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Complementary Tree Domination number and Distance two Complementary Tree Domination number of Cartesian Product of Graphs

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***Abstract:** A set D of a graph G = (V, E) is a dominating set, if every vertex in V(G) – D is adjacent to some vertex in D. The domination number γ (G) of G is the minimum cardinality of a dominating set. A dominating set D $\subseteq$ V(G) is said to be a complementary tree dominating set (ctd-set), if the induced subgraph<V(G) - D>is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}(G)$.A dominating set D is called a distance two complementary tree dominating set, if for each u $\in$V(G) – D, there exists a vertex v $\in$ D such that d(u,v) $\leq$ 2 and also <V(G) – D> is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by γ_{d2ctd}(G).In this paper, complementary tree domination numbers and distance two complementary tree domination number of cartesian product of some standard graphs are found.*

***Key words:** Domination number, complementary tree domination number, distance two complementary tree domination number, Cartesian product.*

I. INTRODUCTION

Graphs discussed in this paper are finite, undirected and simple connected graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A graph G with p vertices and q edges is denoted by G(p, q). The concept of domination in graphs was introduced by Ore[5]. A set D $\subseteq$V(G) is said to be a dominating set of G, if every vertex in V(G) $-$D is adjacent to some vertex in D. The cardinality of a minimum dominating set in G is called the domination number of G and is denoted by γ (G). Muthammai, Bhanumathi and Vidhya[5] introduced the concept of complementary tree dominating set. A dominating set D$\subseteq$V(G) is said to be a complementary tree dominating set (ctd-set), if the induced subgraph <V(G) - D >is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}(G)$.

The cartesian product of two graphs G_1 and G_2 is the graph, denoted by G_1 x G_2 with V ($G_1\times G_2$) =V (G_1) $\times$ V (G_2) (where x denotes the cartesian product of sets) and two vertices u = (u_1, u_2) and v = (v_1, v_2) in V ($G_1\times G_2$) are adjacent in $G_1\times G_2$ whenever [$u_1 = v_1$ and (u_2, v_2) $\in$ E(G_2)] or [$u_2 = v_2$ and (u_1, v_1) $\in$ E(G_1)].

The concept of distance two complementary tree dominating set is introduced in [4]. A dominating set D is called a distance two complementary tree dominating set, if for each u $\in$V(G) – D, there exists a vertex v $\in$ D such that d(u,v) $\leq$ 2, and also <V(G) – D> is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by γ_{d2ctd}(G).

In this paper, complementary tree domination number and distance two complementary tree domination number of K_m x K_n, K_m x P_n, K_m x C_n, C_3 x P_n, C_4 x P_n, C_5 x P_n, C_6 x P_n, C_3 x C_n, C_4 x C_n, C_5 x C_n and C_6 x C_n are found. Any undefined terms in this paper may be found in Harary[2].

II. COMPLEMENTARY TREE DOMINATION NUMBER OF CARTESIAN PRODUCT OF GRAPHS

A. Theorem 2.1

If G $\cong$ K_m x K_n(m, n $\geq$ 3 and m $\leq$ n), then $\gamma_{ctd}(G) = \begin{cases} m(n-2)+1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$

B. Proof.

Let $G \cong K_m \times K_n$.

Let $V(G) = \cup_{i=1}^{m}\{v_{i1}, v_{i2}, \dots, v_{in}\}$ such that $< \{v_{i1}, v_{i2}, \dots, v_{in}\} > \cong K_n^i$, i =1,2, …, m and $< \{v_{1j}, v_{2j}, \dots, v_{mj}\} > \cong K_m^j$, j=1,2, …, n, where K_n^i is the i^{th} copy of K_n and K_m^j is the j^{th} copy of K_m in $K_m \times K_n$. $|V(G)| = mn$.

C. Case 1: m = n.

et $D' = \left(\cup_{i=1}^{m-1}\{v_{ii}, v_{i,i+1}\}\right) \cup \{v_{m,m}\}$ and $D = V(G) - D'$. Then $V(G) - D = D'$and $|D'| = 2(m-1)+1 = 2m-1$. For i =1, 2,3, …, m-1, the vertices$v_{ii}, v_{i,i+1}$in $V(G) - D$ are adjacent to v_{i1}inD, and the vertex v_{mm} is adjacent to v_{m1} in D. Therefore D is a dominating set of G. $<V(G) - D> \cong P_{2(m-1)+1} = P_{2m-1}$. SinceD is a ctd-set of G. Hence$\gamma_{ctd}(G) \leq |D| = |V(G)| - |D'| = mn-(2m-1) = m(n-2)+1$.

It is to be noted that, any tree in G is a path. Let D' be a γ_{ctd}-set of G. The longest path that can be obtained from the subgraph of G induced by the vertices of $V(G)-D'$is P_{2m-1}.That is,$<V(G)-D'> \cong P_{2m-1}$.D' contains atleast $mn - (2m-1) = m(n-2)+1$ vertices. Therefore$\gamma_{ctd}(G) = |D'| \geq m(n-2)+1$.

Hence $\gamma_{ctd}(G) = m(n-2)+1$.

D. Case 2: m < n.

Let $D' = \cup_{i=1}^{m}\{v_{ii}, v_{i,i+1}\}$ and $D = V(G) - D'$. Then $V(G) - D = D'$and $|D'| = 2m$. The vertices v_{11}, v_{12} are adjacent to v_{1n} and $v_{ii}, v_{i,i+1}$(i = 2, 3, …, m) are adjacent to v_{i1}, (i = 2, 3, …, m) in D. Therefore D is a dominating set of G. Since $<V(G) - D> \cong P_{2m}$,D is a ctd-set of G. Therefore $\gamma_{ctd}(G) \leq |V(G)| - |D'| = mn - 2m = m(n-2)$.

As in case 1, any tree in G is a path. Let D' be γ_{ctd}-set of G. The longest path that can be obtained from the subgraph of G induced by the vertices of $V(G) - D'$ is P_{2m}.

That is$<V(G) - D'> \cong P_{2m}$.Therefore D' contains atleast $mn - 2m = m(n-2)$ vertices. Therefore$\gamma_{ctd}(G) = |D'| \geq m(n-2)$.

Therefore$\gamma_{ctd}(G) = m(n-2)$.

Hence $\gamma_{ctd}(G) = \begin{cases} m(n-2)+1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$.

E. Remark 2.1

The set D defined in Case 1 and Case 2 is also a d2ctd-set of G. Since any vertex u in D which is at distance two from a vertex of D, <(V (G) – D) ∪ {u}> either disconnected or contains cycle.

F. Example 2.1

For the graph G given in Figure 1.a and Figure 1.b, the set of vertices marked with is a minimum ctd-set of $K_m \times K_n$and $\gamma_{ctd}(K_4 \times K_4) = 9$ and $\gamma_{ctd}(K_4 \times K_5) = 12$.

$G \cong K_4 \times K_4$

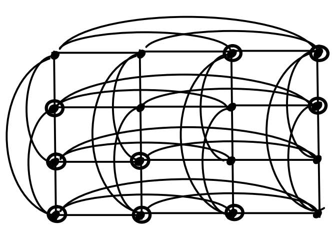

Figure 1.a

$G \cong K_4 \times K_5$

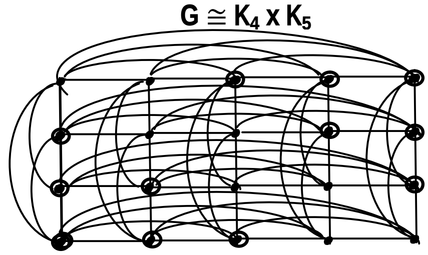

Figure 1.b

G. Theorem 2.2.

If $G \cong K_m \times P_n (4 \leq m \leq n)$, then $\gamma_{ctd}(G) = n(m-2)$.

1) Proof.

Let $G \cong K_m \times P_n$, $m,n \geq 4$.

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Let $V(G) = \cup_{i=1}^{m}\{v_{i1}, v_{i2}, \dots, v_{in}\}$ such that $< \{v_{i1}, v_{i2}, \dots, v_{in}\} > \cong K_n^i$, i =1, 2, …, m and $< \{v_{1j}, v_{2j}, \dots, v_{mj}\} > \cong P_m^j$, j = 1, 2, …, n, where K_n^i is the i^{th} copy of K_n and P_m^j is the j^{th} copy of P_m in $K_m \times P_n$.

$$\text{Let } D' = \begin{cases} \{v_{1n}\} \cup [\cup_{i=1}^{n}\{v_{2i}\}] \cup [\cup_{i=1}^{\frac{n-1}{2}}\{v_{1,2i-1}, v_{3,2i}\}], \text{ if n is odd} \\ [\cup_{i=1}^{n}\{v_{2,i}\}] \cup [\cup_{i=1}^{\frac{n}{2}}\{v_{1,2i-1}, v_{3,2i}\}], \text{ if n is even} \end{cases}.$$

Then $|D'| = 2n$. If $D = V(G) - D'$, then D is a dominating set of G. Also $<V(G) – D> = <D'> \cong P_n \circ K_1$. Therefore D is a ctd-set of G and $\gamma_{ctd}(G) \leq |D| = mn – 2n = n(m – 2)$.

Hence $\gamma_{ctd}(G) \leq n(m – 2)$.

Let D' be a γ_{ctd}-set of G. Since D' is a ctd-set of G, D' contains atleast (m – 2) vertices in each of n copies of K_m. Hence D' contains atleast n(m – 2) vertices. Therefore $\gamma_{ctd}(G) = |D'| \geq n(m – 2)$.

Hence $\gamma_{ctd}(G) = n(m – 2)$.

H. Example 2.2.

For the graph G given in Figure 2, the set of vertices marked with ⊙ is a minimum ctd-set of $K_m \times K_n$ and $\gamma_{ctd}(K_4 \times K_9) = 18$.

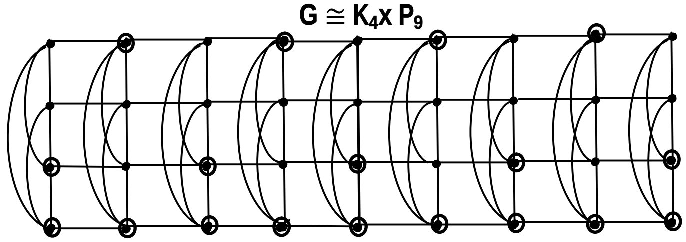

Figure 2

I. Theorem 2.3.

$G \cong C_3 \times P_n$, then $\gamma_{ctd}(G) = n$, $n \geq 1$.

1) Proof.

Let $G \cong C_3 \times P_n$.

Let $V(G) = \cup_{i=1}^{n}\{v_{1i}, v_{2i}, v_{3i}\}$ such that $< \{v_{i1}, v_{i2}, \dots, v_{in}\} > \cong P_n^i$, i =1,2,3 and $< \{v_{1j}, v_{2j}, v_{3j}\} > \cong C_3^j$, j=1,2, …, n, where P_n^i is the i^{th} copy of P_n and C_3^j is the j^{th} copy of C_3 in $C_3 \times P_n$.

$$\text{Let } D = \begin{cases} \{v_{1n}\} \cup [\cup_{i=1}^{\frac{n-1}{2}}\{v_{1,2i-1}, v_{3,2i}\}], \text{ if n is odd} \\ \cup_{i=1}^{\frac{n}{2}}\{v_{1,2i-1}, v_{3,2i}\}, \text{ if n is even} \end{cases}$$

Then D is a dominating set of G. Also $<V(G) – D> \cong P_n \circ K_1$. Therefore D is a ctd-set of G and

$$|D| = \begin{cases} 2\left(\frac{n-1}{2}\right) + 1 = n, \text{ if n is even} \\ 2\left(\frac{n}{2}\right) = n, \quad \text{if n is odd} \end{cases} \text{ and } \gamma_{ctd}(G) \leq |D| = n.$$

Let D' be a γ_{ctd}-set of G. Then D' contains atleast one vertex from each cycle. Since $C_3 \times P_n$ contains n copies of C_3, D' contains atleast n vertices. $\gamma_{ctd}(G) = |D'| \geq n$.

Hence $\gamma_{ctd}(G) = n$, $n \geq 1$.

J. Theorem 2.4.

If $G \cong C_4 \times P_n$, then $\gamma_{ctd}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor$, $n \geq 1$.

Let $G \cong C_4 \times P_n$ and $V(G) = \cup_{i=1}^{n}\{v_{1i}, v_{2i}, v_{3i}, v_{4i}\}$ such that $< \{v_{i1}, v_{i2}, \dots, v_{in}\} > \cong P_n^i$, i =1,2,3,4 and $< \{v_{1j}, v_{2j}, v_{3j}, v_{4j}\} > \cong C_4^j$, j=1,2, …, n, where P_n^i is the i^{th} copy of P_n and C_4^j is the j^{th} copy of C_4 in $C_4 \times P_n$ and $|V(G)| = 4n$.

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Let $D' = \{v_{31}\} \cup [\bigcup_{i=1}^{\frac{n-1}{2}}\{v_{1,2i}, v_{4,2i}, v_{3,2i+1}\}] \cup [\bigcup_{i=1}^{n}\{v_{2i}\}]$.

Then $|D'| = 1 + 3\left(\frac{n-1}{2}\right) + n = \frac{5n-1}{2}$ and $D = V(G) - D'$. Then D is a dominating set of G. Also $<V(G) - D> = <D'>$ is a tree obtained from the path $P_n = <\{v_{2,i},\ i = 1,2,3,\dots,n\}>$, $(n \geq 2)$ by attaching P_3 at each of the vertices $v_{22}, v_{24}, v_{26}, \dots, v_{2,n-1}$ and attaching a pendant edge at each of the vertices $v_{21}, v_{23}\dots, v_{2,n}$. Therefore D is a ctd-set of G, and

$\gamma_{ctd}(G) \leq |D| = |V(G) - D'| = 4n - \left(\frac{5n-1}{2}\right) = \frac{3n+1}{2}$.

Hence $\gamma_{ctd}(G) \leq \frac{3n+1}{2} = \left\lfloor\frac{3n+1}{2}\right\rfloor$.

Let D' be a γ_{ctd}-set of G. The blocks A, B, A^2 and A^2B are constructed as given below.

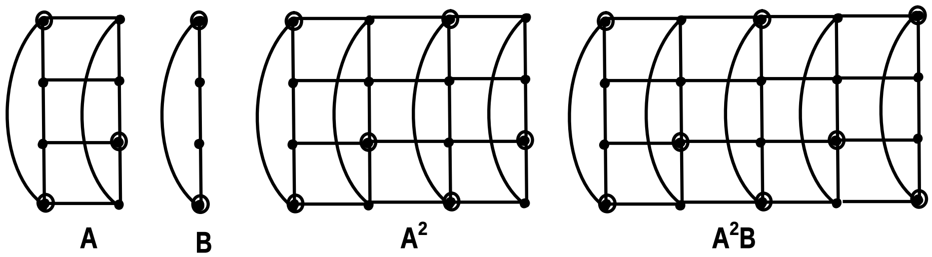

Figure 3

G is obtained by concatenating the blocks $A^{\frac{n-1}{2}}$ and B. That is, $G \cong A^{\frac{n-1}{2}}B$. The vertices with the symbol in each of the blocks represent the vertices that are to be included in D'

Therefore D' contains atleast 3 vertices from each block A of $A^{\frac{n-1}{2}}$ and atleast 2 vertices from block B.

Therefore $\gamma_{ctd}(G) = |D'| \geq 2 + 3\left(\frac{n-1}{2}\right) = \frac{3n+1}{2} = \left\lfloor\frac{3n+1}{2}\right\rfloor$.

Hence $\gamma_{ctd}(G) = \left\lfloor\frac{3n+1}{2}\right\rfloor$, $n \geq 1$.

$, v_{3,2i-1}\}] \cup [\bigcup_{i=1}^{n}\{v_{2i}\}]$ and $D = V(G) - D'$. Then $|D'| = 3\left(\frac{n}{2}\right) + n = \frac{5n}{2}$. Then D is a dominating set of G. Also $<V(G) - D> = <D'>$ is a tree obtained from the path $P_n = <\{v_{2i}, i = 1,2,3,\dots,n\}>$, $(n \geq 2)$ by attaching P_3 at each of the vertices $v_{22}, v_{24}, v_{26}, \dots$, and $v_{2,n}$ and attaching a pendant edge at each of the vertices $v_{21}, v_{23}\dots$, and $v_{2,n-1}$. Therefore D is a ctd-set of G and $\gamma_{ctd}(G) \leq |D| = |V(G) - D'| = 4n - \left(\frac{5n}{2}\right) = \frac{3n}{2}$.

Hence $\gamma_{ctd}(G) \leq \frac{3n}{2} = \left\lfloor\frac{3n+1}{2}\right\rfloor$.

Let D' be a γ_{ctd}-set of G.The block A is constructed as in Case 1.

Then$G \cong A^{\frac{n}{2}}$. The vertices with the symbol in each of the blocks represent the vertices that are to be included in D'.

Therefore D' contains atleast 3 vertices from each block A of $A^{\frac{n}{2}}$. Therefore $\gamma_{ctd}(G) = |D'| \geq 3\left(\frac{n}{2}\right) = \frac{3n}{2} = \left\lfloor\frac{3n+1}{2}\right\rfloor$.

Hence $\gamma_{ctd}(G) = \left\lfloor\frac{3n+1}{2}\right\rfloor$, $n \geq 1$.

K. Theorem 2.5

If $G \cong C_5 \times P_n$, then $\gamma_{ctd}(G) = 2n$, $n \geq 3$.

L. Proof

Let $G \cong C_5 \times P_n$ and $V(G) = \bigcup_{i=1}^{n}\{v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i}\}$ such that $<\{v_{i1}, v_{i2}, \dots, v_{in}\}> \cong P_n^i$, i =1, 2, 3, 4, 5 and $<\{v_{1j}, v_{2j}, v_{3j}, v_{4j}, v_{5j}\}> \cong C_5^j$, j = 1,2, …, n, where P_n^i is the i^{th} copy of P_n and C_5^j is the j^{th} copy of C_5 in $C_5 \times P_n$. $|V(G)| = 5n$.

Let $D = \begin{cases} [\cup_{i=1}^{\frac{n+1}{2}}\{v_{1,2i-1}, v_{5,2i-1}\}] \cup [\cup_{i=2}^{\frac{n-1}{2}}\{v_{3,2i}, v_{4,2i}\}], \text{ if n is odd} \\ \cup_{i=2}^{\frac{n}{2}}\{v_{1,2i-1}, v_{3,2i}, v_{4,2i}, v_{5,2i-1}\}, \text{ if n is even.} \end{cases}$

Then D is a dominating set of G and also <V(G) – D> is a tree obtained from the path $P_n = <\{v_{2,i}, i = 1,2,3,\dots,n\}>$, ($n \geq 2$) by attaching P_3 at each of the vertices $v_{21}, v_{22}, v_{23}, \dots$, and $v_{2,n}$ Therefore D is a ctd-set of G.

and $|D| = \begin{cases} 2\left(\frac{n+1}{2}\right) + 2\left(\frac{n-1}{2}\right) = 2n, \text{ if n is even} \\ 4\left(\frac{n}{2}\right) = 2n, \quad \text{if n is odd.} \end{cases}$

Therefore $\gamma_{ctd}(G) \leq |D'| = 2n$.

Let D' be a γ_{ctd}-set of G. Since $\gamma(C_5) = 2$, D' contains atleast 2 vertices from each of n cycles and hence D'contains atleast 2n vertices. Therefore $\gamma_{ctd}(G) = |D'| \geq 2n$.

Hence $\gamma_{ctd}(G) = 2n, n \geq 1$.

M. Theorem 2.6

If $G \cong C_6 \times P_n$, then $\gamma_{ctd}(G) = \left\lfloor\frac{5n}{2}\right\rfloor$, $n \geq 2$.

1) *Proof.*: Let $G \cong C_6 \times P_n$ and $V(G) = \cup_{i=1}^{n}\{v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i}, v_{6i}\}$ such that $<\{v_{i1}, v_{i2}, \dots, v_{in}\}> \cong P_n^i$, i =1, 2, 3, 4,5,6 and $<\{v_{1j}, v_{2j}, v_{3j}, v_{4j}, v_{5j}, v_{6j}\}> \cong C_6^j$, j = 1, 2, …, n, where P_n^i is the i[th] copy of P_n and C_6^j is the j [th] copy of C_6 in $C_6 \times P_n$ and |V(G)| = 6n.
2) *Case 1:* n is odd.

Let$D' = [\cup_{i=1}^{\frac{n+1}{2}}\{v_{1,2i-1}, v_{5,2i-1}, v_{6,2i-1}\}] \cup [\cup_{i=1}^{n}\{v_{2i}\}][\cup_{i=1}^{\frac{n-1}{2}}\{v_{3,2i}, v_{4,2i}\}]$.

Then $|D'| = 3\left(\frac{n+1}{2}\right) + n + 2\left(\frac{n-1}{2}\right) = \frac{7n+1}{2}$ and $D = V(G) - D'$. Then D is a dominating set of G. Also <V (G) – D > = < D'> is a tree obtained from the path $P_n = <\{v_{2,i}, i = 1,2,3,\dots,n\}>$, ($n \geq 2$) by attaching P_4 at each of the vertices $v_{21}, v_{23}, v_{25}, \dots, v_{2n}$ and attaching P_3 at each of the vertices $v_{22}, v_{24}, \dots, v_{2,n-1}$. Therefore D is a ctd-set of G.

$\gamma_{ctd}(G) \leq |D| = |V(G) - D'| = 6n - \left(\frac{7n+1}{2}\right) = \frac{5n-1}{2}$.

Hence $\gamma_{ctd}(G) \leq \frac{5n-1}{2}$.

Let D' be a γ_{ctd}-set of G. The blocks A,B, A^2 and A^2B are constructed as given below.

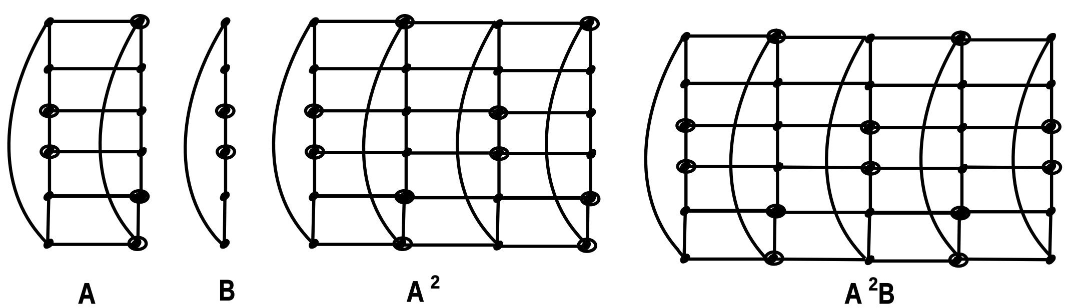

Figure 4

G is obtained by concatenating the blocks $A^{\frac{n-1}{2}}$ and B. That is, $G \cong A^{\frac{n-1}{2}}B$. The vertices with the symbol in each of the blocks represent the vertices that are to be included in D'.

Therefore D' contains atleast 5 vertices from each block A of $A^{\frac{n-1}{2}}$and atleast 2 vertices from block B. Therefore $\gamma_{ctd}(G) = |D'| \geq 5\left(\frac{n-1}{2}\right) + 2 = \frac{5n-1}{2}$ and hence $\gamma_{ctd}(G) = \frac{5n-1}{2} = \left\lfloor\frac{5n}{2}\right\rfloor$.

Case 2: n is even.

Let $D' = [\cup_{i=1}^{\frac{n}{2}}\{v_{1,2i-1}, v_{5,2i-1}, v_{6,2i-1}\}] \cup [\cup_{i=1}^{n}\{v_{2i}\}][\cup_{i=1}^{\frac{n}{2}}\{v_{3,2i}, v_{4,2i}\}]$.

Then $|D'| = 3\left(\frac{n}{2}\right) + n + 2\left(\frac{n}{2}\right) = \frac{7n}{2}$ and $D = V(G) - D'$. Then D is a dominating set of G. Also $<V(G) - D> = <D'>$ is a tree obtained from the path $P_n = <\{v_{2,i},\ i = 1,2,3,\dots,n\}>$, $(n \geq 2)$ by attaching P_4 at each of the vertices $v_{21}, v_{23}, v_{25}, \dots, v_{2,n-1}$ and attaching P_3 at each of the vertices $v_{22}, v_{24} \dots, v_{2n}$. Therefore D is a ctd-set of G.

$\gamma_{ctd}(G) \leq |D| = |V(G) - D'| = 6n - \left(\frac{7n}{2}\right) = \frac{5n}{2}$.

Hence $\gamma_{ctd}(G) \leq \frac{5n}{2}$.

Let D' be a γ_{ctd}-set of G. The block A is constructed as in Case 1.

Then$G \cong A^{\frac{n}{2}}$. The vertices with the symbol in each of the blocks represent the vertices that are to be included in D'.

Therefore D' contains atleast 5 vertices from each block A of $A^{\frac{n}{2}}$.

Therefore $\gamma_{ctd}(G) = |D'| \geq 5\left(\frac{n}{2}\right) = \frac{5n}{2}$ and hence$\gamma_{ctd}(G) = \frac{5n}{2} = \left\lfloor\frac{5n}{2}\right\rfloor$.

Hence $\gamma_{ctd}(G) = \left\lfloor\frac{5n}{2}\right\rfloor$, $n \geq 1$.

N. *Remark 2.2.*

In view of Theorem 2.4, Theorem 2.5, Theorem 2.6, and Theorem 2.7,

1) $\gamma_{ctd}(C_3 \times C_n) = n+1,\ n \geq 2$.
2) $\gamma_{ctd}(C_4 \times C_n) = \left\lfloor\frac{3n+1}{2}\right\rfloor + 1$
3) $\gamma_{ctd}(C_5 \times C_n) = \begin{cases} 2n+1, & \text{if n is even} \\ 2n, & \text{if n is odd.} \end{cases}$
4) $\gamma_{ctd}(C_6 \times C_n) = \left\lfloor\frac{5n}{2}\right\rfloor + 1$.

O. *Remark 2.3.*

1) If $G_1 \cong K_m$ and $G_2 \cong K_n$, then $\gamma_{ctd}(G_1 + G_2) = m + n - 2$.
2) If G_1 and G_2 are any two noncomplete connected graphs of order m and n respectively, with minimum degree atleast two, then $\gamma_{ctd}(G_1 + G_2) \leq m + n - 4$. Equality holds, if $G_1 \cong K_m - e$, $G_2 \cong K_n - e$.
3) For any two connected graphs G_1 and G_2 of order m and n respectively, $\gamma_{ctd}(G_1 \circ G_2) \leq m + n - 4$. Equality holds, if $G_1 \cong P_2$ and $G_2 \cong C_3$.

P. *Distance Two Complementary tree domination number of Cartesian product of graphs*

In the following, distance two complementary tree domination number of $K_m \times K_n$, $K_m \times P_n$, $K_m \times C_n$, $C_3 \times P_n$, $C_4 \times P_n$, $C_5 \times P_n$, $C_6 \times P_n$, $C_3 \times C_n$, $C_4 \times C_n$, $C_5 \times C_n$and $C_6 \times C_n$are given

1) If $G \cong K_m \times K_n$ $(m, n \geq 3$ and $m \leq n)$, then $\gamma_{d2ctd}(G) = \begin{cases} m(n-2) + 1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$
2) If $G \cong K_m \times P_n$ $(4 \leq m \leq n)$, then $\gamma_{d2ctd}(G) = n(m-2)$.
3) $\gamma_{d2ctd}(K_m \times C_n) = n(m-2) + 1$.
4) If $G \cong C_3 \times P_n$, then $\gamma_{d2ctd}(G) = n,\ n \geq 1$.
5) If $G \cong C_4 \times P_n$, then $\gamma_{d2ctd}(G) = \left\lfloor\frac{3n+1}{2}\right\rfloor$, $n \geq 1$.
6) If $G \cong C_5 \times P_n$, then $\gamma_{d2ctd}(G) = 2n,\ n \geq 3$.
7) If $G \cong C_6 \times P_n$, then $\gamma_{d2ctd}(G) = \left\lfloor\frac{5n}{2}\right\rfloor$, $n \geq 1$.
8) $\gamma_{d2ctd}(C_3 \times C_n) = n+1,\ n \geq 2$.
9) $\gamma_{d2ctd}(C_4 \times C_n) = \left\lfloor\frac{3n+1}{2}\right\rfloor + 1$
10) $\gamma_{d2ctd}(C_5 \times C_n) = \begin{cases} 2n+1, & \text{if n is even} \\ 2n, & \text{if n is odd.} \end{cases}$

11) $\gamma_{d2ctd}(C_6 \times C_n) = \left\lfloor \frac{5n}{2} \right\rfloor + 1, n \geq 2.$

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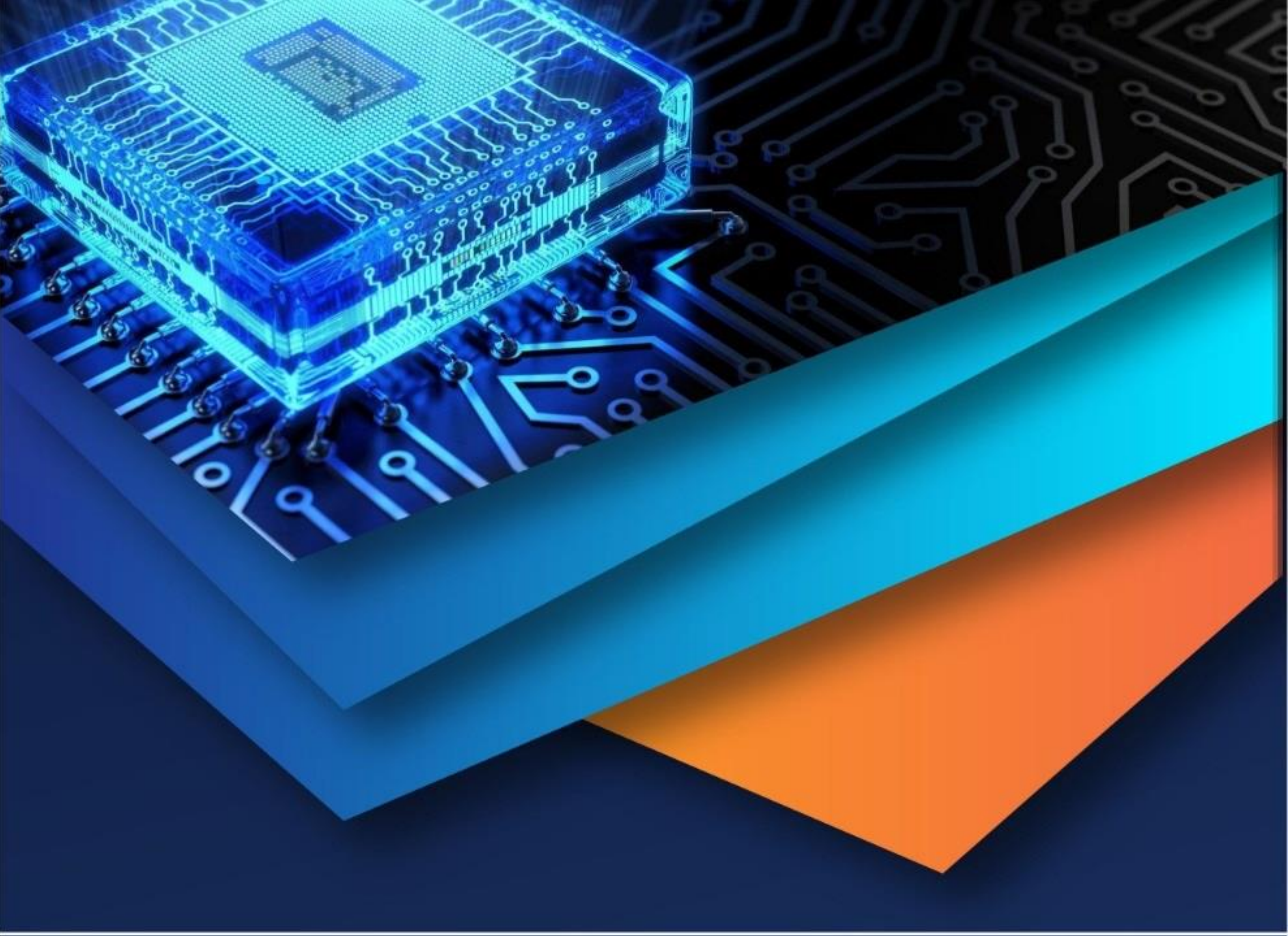

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