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Analysis of an M/M/1 Queue with Two Vacation Policies

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Abstract: We consider M/M/1 queue with single vacation and multiple working vacation. We derive the stochastic decomposition structures of the stationary queue length and waiting time, and obtain the distribution of additional queue length and additional delay by using quasi birth and death process and matrix-geometric solution method.
Keywords: M/M/1 queue; Vacation; Working vacation; Stochastic decomposition; Matrix-geometric solution.

I. INTRODUCTION

In this paper, we consider M/M/1 queue with two vacation policies i.e single vacation and multiple working vacation. As a regular busy period ends, a vacation is taken, during which the server completely stops the service to any of the new arrivals. The server will enter into a working vacation if there are still no customers in the system when the vacation ends. During the working vacation, the server operates with variation in service rate but does not completely halts the service. On returning of the server from his working vacation and finds the system non-empty, the system will be resumed to a regular busy period, otherwise the server immediately goes for another working vacation.

Queuing systems with vacations have been studied by various researchers for their profound applications in many real life situations such as telecommunication, computer networks, production systems, and so on. The survey papers of Doshi[1] and Teghem[2] the monograph of Takagi [3] and Tian and Zhang [4] acts as a reference for the readers. In these studies, it is assumed that the server completely ceases service during a vacation. However, there are many situations where the server does not remain completely inactive during a vacation. But provides service to the queue at a lower rate. This idea was first utilized by Servi and Finn [5] and introduced a class of semi vacation polices. This type of vacation is called a working vacation (WV). Servi and Finn [5] analyzed M/M/1 queue with multiple working vacations policy and derived the PGF for the number of customers in the system and LST for waiting time distributions and utilized results to analyze the system performance of gateway router in fiber communication networks. Later M/M/1 with multiple working vacation model was also studied by Liu, Xu and Tian[6] to obtain explicit expressions of the performance measures and their stochastic decomposition by using the matrix-geometric method. Subsequently, by applying the same method, M/M/1 queue with single working vacation was analyzed by Tian and Zhao [7] and obtained various steady state indicators. Moreover, Kim, Choi and Chae [8], Wu and Takagi [9] and Li et al. [10] extended the work of Finn [5] to an M/G/1/WV queue . Baba [11] first analyzed the GI/M/1 queue with general arrival process and multiple working vacations by utilizing the matrix-geometric solution method. Later, Li and Tian [12] investigated the GI/M/1 queue with single working vacation. Banik et al. [13] analyzed a GI/M/1/N queuing system with limited waiting space and working vacation.

The model in this paper can be utilized for building server maintenance model. When the regular busy period ends, the server goes for a vacation in order to make the server maintenance. After the vacation period, if there are customers staying in the system, the server may resume to the regular service period for the more benefits. Otherwise, the server goes for the working vacation waiting for the arriving works, in which the server operates on the customers with a lower service rate in order to economize operation cost and energy consumption. The rest of the paper is organized as follows. In section 2, we provide the description of the model as a quasi-birth-death process and establish the rate matrix to ensure the existence of analytic solutions in various systems. In section 3, stationary queue length is obtained. Section 4 presents the stochastic decomposition structures for the queue length and waiting time of the customer. Finally the numerical examples are presented in section 5.

II. MODEL DESCRIPTION

A. *The queuing system we consider here is described explicitly as follows*

1) Customer's arrival to the system occurs according to the Poisson process with rate λ, and service times during a regular service period follow the exponential distribution with rate μ_b.

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2) The service times during a working vacation is exponentially distribution with rate μ_v ($\mu_v < \mu_b$).The durations of the vacation period and working vacation period are exponential distributed with mean θ and γ ,respectively.
3) The two vacation terminologies are described as follows: After a regular service period, the server goes for a vacation. If there are customers in the system at a vacation completion instant, the server immediately serve the customers i.e. it changes its service rate from μ_v to μ_b, and new regular busy period starts. Otherwise, the system enters into a working vacation. The server continues to take working vacation till it finds atleast one customer waiting in the queue at a vacation completion epoch, and another regular service period will start.
4) We assume that the inter-arrival times, service times during a service period, service times during a working vacation Period, working vacation times and vacation times are mutually independent.In addition, the service discipline is First in First out.

Let L(t) denote the number of customers in the system at time t and

$$J(t)=\begin{cases}0 & \text{the server is in the vacation period at time t,}\\ 1 & \text{the server is in the working vacation period at time t,}\\ 2 & \text{the server is in the regular service period at time t.}\end{cases}$$

Clearly the process $\{(L(t),J(t)),t\geq 0\}$ is a Markov chain with state space

$$S=\{\{(0,0)(0,1)\}\bigcup\{(k,j)\},k=1,2,...,j=0,1,2\}$$

Using the lexicographical order for the states, the infinitesimal generator for the Markov process can be written as a block-partitioned matrix

$$Q=\begin{pmatrix} A_{0,0} & A_{01} & & & \\ A_{1,0} & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & A_2 & A_1 & A_0 \\ & & \vdots & \vdots & \vdots \end{pmatrix}$$

where

$$A_{0,0}=\begin{pmatrix}-(\lambda+\theta) & \theta \\ 0 & -\lambda\end{pmatrix},\ A_{0,1}=\begin{pmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\end{pmatrix},\ A_{10}=\begin{pmatrix}0 & 0\\ 0 & \mu_v \\ \mu_b & 0\end{pmatrix}$$

$$A_0=\begin{pmatrix}\lambda & & \\ & \lambda & \\ & & \lambda\end{pmatrix},\ A_1=\begin{pmatrix}-(\lambda+\theta) & & \theta\\ & -(\lambda+\theta+\gamma) & \gamma \\ & & -(\lambda+\mu_b)\end{pmatrix}$$

$$A_2=\begin{pmatrix}0 & & \\ & \mu_v & \\ & & \mu_b\end{pmatrix}$$

The matrix structure of Q indicates that the Markov Chain $\{(L(t),J(t)),t\geq 0\}$ is a quasi birth and death process. To analyze the QBD process, we first need to get the rate matrix, denoted by R, which is the minimal non-negative solution of the matrix quadratic equation

$$R^2A_2+RA_1+A_0=0 \quad (1)$$

The following lemma presents the explicit soluion of R.

Lemma 1.If $\frac{\lambda}{\mu_b}<1$, the minimal non-negative solution of matrix equation (1) has the following expression:

$$R=\begin{pmatrix} \alpha & 0 & \rho \\ & r & \frac{\gamma r}{\mu_b(1-r)} \\ & & \rho \end{pmatrix} \quad (2)$$

$$r=\frac{1}{2\mu_v}\left(\lambda+\gamma+\mu_v-\sqrt{(\lambda+\gamma+\mu_v)^2-4\lambda\mu_v}\right) \quad (3)$$

$$\alpha=\frac{\lambda}{\lambda+\theta}, \qquad \rho=\frac{\lambda}{\mu_b} \quad (4)$$

Proof. Since the matrices A_0, A_1 and A_2 of (1) are all upper triangular, therefore the matrix R is upper triangular. Hence, we can consider that the matrix R has the same structure as

$$R=\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ & r_{22} & r_{23} \\ & & r_{33} \end{pmatrix}$$

Substituting R^2 and R into (1), we get

$$\begin{aligned}
&-(\lambda+\theta)r_{11}+\lambda=0 \\
&\mu_v(r_{11}r_{12}+r_{12}r_{22})-(\lambda+\gamma+\mu_v)r_{12}=0 \\
&\mu_b(r_{11}r_{13}+r_{12}r_{23}+r_{13}r_{33})+\theta r_{11}+\gamma r_{12}-r_{13}(\lambda+\mu_b)=0 \qquad (5) \\
&\mu_v r_{22}^2-(\lambda+\gamma+\mu_v)r_{22}+\lambda=0 \\
&\mu_b(r_{22}r_{23}+r_{23}r_{33})+\gamma r_{22}-(\lambda+\mu_b)r_{23}=0 \\
&\mu_b r_{33}^2-(\lambda+\mu_b)r_{22}+\lambda=0
\end{aligned}$$

From the above set of equations, we can obtain the minimal non-negative solution of (1) by using the fact that $\mu_v z^2-(\lambda+\gamma+\mu_v)z+\lambda=0$ has a unique root

$$r=\frac{1}{2\mu_v}\left(\lambda+\gamma+\mu_v-\sqrt{(\lambda+\gamma+\mu_v)^2-4\lambda\mu_v}\right)$$

in interval (0,1). Substituting $r_{22}=r$ and $r_{33}=\rho$ into the fifth equation of (5), we get $r_{23}=\frac{\gamma r}{\mu_b(1-r)}$. From the first equation, we get $r_{11}=\frac{\lambda}{\lambda+\theta}=\alpha$ .Also from the third equation, we get $r_{13}=\rho$ by $r_{11}=\alpha$, $r_{12}=0$ and $r_{33}=\rho$, then the proof is complete.

Lemma 2.r satisfies the following expression

$$\lambda+\gamma+\mu_v(1-r)=\mu_v+\frac{\gamma}{1-r}=\frac{\lambda}{r} \qquad (6)$$

Proof. From (5), rsatisfies the following equation

$$\mu_v z^2 - (\lambda + \gamma + \mu_v) z + \lambda = 0$$

Equivalently, we get

$$\mu_v + \frac{\gamma}{1-r} = \frac{\lambda}{r}$$

which completes the proof. Based on the modified method in section 1.5 of Neuts[14], for the infinitesimal generator Q, the QBD process $\{(L(t), J(t)), t \geq 0\}$ is ergodiciff the spectral radius sp(R) of the rate matrix R satisfies $sp(R) < 1$ and the linear system of equations $(\pi_{00}, \pi_{01}, \pi_{10}, \pi_{11}, \pi_{12}) B[R] = 0$ has a positive solution, where

$$B[R] = \begin{pmatrix} A_{00} & A_{01} \\ A_{10} & A_1 + RA_2 \end{pmatrix} = \begin{bmatrix} -(\lambda+\theta) & \theta & \lambda & 0 & 0 \\ 0 & -\lambda & 0 & \lambda & 0 \\ 0 & 0 & -(\lambda+\theta) & 0 & (\lambda+\theta) \\ 0 & \mu_v & 0 & \frac{-\lambda}{r} & \frac{\gamma}{1-r} \\ \mu_b & 0 & 0 & 0 & -\mu_b \end{bmatrix} \quad (7)$$

Lemma 3. The linear system of equations $(\pi_{00}, \pi_{01}, \pi_{10}, \pi_{11}, \pi_{12}) B[R] = 0$ has a positive solution as follows

$$\pi_{01} = \frac{(1-r)\theta}{r\gamma}\pi_{00}, \quad \pi_{10} = \alpha\pi_{00}, \quad \pi_{11} = \frac{(1-r)\theta}{\gamma}\pi_{00}, \quad \pi_{12} = \frac{\lambda+\theta}{\mu_b}\pi_{00} \qquad (8)$$

where π_{00} is a constant that can be determined by using the normalization condition.

Proof. The matrix equation $(\pi_{00}, \pi_{01}, \pi_{10}, \pi_{11}, \pi_{12}) B[R] = 0$ can be rewritten as

$$\begin{aligned} &-(\lambda+\theta)\pi_{00} + \mu_b \pi_{12} = 0 \\ &\theta\pi_{00} - \lambda\pi_{01} + \mu_v \pi_{11} = 0 \\ &\lambda\pi_{00} - (\lambda+\theta)\pi_{10} = 0 \qquad (9) \\ &\lambda\pi_{01} - \frac{\lambda}{r}\pi_{11} = 0 \\ &(\lambda+\theta)\pi_{10} + \frac{\gamma}{1-r}\pi_{11} - \mu_b \pi_{12} = 0 \end{aligned}$$

Solving the above set of equations in terms of π_{00}, we can obtain (8) and the proof is completed.

From Lemmas (1) and (3), a necessary and sufficient condition for the QBD process to be positive recurrent is given as follows.

Theorem 1.The QBD process $\{(L(t), J(t)), t \geq 0\}$ is positive recurrent if and only if $\rho < 1$.

Proof.According to the section 1.5 of Neuts [14], the QBD process $\{(L(t), J(t)), t \geq 0\}$ is positive recurrent iff the spectral radius of rate matrix

$$sp(R) = min(r, \alpha, \rho) < 1$$

and the linear systems of equations

$$(\pi_{00}, \pi_{01}, \pi_{10}, \pi_{11}, \pi_{12}) B[R] = 0$$

hasa positive solution. By lemma 1 and lemma 3, it is clear that the above conditions are satisfied iff $\rho < 1$.

III. STATIONARY DISTRIBUTION OF QUEUE LENGTH

Let (L,J) be the stationary limit of the QBD process $\{(L(t), J(t)), t \geq 0\}$ and define

$$\pi_{kj} = P\{L = k, J = j\} = \lim_{t\to\infty}\{(L(t) = k, J(t) = j\}, \quad (k, j) \in S,$$

$$\pi_0 = (\pi_{00}, \pi_{01})$$

$$\pi_k = (\pi_{k0}, \pi_{k1}, \pi_{k2}), \quad k \geq 1$$

Theorem 2.If $\rho < 1$, the joint probability distribution of (L,J) is

$$\pi_{k0} = \pi_{00}\alpha^k, \ k \geq 0, \ \pi_{k1} = \pi_{00}\frac{(1-r)\theta}{r\gamma}r^k, \ k \geq 0,$$

$$\pi_{k2} = \pi_{00}\left[\rho\sum_{j=0}^{k-1}\alpha^j\rho^{k-1-j} + \frac{\theta}{\mu_b}\sum_{j=0}^{k-1}r^j\rho^{k-1-j}\right], \ k \geq 1 \ (10)$$

Where

$$\pi_{00} = (1-r)(1-\alpha)(1-\rho)\left[(1-\rho)(1-r) + \frac{\theta}{r\gamma}(1-r)(1-\alpha)(1-\rho) + \rho(1-r) + \frac{\theta}{\mu_b(1-\alpha)}\right]^{-1} \quad (11)$$

Proof. By applying matrix geometric solution method [14], we obtain

$$\pi_k = (\pi_{k0}, \pi_{k1}, \pi_{k2}) = (\pi_{10}, \pi_{11}, \pi_{12})R^{k-1}, \ k \geq 1 \ (12)$$

$$(\pi_{00}, \pi_{01}, \pi_{10}, \pi_{11}, \pi_{12})B[R] = 0 \ (13)$$

From (2), we get

$$R^k = \begin{bmatrix} \alpha^k & 0 & \rho\sum_{j=0}^{k-1}\alpha^j\rho^{k-1-j} \\ 0 & r^k & \frac{\gamma r}{\mu_b(1-r)}\sum_{j=0}^{k-1}\alpha^j\rho^{k-1-j} \\ 0 & 0 & \rho^k \end{bmatrix} (14)$$

Substituting the matrix expression R^{k-1} into (12) and noting that π_{00} can be derived by using the normalization condition and hence the proof is completed.

From theorem 2, the probabilities that the server in various state are obtained as follows:

$$P\{\text{the server is in the vacation period}\} = \sum_{k=0}^{\infty}\pi_{k0} = \pi_{00}\frac{1}{1-\alpha} \ (15)$$

$$P\{\text{the server is in working vacation period}\} = \sum_{k=0}^{\infty}\pi_{k1} = \pi_{00}\frac{\theta}{r\gamma} \ (16)$$

$$P\{\text{the server is in working regular service period}\} = \sum_{k=1}^{\infty}\pi_{k1} = \pi_{00}\left[\frac{\rho}{(1-\alpha)(1-\alpha)} + \frac{\theta}{\mu_b(1-r)(1-\rho)}\right] \quad (17)$$

Where π_{00} is given by (11).

IV. STOCHASTIC DECOMPOSITIONS

In order to have a better comparison with the already existing models, we often try to decompose the quantities of interest into various factors. A stochastic decomposition property plays a vital role in vacation queuing models and points out the effects of system vacation on its performance measures like queue length and sojourn time. We can try to do the same for the system under consideration.

Theorem 3.If $\rho < 1$ and $\mu_b > \mu_v$, the stationary queue length L in system can be decomposed into sum of two independent random variables: $L = L_0 + L_d$, where L_0 is the queue length of the classical M/M/1 queue in steady state and follows a geometric distribution with parameter 1- ρ and the additional queue length L_d due to the effect of the two vacation policies has the following PGF

$$L_d(z) = K^*\left[(1-\alpha)(1-r) + \frac{\theta}{r\gamma}(1-r)(1-\alpha)(1-r) + \frac{(1-\alpha)r\theta}{\lambda - r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{z(1-r)}{1-rz} + \alpha(1-r)\frac{z(1-\alpha)}{1-\alpha z}\right] \quad (18)$$

where

$$K^* = \left[(1-\alpha)(1-r) + \frac{\theta}{r\gamma}(1-r)(1-\alpha)(1-r) + \frac{(1-\alpha)r\theta}{\lambda - r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right) + \alpha(1-r)\right]^{-1} \quad (19)$$

Proof. By theorem 2, the PGF of L can be written as

$$L(z) = \sum_{k=0}^{\infty}\pi_{k0}z^k + \sum_{k=0}^{\infty}\pi_{k1}z^k + \sum_{k=1}^{\infty}\pi_{k2}z^k$$

$$= \pi_{00}\left[\frac{1}{1-\alpha z} + \frac{\theta}{\gamma}\frac{1-r}{r}\frac{1}{1-rz} + \frac{\rho}{1-\rho z}\frac{z}{1-\alpha z} + \frac{\theta}{\mu_b}\frac{1}{1-\alpha z}\frac{z}{1-rz}\right]$$

$$= \frac{1-\rho}{1-\rho z}K^*\Big\{(1-\alpha)(1-r)\frac{1-\rho z}{1-\alpha z} + (1-\alpha)(1-r)\frac{\theta}{\gamma}\frac{1-r}{r}\frac{1-\rho z}{1-rz}$$

$$1-\alpha)(1-r)\rho\frac{z}{1-\alpha z} + (1-\alpha)(1-r)\frac{\theta}{\mu_b}\frac{z}{1-rz}\Big\}$$

$$= \frac{1-\rho}{1-\rho z}L_d(z) \quad (20)$$

Where

$$K^* = \left[(1-\rho)(1-r) + \frac{\theta}{r\gamma}(1-\alpha)(1-\rho)(1-r) + \rho(1-r) + \frac{\theta}{\mu_b}(1-\alpha)\right]^{-1} \quad (21)$$

Note that

$$\frac{1-r}{1-rz}(1-\rho z) = (1-r) + (r-\rho)\frac{z(1-r)}{1-rz},$$

$$\frac{1-\alpha}{1-\alpha z}(1-\rho z) = (1-\alpha) + (\alpha-\rho)\frac{z(1-\alpha)}{1-\alpha z}.$$

Therefore $L_d(z)$ takes the form

$$L_d(z) = K^*\Big\{(1-\alpha)(1-r) + \frac{1-r}{r}\frac{\theta}{\gamma}(1-\alpha)(1-r) + \big[(1-r)(\alpha-\rho) + \rho(1-r)\big]\frac{z(1-\alpha)}{1-\alpha z}$$

$$+\left[\frac{\theta}{r\gamma}(1-\alpha)(1-r)(r-\rho) + \frac{\theta}{\mu_b}(1-\alpha)\right]\frac{z(1-r)}{1-rz}\Big\} \quad (22)$$

From (6), we ave

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$$\frac{\gamma r}{\mu_b(1-r)} = \rho - r\frac{\mu_v}{\mu_b} \quad (23)$$

Using the above relation, we obtain

$$L_d(z) = K^*\Big[(1-\alpha)(1-r) + \frac{\theta}{r\gamma}(1-r)(1-\alpha)(1-r) + \frac{(1-\alpha)r\theta}{\lambda - r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{z(1-r)}{1-rz} + \alpha(1-r)\frac{z(1-\alpha)}{1-\alpha z}\Big] \quad (24)$$

From (21), K^* can be expressed as

$$K^* = \left[(1-\alpha)(1-r) + (1-r)(\alpha-\rho) + \frac{\theta}{r\gamma}(1-r)\big((1-\alpha)(1-\rho) + (1-\alpha)(r-\rho)\big) + \rho(1-r) + \frac{\theta}{\mu_b}(1-\alpha)\right]^{-1}$$

$$= \Big[(1-\alpha)(1-r) + \frac{\theta}{r\gamma}(1-r)(1-\alpha)(1-r) + \alpha(1-r) + \theta\frac{1-\alpha}{\lambda - r\mu_v} r\left(1-\frac{\mu_v}{\mu_b}\right)\Big]^{-1} \quad (25)$$

Hence the above relation shows that $L_d(z)$ is a PGF.

Based on stochastic decomposition in theorem 4.1, we can easily obtain

$$E(L_d) = K^*\left[\theta\frac{1-\alpha}{\lambda - r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{r}{1-r} + (1-r)\frac{\alpha}{1-\alpha}\right] \quad (26)$$

$$E(L) = \frac{\rho}{1-\rho} + K^*\left[\theta\frac{1-\alpha}{\lambda - r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{r}{1-r} + (1-r)\frac{\alpha}{1-\alpha}\right] \quad (27)$$

Theorem 4.If $\rho<1$ and $\mu_b > \mu_v$, the stationary sojourn time S of an arrival can be decomposed into the sum of two independent variables: $S = S_0 + S_d$, where S_0 is the sojourn time of an arrival in a corresponding classical M/M/1 queue and is exponentially distributed with parameter μ_b - λ and S_d is the additional delay with the LST given by

$$S_d^*(s) = K^*\Big[(1-r)(1-\alpha)\theta\left(\frac{1-r}{r\gamma} - \frac{1-\frac{\mu_v}{\mu_b}}{\lambda - r\mu_v}\right) + \frac{\theta(1-\alpha)}{\lambda - r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{\frac{\lambda}{r}-\lambda}{\frac{\lambda}{r}-\lambda+s} + (1-r)\frac{\frac{\lambda}{\alpha}-\lambda}{\frac{\lambda}{\alpha}-\lambda+s}\Big] \quad (28)$$

Where

$$K^* = \left[(1-r)(1-\alpha)\theta\left(\frac{1-r}{r\gamma} - \frac{1-\frac{\mu_v}{\mu_b}}{\lambda - r\mu_v}\right) + \frac{\theta(1-\alpha)}{\lambda - r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right) + (1-r)\right]$$

Proof. The classical relation between the PGF of L and LST \cite{15}, of sojourn time S is

$$L(z) = S^*(\lambda(1-z))$$

From theorem 4, we get

$$L(z) = \frac{1-\rho}{1-\rho z}K^*\Big[(1-r)(1-\alpha)\left(1+\frac{\theta}{\gamma}\frac{1-r}{r}\right) + \frac{r(1-\alpha)\theta}{\lambda - r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{(1-r)z}{1-rz} + \alpha(1-r)\frac{(1-\alpha)z}{1-\alpha z}\Big] \quad (29)$$

Taking $z=1-\frac{s}{\lambda}$ in (29), we obtain

$$S^*(s)=\frac{\mu_b(1-\rho)}{\mu_b(1-\rho)+s}K^*\left[(1-r)(1-\alpha)\theta\left(\frac{1-r}{r\gamma}-\frac{1-\frac{\mu_v}{\mu_b}}{\lambda-r\mu_v}\right)+\frac{\theta(1-\alpha)}{\lambda-r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{\frac{\lambda}{r}-\lambda}{\frac{\lambda}{r}-\lambda+s}+(1-r)\frac{\frac{\lambda}{\alpha}-\lambda}{\frac{\lambda}{\alpha}-\lambda+s}\right]$$

$$=\frac{\mu_b(1-\rho)}{\mu_b(1-\rho)+s}S_d^*(s) \quad (30)$$

From (25), we have

$$K^*=\left[(1-\alpha)(1-r)+\frac{\theta}{r\gamma}(1-r)(1-\alpha)(1-r)+(\alpha-1+1)(1-r)+\theta\frac{1-\alpha}{\lambda-r\mu_v}(r-1+1)\left(1-\frac{\mu_v}{\mu_b}\right)\right]^{-1}$$

$$=\left[(1-r)(1-\alpha)\theta\left(\frac{1-r}{r\gamma}-\frac{1-\frac{\mu_v}{\mu_b}}{\lambda-r\mu_v}\right)+\frac{\theta(1-\alpha)}{\lambda-r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)+(1-r)\right]^{-1} \quad (31)$$

Therefore $S_d^*(s)$ is a LST.

We can easily obtain

$$E(S_d)=K^*\left[\frac{\theta(1-\alpha)}{\lambda-r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{r}{\lambda(1-r)}+(1-r)\frac{\alpha}{\lambda(1-\alpha)}\right]=\frac{1}{\lambda}E(L_d) \quad (32)$$

$$E(S)=\frac{1}{\mu_b(1-\rho)}+K^*\left[\frac{\theta(1-\alpha)}{\lambda-r\mu_v}\left(1-\frac{\mu_v}{\mu_b}\right)\frac{r}{\lambda(1-r)}+(1-r)\frac{\alpha}{\lambda(1-\alpha)}\right] \quad (33)$$

V. NUMERICAL RESULTS

In this section, we present some numerical examples to investigate the effects of various parameters on the mean queue length and mean sojourn time. The main findings in this study are itemized as

A. Figs. 5.1 (a) and 5.1 (b) demonstrates the impact of vacation service rate μv on the mean queue length and mean sojourn time with μ_b =1, γ =0.3 and θ =0.4. If μv is fixed, it is obvious that, the higher rho is, the larger the mean queue length and mean sojourn time become. We also observe that increased μv leads to the smaller values of $E(L)$ and $E(S)$.

B. Assuming that μ_b =1, μ_v =0.3, θ =0.4, Figs. 5.2 (a) and 5.2 (b) displays the variation of the mean queue length and mean sojourn time against γ for different values of ρ. We can find that the mean queue length and mean sojourn time decrease and both converge to fixed constants as γ increases, as we expect.

C. In Figs. 5.3 (a) and 5.3 (b), we assume that μ_b =1, μ_v =0.3, γ =0.3 and plot the mean queue length and mean sojourn time, with θ varying for different values of rho. A similar property as the Figs. 5.2 (a) and 5.2 (b) can be found that the mean queue length and mean sojourn time both decrease and converges evidently to fixed constants as θ increases.

D. Fig. 5.4 gives the comparison between M/M/1/SV+MWV, M/M/1 with single vacation and M/M/1 with multiple working vacation in terms of their mean queue length, with μ_v varying, where μ_b =5, λ =0.5, γ =0.3, θ =0.5.

According to the numerical analysis of the mean queue length and mean sojourn time, we find that our queuing system has some reasonable practical implications. So, based on the particular problems, the service companies can choose the reasonable vacation rate, working vacation rate and service rate so that the companies can work more flexibly and effectively.

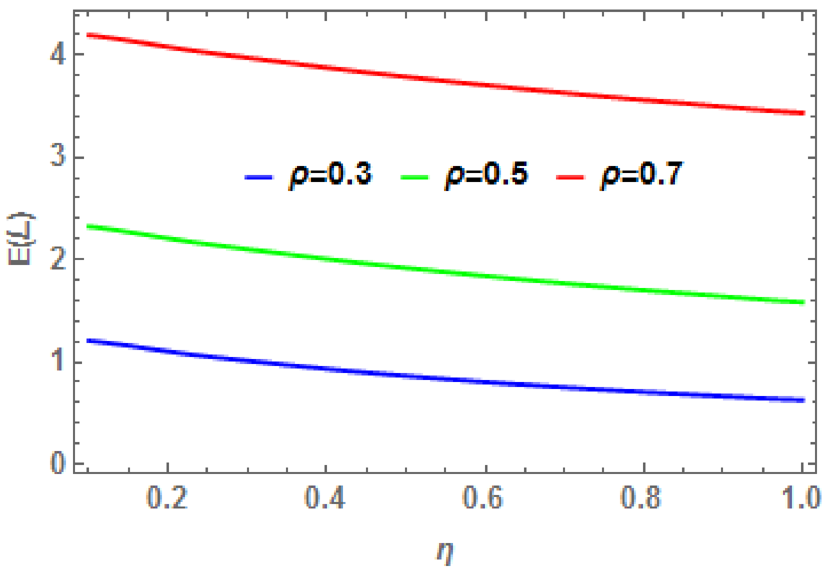


Figure 1

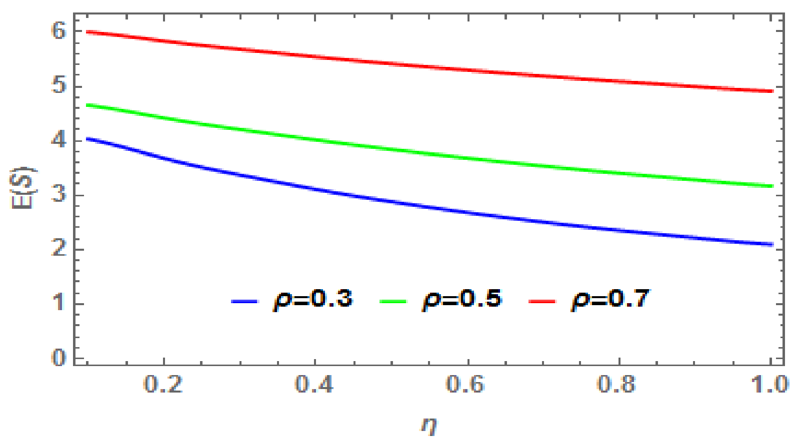


Figure 2

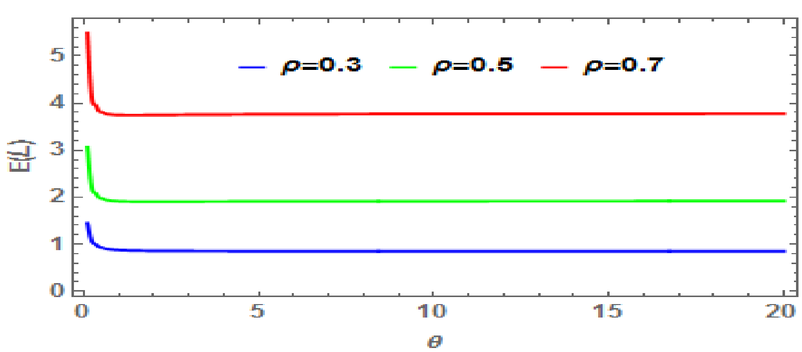


Figure 3

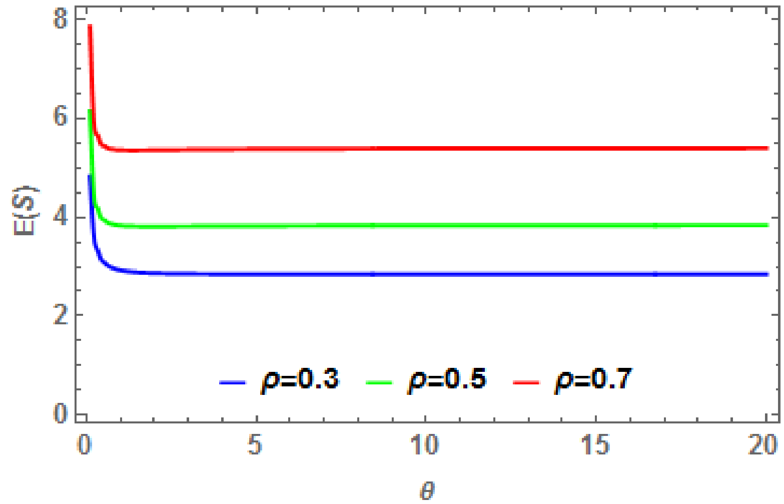
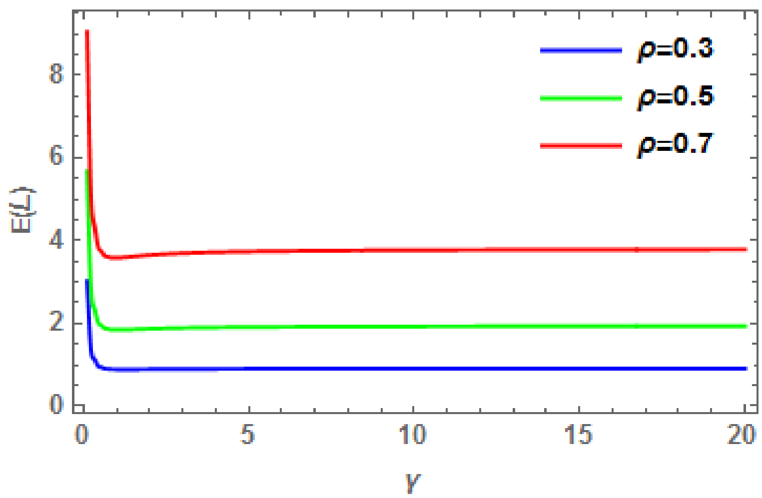


Figure 4



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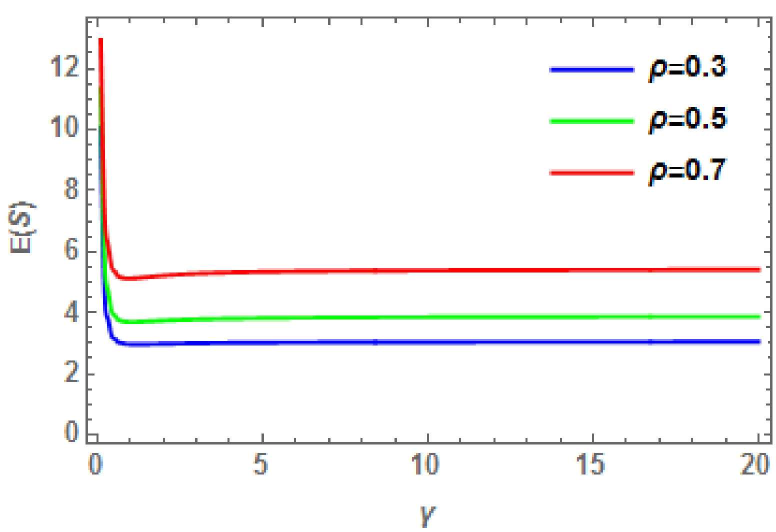

Figure 6

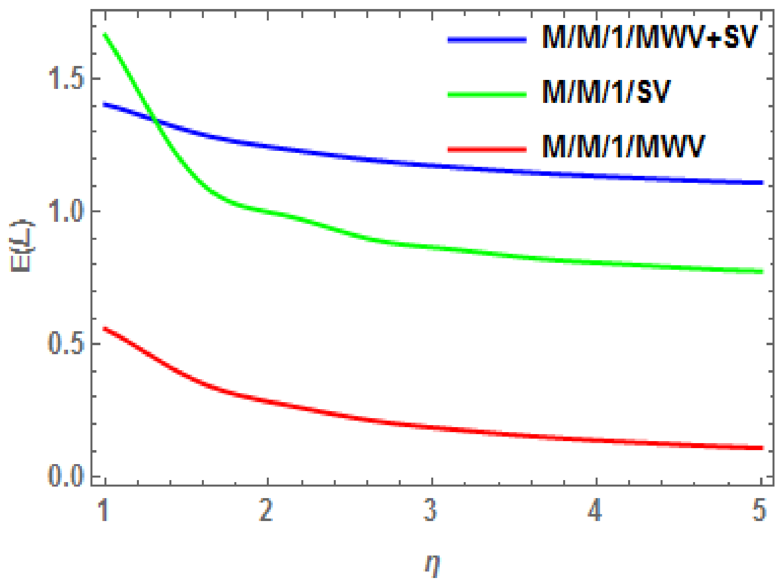

Figure7

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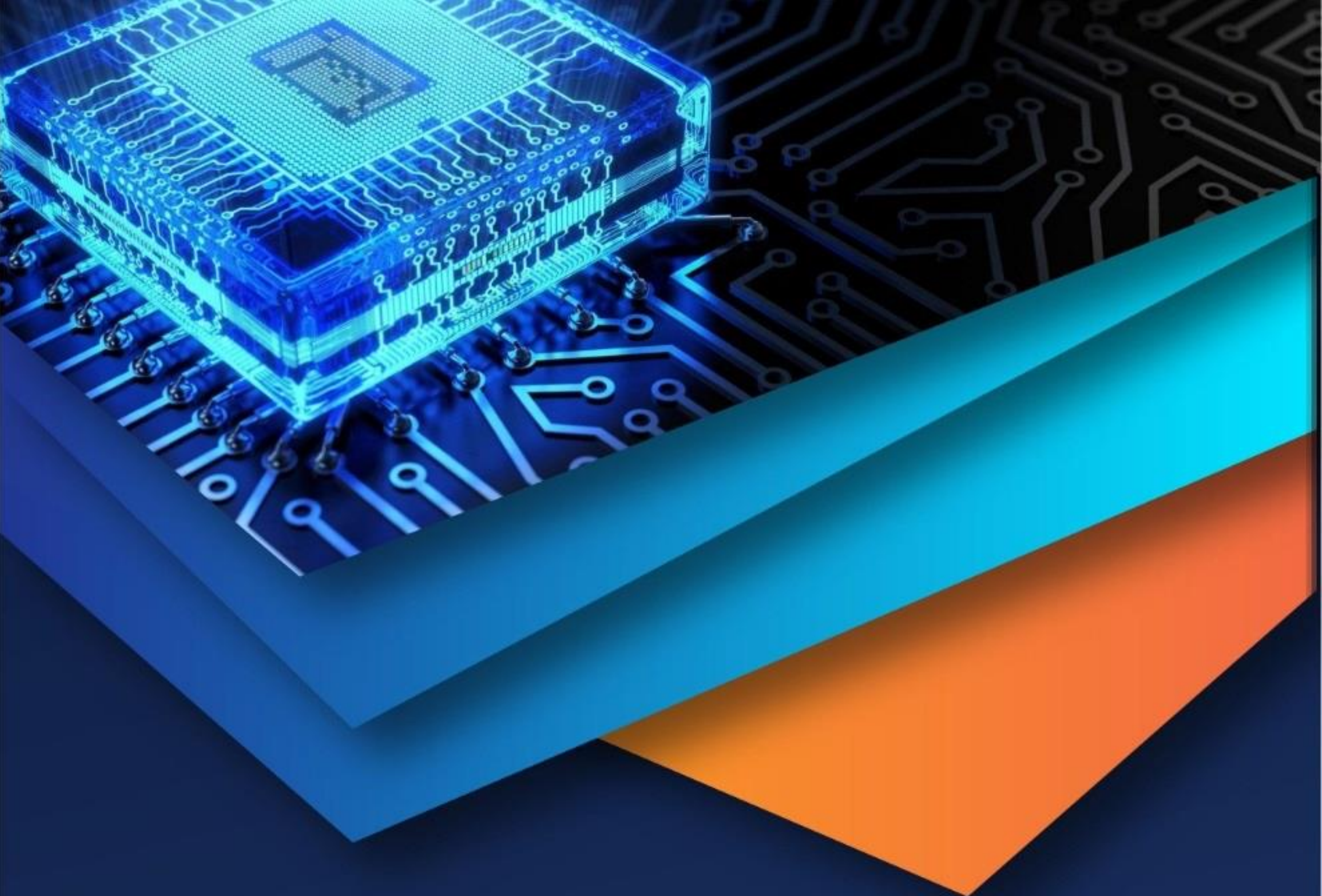

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