm{Pro}_{n-2}, \mathrm{Pro}_n, \mathrm{Pro}_{2n-2}+2n+3)$ is a Dio 3-tuple with property $D(2n^2-2n-1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

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Table 3

n	(a,b,c)	$D(n)$
1	(0,2,5)	-1
2	(0,6,13)	3
3	(2,12,29)	11
4	(6,20,53)	23
5	(12,30,85)	39

We present below, a few Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 4

a	b	c	$D(n)$
Pro_{n-2}	Pro_{n}	$\mathrm{Pro}_{2n-2}+2n+5$	$D(4n^2-4n+2)$
Pro_{n-2}	Pro_{n}	$\mathrm{Pro}_{2n-2}+2n+7$	$D(6n^2-6n+7)$
Pro_{n-2}	Pro_{n}	$\mathrm{Pro}_{2n-2}+2n+9$	$D(8n^2-8n+14)$
Pro_{n-2}	Pro_{n}	$\mathrm{Pro}_{2n-2}+2n+11$	$D(10n^2-10n+23)$
Pro_{n-2}	Pro_{n}	$\mathrm{Pro}_{2n-2}+2n+13$	$D(12n^2-12n+34)$

3) Case 3: Construction of Dio 3-tuples for Pronic number of rank $n-2$ and $n-1$.

Let $a=\mathrm{Pro}_{n-2}$, $b=\mathrm{Pro}_{n-1}$ be Pronic number of rank $n-2$ and $n-1$ respectively such that $ab+(a+b)+(-n^2+2n-1)$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac+(a+c)+(-n^2+2n-1)=\beta^2 \tag{9}$$

$$bc+(b+c)+(-n^2+2n-1)=\gamma^2 \tag{10}$$

On solving equations (9) and (10), we get

$$(a-b)+(-n^2+2n-1)(b-a)=(b+1)\beta^2-(a+1)\gamma^2 \tag{11}$$

Assume $\beta=x+(a+1)y$ and $\gamma=x+(b+1)y$ and it reduces to

$$x^2=(a+1)(b+1)y^2+(-n^2+2n-2) \tag{12}$$

The initial solution of the equation (12) is given by

$$x_0=n^2-2n+1,\quad y_0=1$$

Therefore, $\beta=2n^2-5n+4$

On substituting the values of a and β in equation (9), we get

$$c=4n^2-8n+5=\mathrm{Pro}_{2n-2}-2n+3$$

Hence, The triple $(\mathrm{Pro}_{n-2},\mathrm{Pro}_{n-1},\mathrm{Pro}_{2n-2}-2n+3)$ is a Dio 3-tuple with property $D(-n^2+2n-1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 5

n	(a,b,c)	$D(n)$
1	(0,0,1)	0
2	(0,2,5)	-1
3	(2,6,17)	-4
4	(6,12,37)	-9
5	(12,20,65)	-16

We present below, a few Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 6

a	b	c	$D(n)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2}-2n+5$	$D(n^2-2n+2)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2}-2n+7$	$D(3n^2-6n+7)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2}-2n+9$	$D(5n^2-10n+14)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2}-2n+11$	$D(7n^2-14n+23)$
Pro_{n-2}	Pro_{n-1}	$\mathrm{Pro}_{2n-2}-2n+13$	$D(9n^2-18n+34)$

III. CONCLUSION

In this paper we have presented a few examples of constructing a special Dio 3-tuples for Pronic number of different ranks with suitable properties. To conclude one may search for Dio 3-tuples for higher order Pronic number with their corresponding suitable properties.

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