iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 Issue: XII Month of publication: December 2017

DOI:

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

Distance Two Complementary Tree Domination Number of a Graph

S. Muthammai[1], G. Ananthavalli[2]
[1]Alagappa Government Arts College, Karaikudi.
[2]Government Arts College for Women (Autonomous), Pudukkottai-622001, India.

Abstract: A set D of a graph G = (V, E) is a dominating set, if every vertex in V(G) - D is adjacent to some vertex in D. The domination number γ(G) of G is the minimum cardinality of a dominating set. A dominating set D is called a distance two complementary tree dominating set,if for each u ∈ V – D, d(u,v) ≤ 2 for some v in D and also <V – D> is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by γ_{d2ctd}(G). In this paper, bounds for γ_{d2ctd}(G) and its exact values for some particular classes of graphs are found. Some results on distance two complementary tree domination number are also established.
Keywords: complementary tree domination number, distance two complementary tree domination number.

I. INTRODUCTION

Graphs discussed in this paper are finite, undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A graph with p vertices and q edges is denoted by G(p, q). The concept of domination in graphs was introduced by Ore[6]. A set D ⊆V(G) is said to be a dominating set of G, if every vertex in V(G) −D is adjacent to some vertex in D. The cardinality of a minimum dominating set in G is called the domination number of G and is denoted by γ(G). The problem of finding a minimal distance k-dominating set (call k-basis) was considered by Slater[7] with special reference to communication networks while the distance k-dominating set was defined by Henning etal. [4]. For an integer k ≥1, a set D ⊆ V (G) is a distance k-dominating set of G if every vertex in V(G) – D is within distance k from some vertex v ∈ D. The minimum cardinality among all distance k-dominating sets of G is called the distance k-domination number of G and is denoted by γ_k(G).Muthammai, Bhanumathi and Vidhya[5] introduced the concept of complementary tree dominating set. A dominating set D ⊆V(G) is said to be a complementary tree dominating set (ctd-set) if the induced subgraph <V(G) −D >is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by γ_{ctd}(G). Any undefined terms in this paper may be found in Harary[1].

In this paper, bounds forγ_{d2ctd}(G) and its exact values for some particular classes of graphs are found. Also, the graphs for whichγ_{d2ctd}(G) = 1, 2, p – 1 or p – 2 are characterized.

II. PRIOR RESULTS

Theorem 2.1[5] For any connected graph G, $\gamma(G) \le \gamma_{ctd}(G)$.

Theorem 2.2[5] For any connected graph G with p ≥ 2, $\gamma_{ctd}(G) \le p - 1$.

Theorem 2.3[5] Let G be a connected graph with p ≥ 4. Then $\gamma_{ctd}(G) = p - 1$ if and only if G is a star on p vertices.

Theorem 2.4[5] Let Tbe a tree with p vertices which is not a star. Then, $\gamma_{ctd}(T) = p - 2$ (p ≥ 5) if and only if T is a path or T is obtained from a path by attaching pendant edges at atleastone of the end vertices.

Theorem 2.5[5]Let Gbe a connected graph containing a cycle. Then, $\gamma_{ctd}(G) = p - 2$ (p ≥ 5)if and only if G is isomorphic to one of the following graphs C_p,$K_p$or G is the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph.

III. MAIN RESULTS

In this section, a new parameter called distance two complementary tree domination numberis defined, bounds and exact values of this parameter are found.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

A. Definition 3.1

A subset $D \subseteq V(G)$ is said to be a distance two complementary tree dominating set (d2ctd-set), if for each $u \in V - D$, $d(u,v) \leq 2$ for some v in D and also $\langle V - D \rangle$ is a tree. The minimum cardinality of adistance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by$\gamma_{d2ctd}(G)$.

1) *Observation 3.1*

a) Since any ctd-set is a d2ctd-set, $\gamma_{d2ctd}(G) \leq \gamma_{ctd}(G)$.
   Equality holds if, $G \cong P_n o K_1$.

b) Since d2ctd-set is a distance two dominating set, $\gamma_2(G) \leq \gamma_{d2ctd}(G)$.
   Equality holds if, $G \cong P_5, C_5.$

c) For any connected graph G with p vertices, $1 \leq \gamma_{d2ctd}(G) \leq p - 1$, since $\gamma_{ctd}(G) \leq p - 1$.

2) *Theorem 3.1:* For any connected graph G, $\gamma_{d2ctd}(G) = p - 1$, $p \geq 2$ if and only if $G \cong K_2$.

3) *Proof.:*By the Observation 3.1, $\gamma_{d2ctd}(G) \leq \gamma_{ctd}(G)$.

If $\gamma_{d2ctd}(G) = p - 1$, then $\gamma_{ctd}(G) \geq p - 1$. But $\gamma_{ctd}(G) \leq p - 1$and $\gamma_{ctd}(G) = p - 1$ if and only if $G \cong K_{1, p-1}$.If $p \geq 3$, then $\gamma_{d2ctd}(K_{1, p-1}) = 1 < p - 1$.

Therefore $\gamma_{d2ctd}(G) = p - 1$, if $p = 2$, Hence $G \cong K_2$.

Conversely if $G \cong K_2$,then$\gamma_{d2ctd}(G) = 1 = p - 1$.

B. Observation 3.2

1) For any path P_p on p vertices, $\gamma_{d2ctd}(G) = p - 4$, $p \geq 6$.

$\gamma_{d2ctd}(P_3) = 1$, $\gamma_{d2ctd}(P_4) = \gamma_{d2ctd}(P_5) = 2$.

2) For any cycle $C_p$on p vertices, $\gamma_{d2ctd}(C_p) = p - 4$, $p \geq 5$.

$\gamma_{d2ctd}(C_3) = \gamma_{d2ctd}(C_4) = 1$.

If v_1, v_2, v_3, v_4 be any four consecutive vertices of degree 2 in P_p(or C_p), $V(P_p)$ - $\{v_1, v_2, v_3, v_4\}$ (or $V(C_p)$ -$\{v_1, v_2, v_3, v_4\}$) is a γ_{d2ctd}-set of P_p(or C_p).

3) For any star $K_{1, p-1}$, $\gamma_{d2ctd}(K_{1, p-1}) = 1$.
4) For any complete graph K_p, $\gamma_{d2ctd}(K_p) = p - 2$, $p \geq 3$.
5) For any complete bipartite graph $K_{m, n}$,$\gamma_{d2ctd}(K_{m, n}) = m - 1$, $n \geq m \geq 2$.
6) $\gamma_{d2ctd}(\overline{mK_2}) = 2m - 3$, $m \geq 2$.
7) For the graph $K_p - e$, $\gamma_{d2ctd}(K_p - e) = p - 3$, where e is an edge in K_p.
8) For the graph $\underline{K_{m,n} - e}$, $\gamma_{d2ctd}(\underline{K_{m,n} - e}) = \min\{m, n\} - 1$.
9) For the graph $\overline{K_{m,n} - e}$,$\gamma_{d2ctd}(\overline{K_{m,n} - e}) = m + n - 4$.
10) $\gamma_{d2ctd}(P_n \circ K_1) = n$.
11) $\gamma_{d2ctd}(C_n \circ K_1) = n - 1$, $n \geq 3$.

C. Observation 3.3

1) For the Fan F_p, $\gamma_{d2ctd}(F_p) = 1$, where $F_p = P_{p-1} + K_1$ ($p \geq 3$).
2) For the Wheel W_p, $\gamma_{d2ctd}(W_p) = 2$, where $W_p = C_{p-1} + K_1$ ($p \geq 3$).

D. Definition 3.2

The one point union $C_n^{(t)}$ of t-copies of cycle C_n is the graph obtained by taking a new vertex u as a common vertex such that any two distinct cycles C_i and C_j are edge disjoint and do not have any vertex in common except u.

E. Observation 3.4

For $t \geq 2$ and $n \geq 4$, $\gamma_{ctnd}(C_n^{(t)}) = \begin{cases} t, & \text{if } n \leq 5 \\ (n-5)t + 1, & \text{if } n > 5. \end{cases}$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

F. Definition 3.3

Let $G_1, G_2, \ldots, G_k$ be k copies of a graph G, where $k \geq 2$. G(k) is a graph obtained by adding an edge from G_i to G_{i+1}; i = 1, 2, 3, …, k-1 and the graph G(k) is called the path union of k copies of the graph G.

G. Observation 3.5

Let $C_n(t)$, $t \geq 2$, $n \geq 3$ be the path union of t cycles of length n. Then $\gamma_{ctnd}(C_n(t)) = \begin{cases} t, & \text{if } n \leq 5 \\ (n-4)t, & \text{if } n > 5. \end{cases}$

H. Definition 3.4

A t-ply $P_t(u,v)$ is a graph with t paths joining vertices u and v, each of length atleast two and no two paths have a vertex in common except the end vertices u and v in $P_t(u,v)$.

I. Observation 3.6

$\gamma_{ctnd}(P_t(u, v)) = p - 2t$, where p is the number of vertices in $P_t(u,v)$.

J. Observation 3.7

For the graph G, there is no relationship between $\gamma(G)$ and $\gamma_{d2ctd}(G)$ is not true in general. This is illustrated by the following example.

K. Example 3.1.

For the graph given in Figure 3.1, $\{v_1, v_2, v_4\}$ is a γ-set of G and hence $\gamma(G) = 3$ and $\{v_1, v_2\}$ is a γ_{d2ctd}-set of Gand $\gamma_{d2ctd}(G) = 2$. Therefore $\gamma(G) > \gamma_{d2ctd}(G)$.

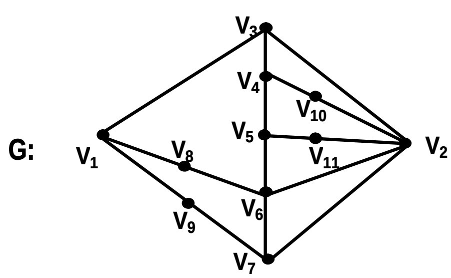


Figure 3.1

For the graph given in Figure 3.2, $\{v_3, v_{10}\}$ is a γ-set of G and hence $\gamma(G) = 2$ and $\{v_1, v_2\}$ is a γ_{d2ctd}-set of G and $\gamma_{d2ctd}(G) = 2$. Therefore $\gamma(G) = \gamma_{d2ctd}(G)$.

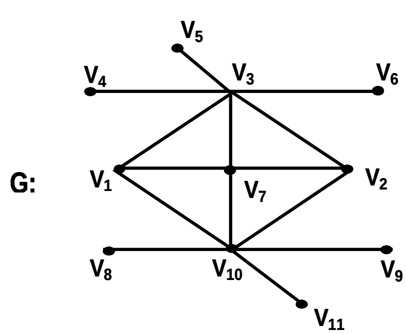


Figure 3.2

For the graph given in Figure 3.3, $\{v_7\}$ is a γ-set of G and hence $\gamma(G) = 1$ and $\{ v_1, v_7 \}$ is a γ_{d2ctd}-set of G and $\gamma_{d2ctd}(G) = 2$. Therefore $\gamma(G) < \gamma_{d2ctd}(G)$.

G:

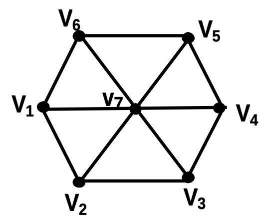


Figure 3.3

L. Theorem 3.2

Let G be a connected graph. Thenγ_{d2ctd}(G) = 1 if and only if there exists a vertex u ∈ G such that G – u is a tree and d(u,v) ≤ 2, for all v ∈ G – u.

M. Proof.

Let there exist vertex u ∈ G such that G – u is a tree and d(u,v) ≤ 2,for all v ∈ G – u. Let D = {u}.Since d(u,v) ≤ 2, for all v ∈ G – u, D is a 2-distance dominating set of G. Also, since G – u is a tree, D is a complementary tree dominating set. Therefore D is a d2ctd-set of G and hence γ_{d2ctd}(G) = 1.

Conversely, assume γ_{d2ctd}(G) = 1. Let D be a d2ctd-set of G such that |D| = 1. Then <V(G) – D> is a tree and all the vertices of V(G) – D are at a distance ≤2 from the vertex of D.

N. Example 3.2

For the graphs given in Figure 3.4,γ_{d2ctd}(G) = 1.

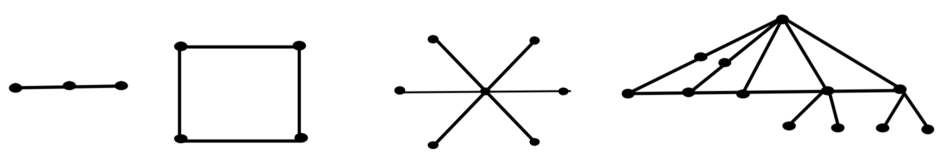


Figure 3.4

O. Remark 3.1

Let v be a support of G with minimum number t of pendant vertices $v_1,v_2, \dots, v_t$ such that T = G - {$v_1,v_2, \dots, v_t$} is a tree and each vertex of T is of distance atleast 2 from v. Then γ_{d2ctd}(G) ≤ t + 1.

P. Theorem 3.3

Let G be connected graph with atleast three vertices. Then γ_{d2ctd}(G) = 2 if and only if there exist two vertices u and v such that G – {u,v} is a tree and each vertex in G – {u,v} is at distance at most 2 from atleast one u and v and if atleast one of the following holds

(i) G – {u} or G – {v} is not a tree.

(ii) d(u,v) ≥3.

*1) Proof.*Assume γ_{d2ctd}(G) = 2.

Then there exists a d2ctd-set D of G such that |D| = 2.

Let D = {u,v}, where u, v∈ V(G).

Then G – {u,v} is a tree and each vertex in G – {u,v} is at distance atmost 2 from atleast one of u and v. If G – {u} or G – {v} is tree or d(u,v)≤ 2, then {u} or {v} is a d2ctd-set of G.

Therefore G – {u} or G – {v} is not a tree and d(u,v) ≥ 3.

Conversely, if the conditions given in the theorem holds, then D = {u,v} is a d2ctd-set of G and henceγ_{d2ctd}(G) ≤ 2. Also γ_{d2ctd}(G) ≠1. Henceγ_{d2ctd}(G) = 2.

Q. Example 3.3

For the graphs given in Figure 3.5,γ_{d2ctd}(G) = 2.

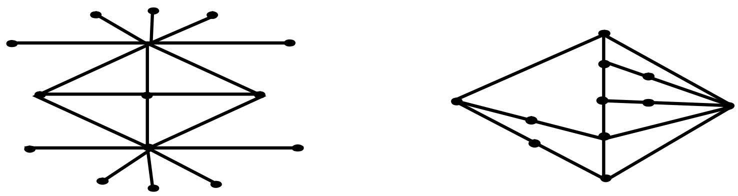


Figure 3.5

R. Theorem 3.4

For any connected graph G with atleast three vertices $\gamma_{d2ctd}(G) = p - 2$ if and only if $G \cong P_3, P_4, K_p, p \geq 3$.

1) *Proof.* Assume $\gamma_{d2ctd}(G) = p - 2$.

Since by the Observation 3.1, $\gamma_{d2ctd}(G) \leq \gamma_{ctd}(G)$. That is, $p - 2 \leq \gamma_{ctd}(G)$. Therefore, $\gamma_{ctd}(G) = p - 1$ or $p - 2$, Since $\gamma_{ctd}(G) \leq p - 1$.

2) *Case 1.* $\gamma_{ctd}(G) = p - 1$.

Then $G \cong K_{1, p-1}, p \geq 3$

But $\gamma_{d2ctd}(G) = 1 = p - 2$ implies $p = 3$.

Therefore $G \cong K_{1, 2} \cong P_3$.

3) *Case 2.* $\gamma_{d2ctd}(G) = p - 2$.

a) *Subcase 2.1.* G is a tree.

By Theorem 2.4, G is a path or G is a tree obtained from a path by attaching pendant edges at atleast one of the end vertices.

If G is a path P_p, then $\gamma_{d2ctd}(G) = \begin{cases} p-4 & \text{if } p \geq 6 \\ 2 & \text{if } p = 4,5 \\ 1 & \text{if } p = 3. \end{cases}$

$\gamma_{d2ctd}(G) = p - 2$ implies $G \cong P_4$ or P_3.

Let G be a tree obtained from a path P_n, $n<p$ by attaching pendant edges at atleast one of the end vertices.

If $n \leq 4$, then $\gamma_{d2ctd}(G) = 2 \neq p - 2$.

If $n \geq 5$, then $\gamma_{d2ctd}(G) = p - 5$.

b) *Subcase 2.2.*

By Theorem 2.5, G is isomorphic to (i) C_p (ii) K_p or (iii) G is the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph.

If $G \cong C_p$, $\gamma_{d2ctd}(G) = \begin{cases} p-4, & \text{if } p \geq 5 \\ 1 & \text{if } p = 3, 4. \end{cases}$

Therefore $G \cong C_3$.

If $G \cong K_p$, then $\gamma_{d2ctd}(G) = p - 2$.

Let G be the graph obtained from a complete graph by attaching pendant edges at atleast one of the vertices of the complete graph K_n, $n <p$. Then G contains an induced P_3 and $V(G) - V(P_3)$ is a d2ctd-set of G and hence $\gamma_{d2ctd}(G) \leq p - 3$.

From Case 1 and Case 2, $G \cong P_3, P_4, K_p, p \geq 3$.

c) *Remark 3.2*

Let G be a graph such that both G and its complement $\overline{G}$ are connected. Then,

a) $4 \leq \gamma_{d2ctd}(G) + \gamma_{d2ctd}(\overline{G}) \leq 2(p - 2)$

b) $4 \leq \gamma_{d2ctd}(G) . \gamma_{d2ctd}(\overline{G}) \leq (p - 2)^2$.

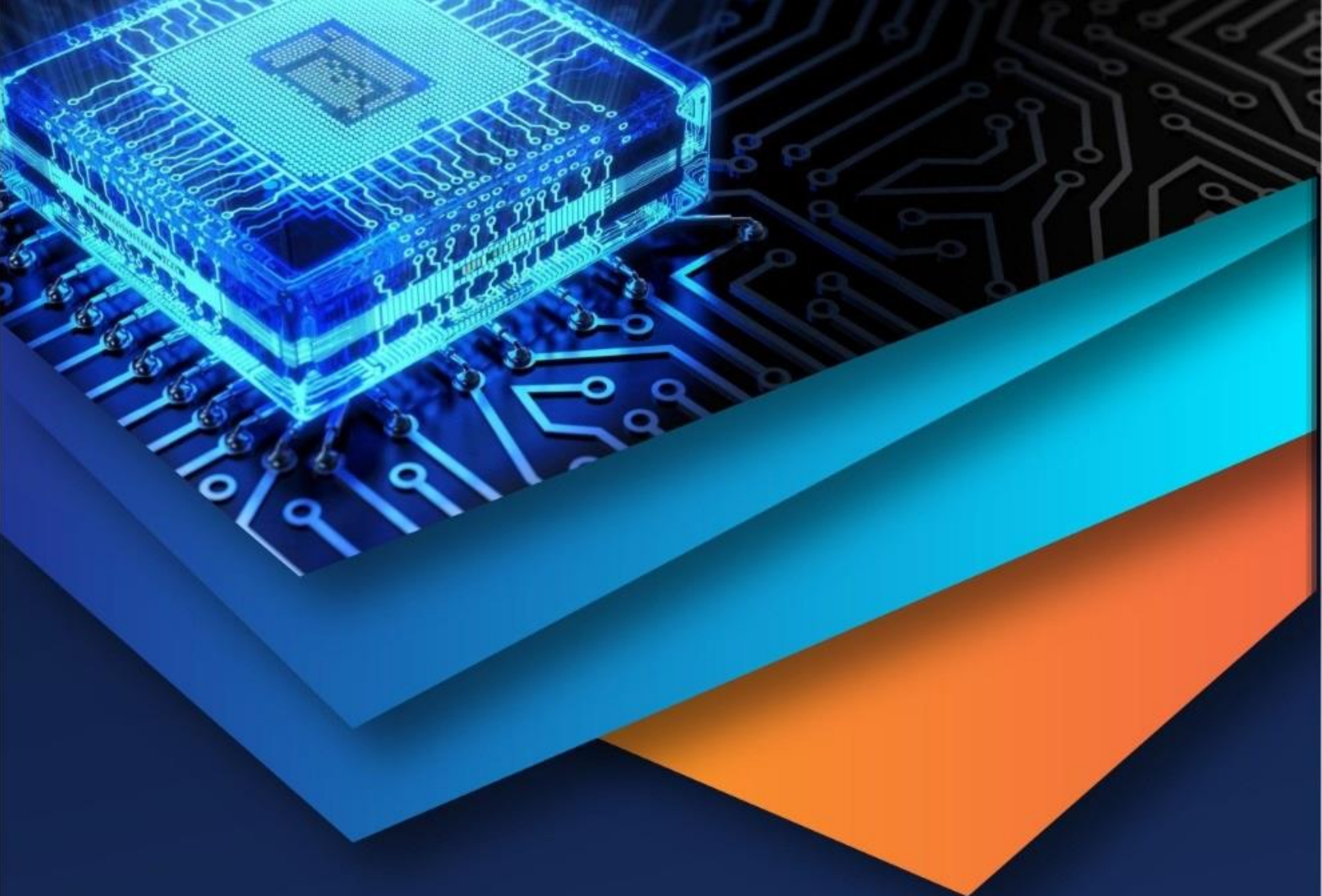
Both lower and upper bounds are attained, if G is a path on 4 vertices.

REFERENCES

[1] F. Harary, Graph Theory, Addison-Wesley, Reading Mass, 1972

[2] T. W. Haynes, S. T. Hedetnimi, P. J. Slater, Fundamentals of Domination in Graphs, Marcel Decker, (1998).

[3] M.A. Henning, Distance domination in Graphs, in : T.W. Haynes, S.T. Hedetnimi, P. J. Slater Ed., Domination in Graphs : Advanced Topics, Marcel Dekkar, NewYork, (1998), 321-349.
[4] M.A. Henning, O.R Swart and H. C. Swart, Bounds on Distance Domination Parameters, J.Combin . Inform. System Sci., 16, (1991), 11-18.
[5] S. Muthammai, M. Bhanumathi and P. Vidhya, Complementary tree domination in graphs, International Mathematical Forum, Vol. 6, 2011, 26, 1273–1283
[6] O. Ore, Theory of graphs, Amer. Math. Soc. Colloq. Publ., 38, (1962).
[7] P. J. Slater, R-Domination in Graphs, Journal of Association for Computer Machinery, 23(3), (1976), 446-450.



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429