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Distance Two Complementary Tree Domination Number of a Graph

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Abstract: A set D of a graph G = (V, E) is a dominating set, if every vertex in V(G) - D is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set. A dominating set D is called a distance two complementary tree dominating set, if for each $u \in V - D$, $d(u,v) \leq 2$ for some v in D and also $\langle V - D \rangle$ is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by $\gamma_{d2ctd}(G)$. In this paper, bounds for $\gamma_{d2ctd}(G)$ and its exact values for some particular classes of graphs are found. Some results on distance two complementary tree domination number are also established. Keywords: complementary tree domination number, distance two complementary tree domination number.

I. INTRODUCTION

Graphs discussed in this paper are finite, undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A graph with p vertices and q edges is denoted by G(p, q). The concept of domination in graphs was introduced by Ore[6]. A set $D \subseteq V(G)$ is said to be a dominating set of G, if every vertex in V(G) - D is adjacent to some vertex in D. The cardinality of a minimum dominating set in G is called the domination number of G and is denoted by $\gamma(G)$. The problem of finding a minimal distance k-dominating set (call k-basis) was considered by $\gamma(G)$ with special reference to communication networks while the distance k-dominating set was defined by Henning etal. [4]. For an integer $\gamma(G)$ and $\gamma(G)$ is a distance k-dominating set of G if every vertex in $\gamma(G)$ is within distance k from some vertex $\gamma(G)$. The minimum cardinality among all distance k-dominating sets of G is called the distance k-domination number of G and is denoted by $\gamma(G)$. Muthammai, Bhanumathi and Vidhya[5] introduced the concept of complementary tree dominating set. A dominating set $\gamma(G)$ is said to be a complementary tree dominating set (ctd-set) if the induced subgraph $\gamma(G)$ is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by $\gamma(G)$. Any undefined terms in this paper may be found in Harary[1].

In this paper, bounds for $\gamma_{d2ctd}(G)$ and its exact values for some particular classes of graphs are found. Also, the graphs for which $\gamma_{d2ctd}(G) = 1, 2, p-1$ or p-2 are characterized.

II. PRIOR RESULTS

Theorem 2.1[5] For any connected graph $G, \gamma(G) \leq \gamma_{ctd}(G)$.

Theorem 2.2[5] For any connected graph G with $p \ge 2$, $\gamma_{ctd}(G) \le p-1$.

Theorem 2.3[5] Let G be a connected graph with $p \ge 4$. Then $\gamma_{ctd}(G) = p-1$ if and only if G is a star on p vertices.

Theorem 2.4[5] Let Tbe a tree with p vertices which is not a star. Then, $\gamma_{ctd}(T) = p - 2$ ($p \ge 5$) if and only if T is a path or T is obtained from a path by attaching pendant edges at atleastone of the end vertices.

Theorem 2.5[5]Let Gbe a connected graph containing a cycle. Then, $\gamma_{ctd}(G) = p - 2$ ($p \ge 5$)if and only if G is isomorphic to one of the following graphs C_p , K_p or G is the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph.

III. MAIN RESULTS

In this section, a new parameter called distance two complementary tree domination number is defined, bounds and exact values of this parameter are found.



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A. Definition 3.1

A subset $D \subseteq V(G)$ is said to be a distance two complementary tree dominating set (d2ctd-set), if for each $u \in V - D$, $d(u,v) \le 2$ for some v in D and also v - D is a tree. The minimum cardinality of adistance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by $\gamma_{d2ctd}(G)$.

- 1) Observation 3.1
- a) Since any ctd-set is a d2ctd-set, $\gamma_{d2ctd}(G) \le \gamma_{ctd}(G)$.

Equality holds if, $G \cong P_n o K_1$.

b) Since d2ctd-set is a distance two dominating set, $\gamma_2(G) \le \gamma_{d2ctd}(G)$.

Equality holds if, $G \cong P_5$ C_5

- c) For any connected graph G with p vertices, $1 \le \gamma_{d2ctd}(G) \le p-1$, since $\gamma_{ctd}(G) \le p-1$.
- 2) Theorem 3.1: For any connected graph G, $\gamma_{d2ctd}(G) = p 1$, $p \ge 2$ if and only if $G \cong K_2$.
- 3) Proof.:By the Observation 3.1, $\gamma_{d2ctd}(G) \le \gamma_{ctd}(G)$.

If $\gamma_{d2ctd}(G) = p-1$, then $\gamma_{ctd}(G) \ge p-1$. But $\gamma_{ctd}(G) \le p-1$ and $\gamma_{ctd}(G) = p-1$ if and only if $G \cong K_{1, p-1}$. If $p \ge 3$, then $\gamma_{d2ctd}(K_{1, p-1}) = 1 < p-1$.

Therefore $\gamma_{d2ctd}(G) = p - 1$, if p = 2, Hence $G \cong K_2$.

Conversely if $G \cong K_2$, then $\gamma_{d2ctd}(G) = 1 = p - 1$.

- B. Observation 3.2
- 1) For any path P_p on p vertices, $\gamma_{d2ctd}(G) = p 4$, $p \ge 6$.

 $\gamma_{d2ctd}(P_3) = 1, \gamma_{d2ctd}(P_4) = \gamma_{d2ctd}(P_5) = 2.$

2) For any cycle C_p on p vertices, $\gamma_{d2ctd}(C_p) = p - 4$, $p \ge 5$.

 $\gamma_{d2ctd}(C_3) = \gamma_{d2ctd}(C_4) = 1.$

If v_1, v_2, v_3, v_4 be any four consecutive vertices of degree 2 in P_p (or C_p), $V(P_p)$ - $\{v_1, v_2, v_3, v_4\}$ (or $V(C_p)$ - $\{v_1, v_2, v_3, v_4\}$) is a γ_{d2ctd} -set of P_p (or P_p).

- 3) For any star $K_{1, p-1}$, $\gamma_{d2ctd}(K_{1, p-1}) = 1$.
- 4) For any complete graph K_p , $\gamma_{d2ctd}(K_p) = p 2$, $p \ge 3$.
- 5) For any complete bipartite graph $K_{m, n}, \gamma_{d2ctd}(K_{m, n}) = m 1, n \ge m \ge 2$.
- 6) $\gamma_{d2ctd}(\overline{mK_2}) = 2m 3, m \ge 2.$
- 7) For the graph $K_p e$, $\gamma_{d2ctd}(K_p e) = p 3$, where e is an edge in K_p .
- 8) For the graph $K_{m,n}-e, \gamma_{d2ctd}(K_{m,n}-e)=min\ \{m,n\}-1.$
- 9) For the graph $K_{m,n}-e$, $\gamma_{d2ctd}(K_{m,n}-e)=m+n-4$.
- 10) $\gamma_{d2ctd}(P_n \circ K_1) = n$.
- 11) $\gamma_{d2ctd}(C_n \circ K_1) = n 1, n \ge 3.$
- C. Observation 3.3
- 1) For the Fan F_p , $\gamma_{d2ctd}(F_p) = 1$, where $F_p = P_{p-1} + K_1$ ($p \ge 3$).
- 2) For the Wheel W_p , $\gamma_{d2ctd}(W_p) = 2$, where $W_p = C_{p-1} + K_1$ $(p \ge 3)$.
- D. Definition 3.2

The one point union $C_n^{(t)}$ of t-copies of cycle C_n is the graph obtained by taking a new vertex u as a common vertex such that any two distinct cycles C_i and C_j are edge disjoint and do not have any vertex in common except u.

E. Observation 3.4

For
$$t \ge 2$$
 and $n \ge 4$, $\gamma_{ctnd}(C_n^{(t)}) = \begin{cases} t, & \text{if } n \le 5\\ (n-5)t+1, & \text{if } n > 5. \end{cases}$



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F. Definition 3.3

Let $G_1, G_2, ..., G_k$ be k copies of a graph G, where $k \ge 2$. G(k) is a graph obtained by adding an edge from G_i to G_{i+1} ; i = 1, 2, 3, ..., k-1 and the graph G(k) is called the path union of k copies of the graph G(k).

G. Observation 3.5

 $\text{Let } C_n(t),\, t\geq 2,\, n\geq 3 \text{ be the path union of } t \text{ cycles of length } n. \text{ Then } \gamma_{ctnd}(C_n(t)) = \begin{cases} t, & \text{if } n\leq 5\\ (n-4)t, & \text{if } n>5. \end{cases}$

H. Definition 3.4

A t-ply $P_t(u,v)$ is a graph with t paths joining vertices u and v, each of length at least two and no two paths have a vertex in common except the end vertices u and v in $P_t(u,v)$.

I. Observation 3.6

 $\gamma_{ctnd}(P_t(u, v)) = p - 2t$, where p is the number of vertices in $P_t(u, v)$.

J. Observation 3.7

For the graph G, there is no relationship between $\gamma(G)$ and $\gamma_{d2ctd}(G)$ is not true in general. This is illustrated by the following example.

K. Example 3.1.

For the graph given in Figure 3.1, $\{v_1, v_2, v_4\}$ is a γ -set of G and hence $\gamma(G) = 3$ and $\{v_1, v_2\}$ is a γ_{d2ctd} -set of Gand $\gamma_{d2ctd}(G) = 2$. Therefore $\gamma(G) > \gamma_{d2ctd}(G)$.

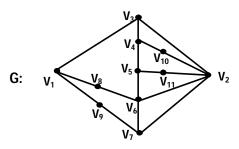


Figure 3.1

For the graph given in Figure 3.2, $\{v_3, v_{10}\}$ is a γ -set of G and hence $\gamma(G) = 2$ and $\{v_1, v_2\}$ is a γ_{d2ctd} -set of G and $\gamma_{d2ctd}(G) = 2$. Therefore $\gamma(G) = \gamma_{d2ctd}(G)$.

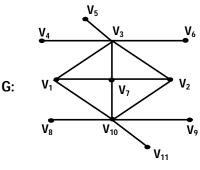


Figure 3.2

For the graph given in Figure 3.3, $\{v_7\}$ is a γ -set of G and hence $\gamma(G) = 1$ and $\{v_1, v_7\}$ is a γ_{d2ctd} -set of G and $\gamma_{d2ctd}(G) = 2$. Therefore $\gamma(G) < \gamma_{d2ctd}(G)$.



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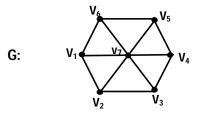


Figure 3.3

L. Theorem 3.2

Let G be a connected graph. Then $\gamma_{d2ctd}(G) = 1$ if and only if there exists a vertex $u \in G$ such that G - u is a tree and $d(u,v) \le 2$, for all $v \in G - u$.

M. Proof.

Let there exist vertex $u \in G$ such that G - u is a tree and $d(u,v) \le 2$, for all $v \in G - u$. Let $D = \{u\}$. Since $d(u,v) \le 2$, for all $v \in G - u$, D is a 2-distance dominating set of G. Also, since G - u is a tree, D is a complementary tree dominating set. Therefore D is a d2ctd-set of G and hence $\gamma_{d2ctd}(G) = 1$.

Conversely, assume $\gamma_{d2ctd}(G) = 1$. Let D be a d2ctd-set of G such that |D| = 1. Then $\langle V(G) - D \rangle$ is a tree and all the vertices of V(G) - D are at a distance ≤ 2 from the vertex of D.

N. Example 3.2

For the graphs given in Figure 3.4, $\gamma_{d2ctd}(G) = 1$.

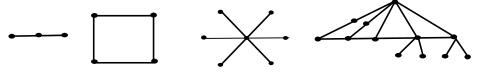


Figure 3.4

O. Remark 3.1

Let v be a support of G with minimum number t of pendant vertices $v_1, v_2, ..., v_t$ such that $T = G - \{v_1, v_2, ..., v_t\}$ is a tree and each vertex of T is of distance at least 2 from v. Then $\gamma_{d2ctd}(G) \le t + 1$.

P. Theorem 3.3

Let G be connected graph with at least three vertices. Then $\gamma_{d2ctd}(G) = 2$ if and only if there exist two vertices u and v such that $G - \{u,v\}$ is a tree and each vertex in $G - \{u,v\}$ is at distance at most 2 from at least one u and v and if at least one of the following holds

- (i) $G \{u\}$ or $G \{v\}$ is not a tree.
- (ii) $d(u,v) \ge 3$.
- 1) Proof. Assume $\gamma_{d2ctd}(G) = 2$.

Then there exists a d2ctd-set D of G such that |D| = 2.

Let $D = \{u, v\}$, where $u, v \in V(G)$.

Then $G - \{u,v\}$ is a tree and each vertex in $G - \{u,v\}$ is at distance atmost 2 from at least one of u and v. If $G - \{u\}$ or $G - \{v\}$ is tree or $d(u,v) \le 2$, then $\{u\}$ or $\{v\}$ is a d2ctd-set of G.

Therefore $G - \{u\}$ or $G - \{v\}$ is not a tree and $d(u,v) \ge 3$.

Conversely, if the conditions given in the theorem holds, then $D = \{u,v\}$ is a d2ctd-set of G and hence $\gamma_{d2ctd}(G) \le 2$. Also $\gamma_{d2ctd}(G) \ne 1$. Hence $\gamma_{d2ctd}(G) = 2$.

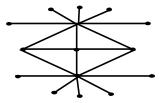
Q. Example 3.3

For the graphs given in Figure 3.5, $\gamma_{d2ctd}(G) = 2$.



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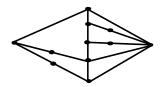


Figure 3.5

R. Theorem 3.4

For any connected graph G with at least three vertices $\gamma_{d2ctd}(G) = p - 2$ if and only if $G \cong P_3, P_4, K_p, p \ge 3$.

1) Proof. Assume $\gamma_{d2ctd}(G) = p - 2$.

Since by the Observation $3.1, \gamma_{d2ctd}(G) \le \gamma_{ctd}(G)$. That is, $p - 2 \le \gamma_{ctd}(G)$. Therefore, $\gamma_{ctd}(G) = p - 1$ or p - 2, Since

 $\gamma_{\rm ctd}(G) \le p - 1.$

2) *Case 1.* $\gamma_{ctd}(G) = p - 1$.

Then $G \cong K_{1, p-1, p} \ge 3$

But $\gamma_{d2ctd}(G) = 1 = p - 2$ implies p = 3.

Therefore $G \cong K_{1,2} \cong P_{3,2}$

- 3) *Case 2.* $\gamma_{d2ctd}(G) = p 2$.
- a) Subcase 2.1.G is a tree.

By Theorem 2.4, G is a path or G is a tree obtained from a path by attaching pendant edges at atleast one of the end vertices.

 $\label{eq:posterior} \text{If G is a path } P_p, \text{then} \\ \gamma_{d2ctd}(G) = \begin{cases} p-4 & \text{if } p \geq 6 \\ 2 & \text{if } p=4,5 \\ 1 & \text{if } p=3. \end{cases}$

 $\gamma_{d2ctd}(G) = p - 2 \text{ implies } G \cong P_4 \text{ or } P_3.$

Let G be a tree obtained from a path P_n, n<p by attaching pendant edges at atleast one of the end vertices.

If $n \le 4$, then $\gamma_{d2ctd}(G) = 2 \ne p - 2$.

If $n \ge 5$, then $\gamma_{d2ctd}(G) = p - 5$.

b) Subcase 2.2.

By Theorem 2.5, G is isomorphic to (i) C_p (ii) K_p or (iii) G is the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph.

$$If \ G \cong C_p, \gamma_{d2ctd}(G) = \left\{ \begin{array}{cc} p-4, & \text{if } p \geq 5 \\ 1 & \text{if } p = 3, 4. \end{array} \right.$$

Therefore $G \cong C_3$.

If $G \cong K_p$, then $\gamma_{d2ctd}(G) = p - 2$.

Let G be the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph K_n , n < p. Then G contains an induced P_3 and $V(G) - V(P_3)$ is a d2ctd-set of G and hence $\gamma_{d2ctd}(G) \le p - 3$.

From Case 1 and Case 2, $G \cong P_3$, P_4 , K_p , $p \ge 3$.

c) Remark 3.2

Let G be a graph such that both G and its complement \overline{G} are connected. Then,

- a) $4 \le \gamma_{d2ctd}(G) + \gamma_{d2ctd}(\overline{G}) \le 2(p-2)$
- b) $4 \le \gamma_{d2ctd}(G) \cdot \gamma_{d2ctd}(\overline{G}) \le (p-2)^2$

Both lower and upper bounds are attained, if G is a path on 4 vertices.

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