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# Complementary Tree Domination number and Distance two Complementary Tree Domination number of Cartesian Product of Graphs

S. Muthammai[1], G. Ananthavalli[2]

[1]*Government Arts and Science College, Kadaladi,Ramanathapuram-623703, India.*

[2] *Government Arts College for Women (Autonomous),Pudukkottai-622001, India.*

*Abstract: A set D of a graph G = (V, E) is a dominating set, if every vertex in V(G) – D is adjacent to some vertex in D. The domination number $\gamma$ (G) of G is the minimum cardinality of a dominating set. A dominating set $D \subseteq V(G)$ is said to be a complementary tree dominating set (ctd-set), if the induced subgraph<V(G) - D>is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}$ (G).A dominating set D is called a distance two complementary tree dominating set, if for each u $\in$V(G) – D, there exists a vertex v $\in$ D such that d(u,v) $\leq$ 2 and also <V(G) – D> is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by $\gamma_{d2ctd}$(G).In this paper, complementary tree domination numbers and distance two complementary tree domination number of cartesian product of some standard graphs are found.*
*Key words: Domination number, complementary tree domination number, distance two complementary tree domination number, Cartesian product.*

## I. INTRODUCTION

Graphs discussed in this paper are finite, undirected and simple connected graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A graph G with p vertices and q edges is denoted by G(p, q). The concept of domination in graphs was introduced by Ore[5]. A set D $\subseteq$V(G) is said to be a dominating set of G, if every vertex in V(G) $-$D is adjacent to some vertex in D. The cardinality of a minimum dominating set in G is called the domination number of G and is denoted by $\gamma$ (G). Muthammai, Bhanumathi and Vidhya[5] introduced the concept of complementary tree dominating set. A dominating set D$\subseteq$ V(G) is said to be a complementary tree dominating set (ctd-set), if the induced subgraph <V(G) - D >is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}$ (G).

The cartesian product of two graphs $G_1$ and $G_2$ is the graph, denoted by $G_1$ x $G_2$ with V ($G_1 \times G_2$) =V ($G_1$) $\times$ V ($G_2$) (where x denotes the cartesian product of sets) and two vertices u = ($u_1$, $u_2$) and v = ($v_1$, $v_2$) in V ($G_1 \times G_2$) are adjacent in $G_1 \times G_2$ whenever [$u_1 = v_1$ and ($u_2$, $v_2$) $\in$ E($G_2$)] or [$u_2 = v_2$ and ($u_1$, $v_1$) $\in$ E($G_1$)].

The concept of distance two complementary tree dominating set is introduced in [4]. A dominating set D is called a distance two complementary tree dominating set, if for each u $\in$V(G) – D, there exists a vertex v $\in$ D such that d(u,v) $\leq$ 2, and also <V(G) – D> is a tree. The minimum cardinality of a distance two complementary tree dominating set is said to be distance two complementary tree domination number of G and is denoted by $\gamma_{d2ctd}$(G).

In this paper, complementary tree domination number and distance two complementary tree domination number of $K_m$ x $K_n$, $K_m$ x $P_n$, $K_m$ x $C_n$, $C_3$ x $P_n$, $C_4$ x $P_n$, $C_5$ x $P_n$, $C_6$ x $P_n$, $C_3$ x $C_n$, $C_4$ x $C_n$, $C_5$ x $C_n$ and $C_6$ x $C_n$ are found. Any undefined terms in this paper may be found in Harary[2].

## II. COMPLEMENTARY TREE DOMINATION NUMBER OF CARTESIAN PRODUCT OF GRAPHS

*A. Theorem 2.1*

If G $\cong$ $K_m$ x $K_n$(m, n $\geq$ 3 and m $\leq$ n), then$\gamma_{ctd}$(G) =$\begin{cases} m(n-2) + 1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$

*B. Proof.*

Let G $\cong$ K$_m$ x K$_n$.

Let $V(G) = \bigcup_{i=1}^{m}\{v_{i1}, v_{i2}, \ldots, v_{in}\}$ such that $< \{v_{i1}, v_{i2}, \ldots, v_{in}\} > \cong K_n^i$, i =1,2, …, m and $< \{v_{1j}, v_{2j}, \ldots, v_{mj}\} > \cong K_m^j$, j=1,2, …, n, where $K_n^i$ is the i$^{th}$ copy of K$_n$ and $K_m^j$ is the j$^{th}$ copy of K$_m$ in K$_m$ x K$_n$. $|V(G)| = mn$.

### C. Case 1: m = n.

et $D' = \left(\bigcup_{i=1}^{m-1}\{v_{ii}, v_{i,i+1}\}\right) \cup \{v_{m,m}\}$ and D = V(G) –D'. Then $V(G) - D = D'$ and $|D'| = 2(m-1) + 1 = 2m - 1$. For i =1, 2,3, …, m-1, the vertices$v_{ii}, v_{i,i+1}$ in V(G) – D are adjacent to $v_{i1}$ in D, and the vertex $v_{mm}$ is adjacent to $v_{m1}$ in D. Therefore D is a dominating set of G. $<V(G) -D> \cong P_{2(m-1)+1} = P_{2m-1}$. Since D is a ctd-set of G. Hence$\gamma_{ctd}(G) \leq |D| = |V(G)| - |D'| = mn - (2m-1) = m(n-2) + 1$.

It is to be noted that, any tree in G is a path. Let D' be a $\gamma_{ctd}$-set of G. The longest path that can be obtained from the subgraph of G induced by the vertices of V(G)–D' is $P_{2m-1}$. That is, $<V(G)–D'> \cong P_{2m-1}$. D' contains atleast mn – (2m – 1) = m(n – 2) + 1 vertices. Therefore$\gamma_{ctd}(G) = |D'| \geq m(n-2) + 1$.

Hence $\gamma_{ctd}(G) = m(n-2) + 1$.

### D. Case 2: m < n.

Let $D' = \bigcup_{i=1}^{m}\{v_{ii}, v_{i,i+1}\}$ and D = V(G) – D'. Then V(G) – D = D' and $|D'| = 2m$. The vertices $v_{11}, v_{12}$ are adjacent to $v_{1n}$ and $v_{ii}, v_{i,i+1}$ (i = 2, 3, …, m) are adjacent to $v_{i1}$, (i = 2, 3, …, m) in D. Therefore D is a dominating set of G. Since <V (G) – D > $\cong P_{2m}$, D is a ctd-set of G. Therefore $\gamma_{ctd}(G) \leq |V(G)| - |D'| = mn - 2m = m(n-2)$.

As in case 1, any tree in G is a path. Let D' be $\gamma_{ctd}$-set of G. The longest path that can be obtained from the subgraph of G induced by the vertices of V(G) – D' is $P_{2m}$.

That is $<V (G) –D'> \cong P_{2m}$. Therefore D' contains atleast mn – 2m = m(n – 2) vertices. Therefore$\gamma_{ctd}(G) = |D'| \geq m(n-2)$.

Therefore$\gamma_{ctd}(G) = m(n-2)$.

Hence $\gamma_{ctd}(G) = \begin{cases} m(n-2) + 1, & \text{if } m = n \\ m(n-2), & \text{if } m < n \end{cases}$.

### E. Remark 2.1

The set D defined in Case 1 and Case 2 is also a d2ctd-set of G. Since any vertex u in D which is at distance two from a vertex of D, <(V (G) – D) $\cup$ {u}> either disconnected or contains cycle.

### F. Example 2.1

For the graph G given in Figure 1.a and Figure 1.b, the set of vertices marked with ● is a minimum ctd-set of K$_m$ x K$_n$ and $\gamma_{ctd}$(K$_4$ x K$_4$) = 9 and $\gamma_{ctd}$(K$_4$ x K$_5$) = 12.

**G $\cong$ K$_4$ x K$_4$**   **G $\cong$ K$_4$ x K$_5$**
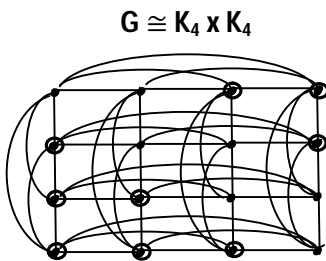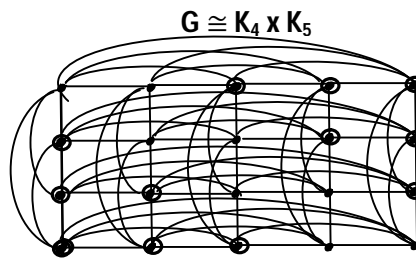


**Figure 1.a**    **Figure 1.b**

### G. Theorem 2.2.

If G $\cong$ K$_m$ x P$_n$(4$\leq$ m $\leq$ n), then $\gamma_{ctd}(G) = n (m-2)$.

*1) Proof.*

Let G $\cong$ K$_m$ x P$_n$, m,n $\geq$ 4.

Let $V(G) = \bigcup_{i=1}^{m}\{v_{i1}, v_{i2}, \ldots, v_{in}\}$ such that $< \{v_{i1}, v_{i2}, \ldots, v_{in}\} > \cong K_n^i$, $i = 1, 2, \ldots, m$ and $< \{v_{1j}, v_{2j}, \ldots, v_{mj}\} > \cong P_m^j$, $j = 1, 2, \ldots,$ n, where $K_n^i$ is the $i^{th}$ copy of $K_n$ and $P_m^j$ is the $j^{th}$ copy of $P_m$ in $K_m \times P_n$.

$$\text{Let } D' = \begin{cases} \{v_{1n}\} \cup [\cup_{i=1}^{n}\{v_{2i}\}] \cup [\cup_{i=1}^{\frac{n-1}{2}}\{v_{1,2i-1}, v_{3,2i}\}], & \text{if n is odd} \\ [\cup_{i=1}^{n}\{v_{2,i}\}] \cup [\cup_{i=1}^{\frac{n}{2}}\{v_{1,2i-1}, v_{3,2i}\}], & \text{if n is even} \end{cases} .$$

Then $|D'| = 2n$. If $D = V(G) - D'$, then D is a dominating set of G. Also $<V(G) - D> = <D'> \cong P_n \circ K_1$. Therefore D is a ctd-set of G and $\gamma_{ctd}(G) \leq |D| = mn - 2n = n(m - 2)$.

Hence $\gamma_{ctd}(G) \leq n(m - 2)$.

Let $D'$ be a $\gamma_{ctd}$-set of G. Since $D'$ is a ctd-set of G, $D'$ contains atleast $(m - 2)$ vertices in each of n copies of $K_m$. Hence $D'$ contains atleast $n(m - 2)$ vertices. Therefore $\gamma_{ctd}(G) = |D'| \geq n(m - 2)$.

Hence $\gamma_{ctd}(G) = n (m - 2)$.

### H.  Example 2.2.

For the graph G given in Figure 2, the set of vertices marked with ◉ is a minimum ctd-set of $K_m \times C_n$ and $\gamma_{ctd}(K_4 \times K_9) = 18$.
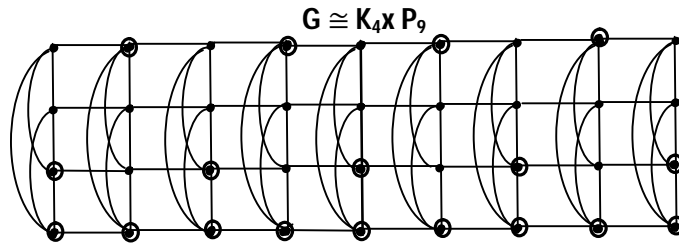


$G \cong K_4 \times P_9$

Figure 2

### I.  Theorem 2.3.

$G \cong C_3 \times P_n$, then $\gamma_{ctd}(G) = n$, $n \geq 1$.

#### 1)  Proof.

Let $G \cong C_3 \times P_n$.

Let $V(G) = \bigcup_{i=1}^{n}\{v_{1i}, v_{2i}, v_{3i}\}$ such that $< \{v_{i1}, v_{i2}, \ldots, v_{in}\} > \cong P_n^i$, $i = 1,2,3$ and $< \{v_{1j}, v_{2j}, v_{3j}\} > \cong C_3^j$, $j = 1,2, \ldots, n$, where $P_n^i$ is the $i^{th}$ copy of $P_n$ and $C_3^j$ is the $j^{th}$ copy of $C_3$ in $C_3 \times P_n$.

$$\text{Let } D = \begin{cases} \{v_{1n}\} \cup [\cup_{i=1}^{\frac{n-1}{2}}\{v_{1,2i-1}, v_{3,2i}\}], & \text{if n is odd} \\ \cup_{i=1}^{\frac{n}{2}}\{v_{1,2i-1}, v_{3,2i}\}, & \text{if n is even} \end{cases}$$

Then D is a dominating set of G. Also $<V(G) - D> \cong P_n \circ K_1$. Therefore D is a ctd-set of G and $|D| =$

$$\begin{cases} 2\left(\frac{n-1}{2}\right) + 1 = n, & \text{if n is even} \\ 2\left(\frac{n}{2}\right) = n, & \text{if n is odd} \end{cases} \text{ and } \gamma_{ctd}(G) \leq |D| = n.$$

Let $D'$ be a $\gamma_{ctd}$-set of G. Then $D'$ contains atleast one vertex from each cycle. Since $C_3 \times P_n$ contains n copies of $C_3$, $D'$ contains atleast n vertices. $\gamma_{ctd}(G) = |D'| \geq n$.

Hence $\gamma_{ctd}(G) = n$, $n \geq 1$.

### J.  Theorem 2.4.

If $G \cong C_4 \times P_n$, then $\gamma_{ctd}(G) = \left\lfloor \frac{3n+1}{2} \right\rfloor$, $n \geq 1$.

Let $G \cong C_4 \times P_n$ and $V(G) = \bigcup_{i=1}^{n}\{v_{1i}, v_{2i}, v_{3i}, v_{4i}\}$ such that $< \{v_{i1}, v_{i2}, \ldots, v_{in}\} > \cong P_n^i$, $i = 1,2,3,4$ and $< \{v_{1j}, v_{2j}, v_{3j}, v_{4j}\} > \cong C_4^j$, $j = 1,2, \ldots, n$, where $P_n^i$ is the $i^{th}$ copy of $P_n$ and $C_4^j$ is the $j^{th}$ copy of $C_4$ in $C_4 \times P_n$ and $|V(G)| = 4n$.

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Let $D^{'} = \{ v_{31} \} \cup [\cup_{i=1}^{\frac{n-1}{2}}\{v_{1,2i}, v_{4,2i}, v_{3,2i+1} \}] \cup [\cup_{i=1}^{n}\{v_{2i} \}]$.

Then $|D^{'}| = 1 + 3\left(\frac{n-1}{2}\right) + n = \frac{5n-1}{2}$ and $D = V(G) - D^{'}$. Then D is a dominating set of G. Also $<V (G) – D > = <D^{'}>$ is a tree obtained from the path $P_n = < \{v_{2,i}, i = 1,2,3,…,n\} >$, $(n \geq 2)$ by attaching $P_3$ at each of the vertices $v_{22}, v_{24}, v_{26}, …, v_{2,n-1}$ and attaching a pendant edge at each of the vertices $v_{21}, v_{23} …, v_{2,n}$. Therefore D is a ctd-set of G, and

$$\gamma_{ctd}(G) \leq |D| = |V(G) - D^{'}| = 4n - \left(\frac{5n-1}{2}\right) = \frac{3n+1}{2}.$$

Hence $\gamma_{ctd}(G) \leq \frac{3n+1}{2} = \left\lceil\frac{3n+1}{2}\right\rceil$.

Let $D^{'}$ be a $\gamma_{ctd}$-set of G. The blocks A, B, $A^2$ and $A^2B$ are constructed as given below.
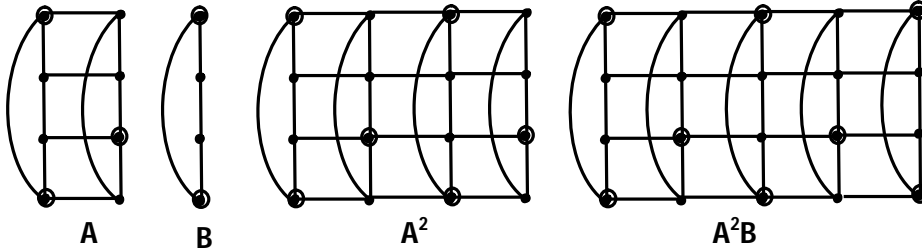


**A**   **B**   **$A^2$**   **$A^2B$**

**Figure 3**

G is obtained by concatenating the blocks $A^{\frac{n-1}{2}}$ and B. That is, $G \cong A^{\frac{n-1}{2}} B$. The vertices with the symbol ⊙ in each of the blocks represent the vertices that are to be included in $D^{'}$

Therefore $D^{'}$ contains atleast 3 vertices from each block A of $A^{\frac{n-1}{2}}$ and atleast 2 vertices from block B.

Therefore $\gamma_{ctd}(G) = |D^{'}| \geq 2 + 3\left(\frac{n-1}{2}\right) = \frac{3n+1}{2} = \left\lceil\frac{3n+1}{2}\right\rceil$.

Hence $\gamma_{ctd}(G) = \left\lceil\frac{3n+1}{2}\right\rceil$, $n \geq 1$.

, $v_{3,2i-1} \}] \cup [\cup_{i=1}^{n}\{v_{2i} \}]$ and $D = V(G) – D^{'}$. Then $|D^{'}| = 3\left(\frac{n}{2}\right) + n = \frac{5n}{2}$. Then D is a dominating set of G. Also $<V (G) – D > = <D^{'}>$ is a tree obtained from the path $P_n = < \{v_{2i}, i = 1,2,3,…,n\} >$, $(n \geq 2)$ by attaching $P_3$ at each of the vertices $v_{22}, v_{24}, v_{26}, …,$ and $v_{2,n}$ and attaching a pendant edge at each of the vertices $v_{21}, v_{23} …,$ and $v_{2,n-1}$. Therefore D is a ctd-set of G and $\gamma_{ctd}(G) \leq |D| = |V(G) - D^{'}| = 4n - \left(\frac{5n}{2}\right) = \frac{3n}{2}$.

Hence $\gamma_{ctd}(G) \leq \frac{3n}{2} = \left\lceil\frac{3n+1}{2}\right\rceil$.

Let $D^{'}$ be a $\gamma_{ctd}$-set of G.The block A is constructed as in Case 1.

Then $G \cong A^{\frac{n}{2}}$. The vertices with the symbol ⊙ in each of the blocks represent the vertices that are to be included in $D^{'}$.

Therefore $D^{'}$ contains atleast 3 vertices from each block A of $A^{\frac{n}{2}}$. Therefore $\gamma_{ctd}(G) = |D^{'}| \geq 3\left(\frac{n}{2}\right) = \frac{3n}{2} = \left\lceil\frac{3n+1}{2}\right\rceil$.

Hence $\gamma_{ctd}(G) = \left\lceil\frac{3n+1}{2}\right\rceil$, $n \geq 1$.

*K. Theorem 2.5*

If $G \cong C_5 \times P_n$, then $\gamma_{ctd}(G) = 2n$, $n \geq 3$.

*L. Proof*

Let $G \cong C_5 \times P_n$ and $V(G) = \cup_{i=1}^{n}\{v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i}\}$ such that $< \{v_{i1}, v_{i2}, … , v_{in}\} > \cong P_n^i$, $i = 1, 2, 3, 4, 5$ and $< \{v_{1j}, v_{2j}, v_{3j}, v_{4j}, v_{5j}\} >$ $\cong C_5^j$, $j = 1,2, …, n$, where $P_n^i$ is the $i^{th}$ copy of $P_n$ and $C_5^j$ is the $j^{th}$ copy of $C_5$ in $C_5 \times P_n$. $|V(G)| = 5n$.

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Let D $=\begin{cases} [U_{i=1}^{\frac{n+1}{2}}\{v_{1,2i-1,}\ v_{5,2i-1}\}] \cup [U_{i=2}^{\frac{n-1}{2}}\{v_{3,2i,}v_{4,2i,}\}], \text{if n is odd} \\ U_{i=2}^{\frac{n}{2}}\{v_{1,2i-1,}\ v_{3,2i,}v_{4,2i,}\ v_{5,2i-1,}\},\ \text{if n is even.}\end{cases}$

Then D is a dominating set of G and also $<V(G) - D>$ is a tree obtained from the path $P_n = < \{v_{2,i},\ i = 1,2,3,\dots,n\} >,$ (n ≥ 2) by attaching $P_3$ at each of the vertices $v_{21}, v_{22}, v_{23},\dots,$ and $v_{2,n}$ Therefore D is a ctd-set of G.

and $|D| = \begin{cases} 2\left(\frac{n+1}{2}\right) + 2\left(\frac{n-1}{2}\right) = 2n,\ \text{if n is even} \\ 4\left(\frac{n}{2}\right) = 2n,\qquad \text{if n is odd.}\end{cases}$

Therefore $\gamma_{ctd}(G) \leq |D'| = 2n.$

Let $D'$ be a $\gamma_{ctd}$-set of G. Since $\gamma(C_5) = 2$, $D'$ contains atleast 2 vertices from each of n cycles and hence $D'$ contains atleast 2n vertices. Therefore $\gamma_{ctd}(G) = |D'| \geq 2n.$

Hence $\gamma_{ctd}(G) = 2n, n \geq 1.$

*M.  Theorem 2.6*

If $G \cong C_6 \text{ x } P_n$, then $\gamma_{ctd}(G) = \left\lfloor\frac{5n}{2}\right\rfloor, n \geq 2.$

1) *Proof.*: Let $G \cong C_6 \text{ x } P_n$ and $V(G) = U_{i=1}^{n}\{v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i}, v_{6i}\}$ such that $< \{v_{i1}, v_{i2},\dots, v_{in}\} > \cong P_n^i,$ i =1, 2, 3, 4,5,6 and $< \{v_{1j}, v_{2j}, v_{3j}, v_{4j}, v_{5j}, v_{6j}\} > \cong C_6^j$, j = 1, 2, …, n, where $P_n^i$ is the i$^{th}$ copy of $P_n$ and $C_6^j$ is the j $^{th}$ copy of $C_6$ in $C_6 \text{ x } P_n$ and $|V(G)| = 6n.$

2) *Case 1:* n is odd.

Let $D'=[U_{i=1}^{\frac{n+1}{2}}\{v_{1,2i-1,}\ v_{5,2i-1,}\ v_{6,2i-1}\}] \cup [U_{i=1}^{n}\{v_{2i}\}][U_{i=1}^{\frac{n-1}{2}}\{v_{3,2i,}\ v_{4,2i}\}].$

Then $|D'| = 3\left(\frac{n+1}{2}\right) + n + 2\left(\frac{n-1}{2}\right) = \frac{7n+1}{2}$ and $D = V(G) - D'$. Then D is a  dominating set of G. Also $<V (G) - D > = < D'>$ is a tree obtained from the path $P_n= < \{v_{2,i},\ i = 1,2,3,\dots,n\} >,$ (n ≥ 2) by attaching $P_4$ at each of the vertices $v_{21}, v_{23}, v_{25},\dots, v_{2n}$ and attaching $P_3$ at each of the vertices $v_{22}, v_{24}\dots, v_{2,n-1}$. Therefore D is a ctd-set of G.

$\gamma_{ctd}(G) \leq |D| = |V(G) - D'| = 6n - \left(\frac{7n+1}{2}\right) = \frac{5n-1}{2}.$

Hence $\gamma_{ctd}(G) \leq \frac{5n-1}{2}.$

Let $D'$ be a $\gamma_{ctd}$-set of G. The blocks A,B, $A^2$ and $A^2B$ are constructed as given below.
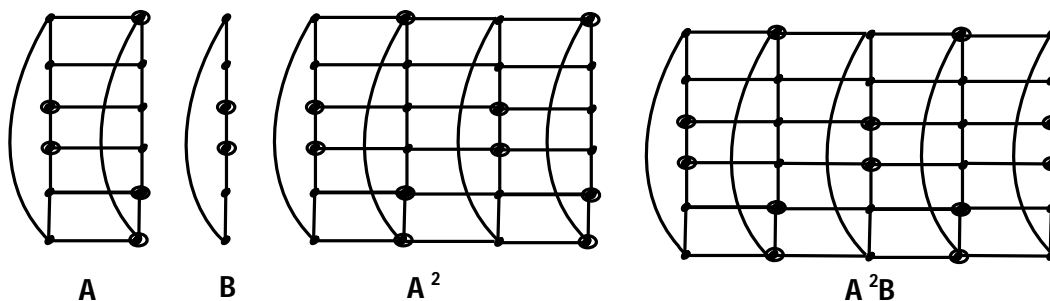


A        B        A$^2$        A$^2$B

Figure 4

G is obtained by concatenating the blocks $A^{\frac{n-1}{2}}$ and B. That is, $G \cong A^{\frac{n-1}{2}}$ B. The vertices with the symbol    in each of the blocks represent the vertices that are to be included in $D'$.

Therefore $D'$ contains atleast 5 vertices from each block A of $A^{\frac{n-1}{2}}$ and atleast 2 vertices from block B. Therefore $\gamma_{ctd}(G) = |D'| \geq 5\left(\frac{n-1}{2}\right) + 2 = \frac{5n-1}{2}$ and hence $\gamma_{ctd}(G) = \frac{5n-1}{2} = \left\lfloor\frac{5n}{2}\right\rfloor.$

Case 2: n is even.

Let $D' = [U_{i=1}^{\frac{n}{2}}\{v_{1,2i-1}, v_{5,2i-1}, v_{6,2i-1}\}] \cup [U_{i=1}^{n}\{v_{2i,}\}][U_{i=1}^{\frac{n}{2}}\{v_{3,2i}, v_{4,2i}\}]$.

Then $|D'| = 3\left(\frac{n}{2}\right) + n + 2\left(\frac{n}{2}\right) = \frac{7n}{2}$ and $D = V(G) - D'$. Then D is a dominating set of G. Also $<V(G) - D> = <D'>$ is a tree obtained from the path $P_n = < \{v_{2,i}, \ i = 1,2,3, \dots, n\} >$, $(n \geq 2)$ by attaching $P_4$ at each of the vertices $v_{21}, v_{23}, v_{25,} \dots, v_{2,n-1}$ and attaching $P_3$ at each of the vertices $v_{22}, v_{24} \dots, v_{2n}$. Therefore D is a ctd-set of G.

$$\gamma_{ctd}(G) \ \leq |D| = |V(G) - D'| = 6n - \left(\frac{7n}{2}\right) = \frac{5n}{2}.$$

Hence $\gamma_{ctd}(G) \ \leq \frac{5n}{2}$.

Let $D'$ be a $\gamma_{ctd}$-set of G. The block A is constructed as in Case 1.

Then $G \cong A^{\frac{n}{2}}$. The vertices with the symbol ⊙ in each of the blocks represent the vertices that are to be included in $D'$.

Therefore $D'$ contains atleast 5 vertices from each block A of $A^{\frac{n}{2}}$.

Therefore $\gamma_{ctd}(G) \ = \ |D'| \geq 5 \left(\frac{n}{2}\right) = \frac{5n}{2}$ and hence $\gamma_{ctd}(G) \ = \ \frac{5n}{2} = \left\lceil\frac{5n}{2}\right\rceil$.

Hence $\gamma_{ctd}(G) \ = \left\lceil\frac{5n}{2}\right\rceil, n \geq 1$.

*N. Remark 2.2.*

In view of Theorem 2.4, Theorem 2.5, Theorem 2.6, and Theorem 2.7,

1) $\gamma_{ctd}( C_3 \times C_n) = n+1, n \geq 2.$

2) $\gamma_{ctd}( C_4 \times C_n) = \left\lceil\frac{3n+1}{2}\right\rceil + 1$

3) $\gamma_{ctd}( C_5 \times C_n) = \begin{cases} 2n + 1, & \text{if n is even} \\ 2n, & \text{if n is odd.} \end{cases}$

4) $\gamma_{ctd}( C_6 \times C_n) = \left\lceil\frac{5n}{2}\right\rceil + 1.$

*O. Remark 2.3.*

1) If $G_1 \cong K_m$ and $G_2 \cong K_n$, then $\gamma_{ctd}(G_1 + G_2) = m + n-2.$

2) If $G_1$ and $G_2$ are any two noncomplete connected graphs of order m and n respectively, with minimum degree atleast two, then $\gamma_{ctd}(G_1 + G_2) \leq m + n - 4$. Equality holds, if $G_1 \cong K_m - e$, $G_2 \cong K_n - e$.

3) For any two connected graphs $G_1$ and $G_2$ of order m and n respectively, $\gamma_{ctd}(G_1 \circ G_2) \leq m + n - 4$. Equality holds, if $G_1 \cong P_2$ and $G_2 \cong C_3.$

*P. Distance Two Complementary tree domination number of Cartesian product of graphs*

In the following, distance two complementary tree domination number of $K_m \times K_n$, $K_m \times P_n$, $K_m \times C_n$, $C_3 \times P_n$, $C_4 \times P_n$, $C_5 \times P_n$, $C_6 \times P_n$, $C_3 \times C_n$, $C_4 \times C_n$, $C_5 \times C_n$ and $C_6 \times C_n$ are given

1) If $G \cong K_m \times K_n$ (m, n $\geq 3$ and m $\leq$ n), then $\gamma_{d2ctd}(G) = \begin{cases} m(n-2) + 1, & \text{if m = n} \\ m(n-2), & \text{if m < n} \end{cases}$

2) If $G \cong K_m \times P_n$ ( $4 \leq m \leq n$), then $\gamma_{d2ctd}(G) = n(m-2).$

3) $\gamma_{d2ctd}( K_m \times C_n) = n(m-2) + 1.$

4) If $G \cong C_3 \times P_n$, then $\gamma_{d2ctd}(G) = n, n \geq 1.$

5) If $G \cong C_4 \times P_n$, then $\gamma_{d2ctd}(G) = \left\lceil\frac{3n+1}{2}\right\rceil, n \geq 1.$

6) If $G \cong C_5 \times P_n$, then $\gamma_{d2ctd}(G) = 2n, n \geq 3.$

7) If $G \cong C_6 \times P_n$, then $\gamma_{d2ctd}(G) = \left\lceil\frac{5n}{2}\right\rceil, n \geq 1.$

8) $\gamma_{d2ctd}(C_3 \times C_n) = n+1, n \geq 2.$

9) $\gamma_{d2ctd}(C_4 \times C_n) = \left\lceil\frac{3n+1}{2}\right\rceil + 1$

10) $\gamma_{d2ctd}(C_5 \times C_n) = \begin{cases} 2n + 1, & \text{if n is even} \\ 2n, & \text{if n is odd.} \end{cases}$

*11)* $\gamma_{d2ctd}(C_6 \times C_n) = \left\lfloor \frac{5n}{2} \right\rfloor + 1,\ n \geq 2.$

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