iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 **Issue:** XII **Month of publication:** December 2017

DOI:

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

Strong LICT Domination in Graphs

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***Abstract:** For any graph $G = (V, E)$, the Lict graph $n(G)$ of a graph $G$is a graph whose set of vertices is the union of the set of edges and cutvertices of G in which two vertices are adjacent if and only if the corresponding edges of G are adjacent and the corresponding cutvertices are incident to the edges. For any two adjacent vertices u and v we say that u strongly dominates $v$if $deg(u) \geq deg(v)$. A dominating set D of a graph $n(G)$ is a strong Lict dominating set if every vertex in $V[n(G)] - D$ is strongly dominated by at least one vertex in D. Strong Lict domination number $\gamma_{Sn}(G)$ of G is the minimum cardinality of strong Lict dominating set of G. In this paper, we study graph theoretic properties of $\gamma_{Sn}(G)$ and many bounds were obtain in terms of elements of G and its relationship with other domination parameters were found.*

***Keywords:** Dominating set/Line graph/Lict graph/Restrained domination/Edge Lict domination/ connected Lict domination/Strong split domination/Strong non split domination/Strong Lict domination.*
Subject Classification number.AMS - 05C69, 05C70.

I. INTRODUCTION

In this paper, all the graphs consider here are simple and finite. For any undefined terms or notation can be found in Harary [2]. In general, we use $< X >$ to denote the subgraph induced by the set of vertices X and $N(v)$ and $N([v])$ denote open (closed) neighborhoods of a vertex v. The minimum distance between any two farthest vertices of a connected G is called the diameter of G and is denoted by $diamG$.

A set $S \subseteq V(G)$ is a dominating set of G, if every vertex in $V - S$ is adjacent to some vertex in S. The minimum cardinality of vertices in such a set is called the domination number of G and is denoted by $\gamma(G)$. A dominating set $S \subseteq V(G)$ is a connected dominating set, if the induced subgraph$< S >$ has no isolated vertices. The connected domination number $\gamma_c(G)$ of G is the minimum cardinality of a connected dominating set of G. A dominating set S of a lictgraph is a restrained dominating set of $n(G)$, if every vertex not in S is adjacent to a vertex in S and to a vertex in $V[n(G)] - S$. The restrained domination number of a lict graph $n(G)$ is denoted by $\gamma_{rn}(G)$ is the minimum cardinality of a restrained dominating set in $n(G)$. The concept of restrained domination in graphs was introduced by Domke [1]and further studied in graph valued functions by M.H.M.[10].

The concept of a dominating set D of a graph G is a strong split dominating set if the induced subgraph$\langle V - D \rangle$ is totally disconnected with at least two vertices. The strong split domination number $\gamma_{ss}(G)$ of graph G is the minimum cardinality of a strong split dominating set of G. Hence the concept of Strong Split Block domination was introduced by M.H. Muddebihal and Nawazoddin U. Patel(see[5]). A concept of a lict dominating set $D \subseteq V[n(G)]$ is said to be dominating set of $n(G)$, if very vertex in $V[n(G)] - D$ is adjacent to some vertex in D. The domination number of $n(G)$ is denoted by $\gamma_n(G)$and is the minimum cardinality of a dominating set in $n(G)$. Analogously, the connected domination number in lict graph is as follows. A dominating set D of lict graph $J = n(G)$ is connected dominating set, if the induced subgraph $\langle D \rangle$ is also connected .The connected domination number of $n(G)$ is the minimum cardinality of a minimal connected dominating set in $n(G)$and is denoted by $\gamma_{nc}(G)$. The Lict domination and connected Lict domination in graphs, introduced by M.H. Muddebihal [8]. A set $D \subseteq V[L(G)]$ is said to be a line dominating set of $L(G)$, if every vertex not in D is adjacent to atleast one vertex in D.The domination number of $L(G)$ is denoted by $\gamma_l(G)$ and is the minimum cardinality of a dominating set in $L(G)$. Analogously, we define edge domination number in lictgraph . A set F of edges of lict graph $J = n(G)$is called edge dominating set of $n(G)$ if every edge in $E[n(G)] - F$ is adjacent to at least one edge in F .The edge domination number $\gamma_n'(G)$ of a graph $n(G)$ is the minimum cardinality of a edge dominating set in $n(G)$. Hence The edge dominating set F is called connected edge dominating set of $n(G)$, if the induced subgraph $\langle F \rangle$ is also connected. The connected edge lict domination number is denoted by $\gamma'_{nc}(G)$ see [8]. Further, edge domination, strong domination, strong split domination and strong non split domination in some graph valued functions were studied see [4,6,7 and 9].

A dominating set D of a graph $B(G)$ is a strong nonsplit block dominating set if the induced subgraph $\langle V[B(G)] - D \rangle$ is complete. The strong nonsplit block domination number $\gamma_{snsb}(G)$ of G is the minimum cardinality of strong nonsplit block dominating set of G. Hence A dominating set D of a graph $n(G)$ is a strong non split Lict dominating set if the induced subgraph $\langle V[n(G)] - D \rangle$ is complete. The strong nonsplitLict domination number $\gamma_{snn}(G)$ of G is the minimum cardinality of strong nonsplitLict dominating set of G. The concept of strongnonsplit Block domination was introduced by M.H. Muddebihal and Nawazoddin U. Patel(see[6]). The concept of strong domination was introduced by Sampath kumar and PushpaLatha in [11]. Given two adjacent vertices u and v we say that u strongly dominates $v$if $\deg(u) \geq \deg(v)$. A set $D \subseteq V(G)$ is strong dominating set of G if very vertex in $V - D$ is strongly dominated by at least one vertex in D. The strong domination number $\gamma_s(G)$ is the minimum cardinality of a strong dominating set of G. A dominating set D of a graph $n(G)$ is a strong Lict dominating set if every vertex in $\langle V[n(G)] - D \rangle$ is strongly dominated by at least one vertex in D. Strong Lict domination number $\gamma_{Sn}(G)$ of G is the minimum cardinality of strong Lict dominating set of G.

In this paper, many bounds on $\gamma_{Sn}(G)$ were obtained in terms of elements of G but not the elements of $n(G)$. Also its relation with other domination parameters were established.

The following figure shows the formation of lict graph $n(G)$ and relation between $\gamma_{Sn}(G)$ and diameter of G.

Diameter of $G = 3\{2\} = \gamma_{Sn} - set$

$$\gamma_{Sn}(G) = 1 = \frac{diam(G) + 2}{5}$$

We need the following theorem for our further results.

Theorem A[3].If G is non-trivial connected graph whose vertices have degree d_i and l_i be the number of edges to which cutvertex C_i belongs in G, the lict graph n(G) has $q + \sum C_i$ vertices and $-q + \sum \left(\frac{d_i^2}{2} + l_i \right)$ edges.

II. MAIN RESULTS

Theorem 1. For any connected (p,q) graph G, $\gamma_{Sn}(G) \geq \left\lceil \frac{diam(G)+2}{5} \right\rceil$.

Proof.Let $S = \{e_1, e_2, e_3, \ldots \ldots \ldots, e_n\} \subseteq E(G)$ be the set of edges which constitute the longest path between two distinct vertices $u, v \in V(G)$ such that $d(u,v) = diam(G)$. Now, let $S_1 \subseteq E(G)$, $\forall e_i \in S_1$ has a maximum edge degree in G. Since $S_1 \subseteq V[n(G)]$ be the minimal set of vertices which covers all the vertices of $n(G)$, then S_1 is a minimal dominating set of $n(G)$. Further if $e_i \in S_1$, deg $(e_i) \geq$ deg $(e_j)$where $e_j \in V[n(G)] - S_1$, then S_1 is a minimal strong lict dominating set. It follows that $|S_1| \geq \left|\frac{S+2}{5}\right|$. Hence $\gamma_{Sn}(G) \geq \left\lceil \frac{diam(G)+2}{5} \right\rceil$.

The next theorem gives a upper bound for γ_{Sn}(T) in terms of the vertices and end vertices of G.

Theorem 2. For any (p,q)tree T, $\gamma_{Sn}(T) \leq p-m$, where m is the number of endvertices in T. Equality holds if $T = K_{1,p}$ with $p \geq 2$ vertices.

Proof. If $diam(T) \leq 3$, then the result is obvious. Let $diam(T) > 3$ and $V_1 = \{v_1, v_2, v_3, ..., v_n\}$ be the set of all end vertices of T with $|V_1| = m$. Further $E' = \{e_1, e_2, e_3, ..., e_j\}$; $C' = \{c_1, c_2, c_3, ..., c_i\}$ be the set of edges and cutvertices in T. In n(G), $V[n(T)] = E'(T) \cup C'(T)$ and in T $\forall e_i$ incident with C_j, $1 \leq j \leq i$ forms a complete induced subgraph as a block in $n(T)$. Hencethe number of blocks in $n(T) = |C'|$. Let $\{e_1, e_2, e_3, ..., e_j\} \in E'(T)$ which are nonendedges of G forms a cutvertices $C_1 = \{c_1, c_2, c_3, ..., c_j\}$ in $n(T)$.Suppose $C_2 \leq C_1$. $\deg(C_k) \geq \deg(C_n) \forall C_k \in C_2$ and $\forall C_n \in V[n(T)] - C_2; 1 \leq k \leq j$. Then $\langle C_k \rangle$ forms a minimal strong dominating set of $n(T)$. Thus $|C_2| = \gamma_{Sn}(T)$. For any nontrivial tree $p > q$ and $|C_2| \leq p-m$ which gives $\gamma_{Sn}(T) \leq p-m$. Further equality holds if $T = K_{1,p}$ then $n(K_{1,p}) = K_{p+1}$ and $\gamma_{Sn}(K_{1,p}) = p-m$.

The following theorem gives lower bound in terms of lict domination.

Theorem 3. For non- trivial connected (p, q) graph G, $\gamma_{Sn}(G) \geq \gamma_n(G)$.

Proof. Let $E_1 = \{e_1, e_2, e_3, ..., e_n\} \subseteq E(G)$, $\deg(e_i) \geq 3$; $1 \leq i \leq n$ and $E_2 = E(G) - E_1$. Since $V[n(G)] = E_1 \cup E_2 \cup C$, $\forall v_i \in C$ is cutvertex G. Then there exists a minimal set $E_1' \subseteq E_1$ which cover all the vertices of $n(G)$. Clearly E' forms a minimal $\gamma - set$ of G. If $\deg(e_j) \geq \deg(e_k), e_k \in V[n(G)] - E_1'$, then E_1' itself is a $\gamma_{Sn} - set$. Otherwise, there exist $e_j \in E_2' \subseteq E_2$ such that $E_1' \cup E_2'$ forms a minimal strong dominating set of $n(G)$. Hence $|E_1' \cup E_2'| \geq |E_1'|$ which gives $\gamma_{Sn}(G) \geq \gamma_n(G)$.

For equality, we can give at least one graph such as, if $G = K_P$, $\gamma_{Sn}(G) = \gamma_n(G)$.

Now we can extend this result for the connected domination in lict graph.

Theorem 4. For any acyclic (p, q) graph G, $\gamma_{Sn}(G) \geq \gamma_{nc}(G)$.

Proof.From the above Theorem, if $\langle E_1' \rangle$ is connected, then the result is true.Otherwise, consider the set $E_3 \subset V[n(G)] - E_1'$ which gives $\langle E_1' \cup E_3 \rangle$ connected. Hence $|E_1' \cup E_2| \geq |E_1' \cup E_3|$ which gives $\gamma_{Sn}(G) \geq \gamma_{nc}(G)$.

Theorem 5.For any acyclic (p, q) graph G, $\gamma_{Sn}(G) + \gamma(G) + m(G) \leq p + \gamma_c(G)$, where m(G) is the maximum number of end vertices of G.

Proof. Let $F' = \{v_1, v_2, v_3, \ldots\ldots, v_m\} \subseteq V(G)$ be the set of all endvertices in $G$with $|F'| = m$. Further, Let $V(G) = \{v_1, v_2, v_3, \ldots\ldots\ldots, v_n\}$ be the set of vertices in G. Suppose there exists a minimal set of vertices $S' = \{v_1, v_2, v_3, \ldots\ldots\ldots, v_k\} \subseteq V(G)$ such that $N[v_i] = V(G), \forall v_i \epsilon S', 1 \leq i \leq k$. Then$S'$ forms a minimal dominating set of G. Suppose the sub graph $< S' >$ has exactly one component. Then$S'$is itself is a connected dominating set of G. Otherwise, if S' has more than one component, then attach the minimal set of vertices S'' of $V(G) - S'$ which are in every $u - w$ path, $\forall u, w \epsilon S$ gives a single component $S_1 = S' \cup S''$. Clearly,S_1 forms a minimal $\gamma_c - set$of G.

Suppose $D = \{v_1, v_2, v_3, \dots\dots\dots, v_m\} \subseteq V[n(G)]$ and $\deg(v_m) \geq \deg(v_k), \forall v_k \in V[n(G)] - D$ and $\forall v_m \in D$ such that $N[v_m] = V(n(G))$. Then D forms a strong dominating set of $n(G)$. Hence it follows that $|D| \cup |S'| \cup |F'| \leq |V(G)| \cup |S_1|$. Clearly $\gamma_{Sn}(G) + \gamma(G) + m(G) \leq p + \gamma_c(G)$.

We need the following theorem to establish the relation between strong lict domination and edge lict domination.

Theorem A[3]. If G is non-trivial connected graph whose vertices have degree d_i and l_i be the number of edges to which cutvertex C_i belongs in G, the lict graph $n(G)$ has $q + \sum C_i$ vertices and $-q + \sum \left(\frac{d_i^2}{2} + l_i \right)$ edges.

Now we have the following theorem.

Theorem 6. For any acyclic (p, q) graph G, $\gamma_{Sn}(G) \leq \gamma_n'(G)$.

Proof.Let $D = \{v_1, v_2, v_3, \dots\dots\dots, v_n\} \subseteq V[n(G)]$ with min $[\Delta\{n(G)\}]$. Suppose there exists a set $D' \subseteq D$ with $diam(u, v) \geq 3, \forall u, v \in D'$ which covers all the vertices in $n(G)$. Then D' forms strong lict dominating set of $n(G)$.By Theorem A, $\left| -q + \sum_{i=1}^{n} \left(\frac{d_i^2}{2} - l_i \right) \right| > |D'|$, and let $E' \subseteq E[n(G)]$, $\forall e_i \in E'$ is adjacent to at least one edge of $E[n(G)] - E'$. Thus E' is a $\gamma_n'(G) - set$. Hence $|E'| \geq |D'|$ gives $\gamma_{Sn}(G) \leq \gamma_n'(G)$.

Next we can extended this result for the connected edge domination in lict graph.

Theorem 7. For any connected (p, q) graph G, $\gamma_{Sn}(G) + \gamma_n'(G) \geq \gamma_{nc}'(G) - 3$.

Proof. Let $F' = \{q_1, q_2, q_3, \dots\dots\dots, q_n\}$ be a minimal edge dominating set of $n(G)$, if $H = E[n(G)] - F'$ and $F_1' = \{q_1, q_2, q_3, \dots\dots\dots, q_i\}$; $\forall\ q_i \in E[n(G)]$, such that $F_1' \in N(F')$ and $H \subset F_1'$ in $n(G)$.Then $\langle F' \cup H \rangle$ is connected. Then $\{F' \cup H\}$ is a connected edge dominating set of $n(G)$.Clearly $|F' \cup H| = \gamma_{nc}'(G)$.

Suppose $F_1 = \{e_1, e_2, e_3, \dots\dots\dots, e_n\}$ be an edge dominating set of G, and let $D_1 = \{v_1, v_2, v_3, \dots\dots\dots, v_n\}$ be the minimal dominating set of $n(G)$. Since $F_1 \in V[n(G)]$ which is also a dominating set of $n(G)$. Then $F_1 = D_1$ and $|D_1| = \gamma[n(G)]$. In $n(G)$, let $F_2 = \{v_1, v_2, \dots, v_m\} \subseteq V(n(G))$ and there exists $D \subseteq F_2$ be the set of vertices with $N[D] = V(n(G))$ and $\forall v_k \in \langle V(n(G)) - D \rangle$, $\deg(v_k) \leq \deg(v_j)$ where $\forall v_j \in D$. Then D forms a strong lict dominating set of G. Otherwise, there exists at least one vertex $\{v\} \in V(n(G)) - D$ such that $\deg(v) > \deg(v_j), \forall v_j \in D$. Clearly $D \cup \{v\}$ forms a minimal $\gamma_{Sn} - set$ of G. Thus $|F'| \cup |D \cup \{v\}| \geq |F' \cup H| - 3$.Hence $\gamma_{Sn}(G) + \gamma_n'(G) \geq \gamma_{nc}'(G) - 3$.

The following theorem relates $\gamma_{Sn}(T)$ and $\gamma_{SSb}(T)$.

Theorem 8.For any connected (p, q) tree T with $p \geq 4$, then $\gamma_{Sn}(T) \leq \gamma_{ssb}(T)$.

Proof.Suppose $B(T) = K_n$. Then by definition of strong split domination, $\gamma_{ssb}(T) - set$ does not exist. Hence $B(T) \neq K_n$. Let $A = \{B_1, B_2, B_3, \dots\dots\dots, B_n\}$ be the blocks of T and $M = \{b_1, b_2, b_3, \dots\dots\dots, b_n\}$ be the block vertices in $B(T)$ corresponding to the blocks of A.

Let $\{B_i\} \subset A$ such that each B_i is an non end block of T. Then $\{b_i\} \subseteq V[B(T)]$ which are vertices corresponding to the set $\{B_i\}$. Since each block is complete in $B(T)$. Again we consider a subset $\{b_i^1\}$ such that $\{b_i^1\} \subset V[B(T)] - \{b_i\}$. Suppose there consists at least one edge then $V[B(T)] - \{b_i^1 \cup b_i\} = \{b_k\}$ where each element of b_k is an isolates. Then $|\{b_i^1 \cup b_i\}| = \gamma_{ssb}(T)$.If $b_i^1 = \emptyset$,then $V[B(T)] - \{b_i\}$ give at least two isolates such that $|b_i| = \gamma_{ssb}(T)$.

Now Suppose let $F=\{e_1,e_2,e_3,........,e_n\}$ be an edge dominating set of T and $C=\{c_1,c_2,c_3,.......,c_n\}$ be the set of cut vertice sin T. Let $D^{'}=\{v_1,v_2,v_3,........,v_n\}$ be the minimal dominating set of $n(T)$ corresponding to F and $|D^{'}|=\gamma[n(G)]$. If for some $c_i \in C$ such that $c_i \notin D^{'}$ in $n(T)$,then $D^{'}=F\cup\{c_i\}$. Otherwise $D^{'}=F$. Further, let $H=\{u_1,u_2,u_3,.........,u_i\}$for some $u_i \in V[n(T)], H \in N(\mathrm{D}^{'})$ and $H \subseteq V[n(T)]-\mathrm{D}^{'}$. Now we consider $H^{'} \subset H$ such that $\langle D^{'} \cup H^{'}\rangle$ is the minimal strong lict dominating set of $n(T)$. Clearly it follows that $|D^{'}\cup H^{'}| \le |b_i|$, which gives $\gamma_{Sn}(T) \le \gamma_{ssb}(T)$.

Theorem 9. For any connected (p,q) tree T, $\gamma_{Sn}(T) \le \gamma_{ss}(T)$.

Proof.letS_1 be a maximum independent set of vertices in T and $S_2 \subset S_1 \forall\, v \in \langle S_2\rangle$ is isolates. Then $(V-S_1)\cup S_2$ is a strong split dominating set of T. Since for each vertex $v \in (V-S_1)\cup S_2$ either v is an isolated vertex in $< (V-S_1)\cup S_2 >$ or there exists a vertex $u \in S_1 - S_2$ and v is adjacent to u, $(V-S_1)\cup S_2$ is minimal. Since S_1 is maximum, $(V-S_1)\cup S_2$ is minimum. Thus $|(V-S_1)\cup S_2| = \gamma_{ss}(T)$.Let $F' = \{e_1,e_2,e_3,\dots,e_n\} \subseteq E(T)$.In$n(T)$,$D' = \{v_1,v_2,v_3,.........,v_n\}$ which corresponds to $\forall e_i \in F'$. Let $\deg(e_i)$,$\forall e_i \in F'$and $\deg(e_j)$ $\forall e_j \in E(T)-F'$such that $\deg(e_i) \ge \deg(e_j)$. Suppose $D'' = \{v_1,v_2,v_3,.........,v_i\} \subseteq D'$and $N[v_k]=V(n(G))$, $\forall v_k \in D^{''}$, $1 \le k \le i$. Then D'' forms a $\gamma_{Sn}-set$. It follows that $|D^{''}| \le |(V-S_1)\cup S_2|$. Hence $\gamma_{Sn}(T) \le \gamma_{ss}(T)$.

Now next theorem gives a upper bound for $\gamma_{Sn}(T)$ in terms of the edges of T.

Theorem 10. For any (p,q)tree T with $p \ge 3$, $\gamma_{Sn}(T) \le q-1$.

Proof.we consider the following cases.

Case 1: Suppose T is a path with $p \ge 3$ vertices. Then

$$\gamma_{Sn} = \left\lfloor \frac{p-1}{2} \right\rfloor \text{ if } p \text{ is even.}$$

$$\gamma_{Sn} = \left\lfloor \frac{p}{2} \right\rfloor \text{ if } p \text{ is odd}$$

Since $\left\lfloor \frac{p-1}{2} \right\rfloor \le q-1$ and $\left\lfloor \frac{p}{2} \right\rfloor \le q-1$

Then one can easily claim that $\gamma_{Sn}(T) \le q-1$.

Case 2: Suppose T is not a path. Then there exists at least one vertex of degree at least three. Let $A=\{e_1,e_2,e_3,........,e_n\}$ be the set of all end edges in G, $B=\{e_1,e_2,e_3,........,e_m\}$ a set of non end edges. Now we consider $B_1=\{e_1,e_2,e_3,........,e_k\} \subseteq B$ $\forall e_i \in B_1, 1 \le j \le k$ have the maximum edge degree and $B_2=\{e_1,e_2,e_3,........,e_p\} \subseteq B$ $\forall e_l \in B_2, 1 \le l \le p$ are the edges which are adjacent to the edges of B_1.Since $E(T)-[\{B_1\}-\mathrm{B}_2]$ is a $\gamma_{Sn}-set$ in $n(T)$, then it is easy to verify that $|E(T)-[\{B_1\}-\{\mathrm{B}_2\}]| \le |E(\mathrm{G})|-1$, which gives $\gamma_{Sn}(T) \le q-1$.

Theorem 11.For any connected (p,q) graph G, $\gamma_{Sn}(\mathrm{G}) \le \gamma(\mathrm{G}) + \gamma_l(\mathrm{G}) - 1$.

Proof. Suppose $S' = \{v_1,v_2,v_3,.........,v_i\} \subseteq V(G)$ be the set of vertices with $\deg(v_i) \ge 2$,suppose exists a set $S_1 \subseteq S'$ of vertices with $dist(u,v) \ge 3$, $\forall u,v \in S_1$which covers all the vertices in G. Then S_1 forms a dominating set of G. Otherwise,if $diam(u,v) < 3$,then there exists at least one vertex $x \notin S_1$ such that $S'' = S_1 \cup \{x\}$ form a minimal $\gamma - set$ of G. Hence $|S^{''}| = \gamma(\mathrm{G})$. Let $C^{'}=\{v_1,v_2,...,v_j\} \subseteq V(L(G))$ be the set of vertices with $dist(u,v) \ge 3$. Suppose there exists a set $D^{'} \subseteq C^{'}$ which covers all the vertices in $L(G)$. Then D' itself is a line dominating setof G. If $dist(u,v)<3$ and $N[D']\neq V(L(G))$, then $D''=D'\cup\{w\}$, where $w\notin N[v]$, $v\in D'$ forms a minimal dominating set of $L(G)$. Hence $|D'\cup\{w\}|=\gamma_l(G)$. Further, let $F'=\{e_1,e_2,e_3,........,e_i\}$ be an edge dominating set of G and $C'=\{c_1,c_2,c_3,.......,c_i\}$ be the set of cut vertice sin G. In $n(G)$, $\{F_1\cup C_1\}\subseteq V[n(G)]$ such that $N[\{F_1\cup C_1\}]=V[n(G)]$ where $F_1\subseteq F'$, $C_1\subset C'$ form a minimal dominating set of $n(G)$. Suppose $\deg(v_i)\geq\deg(u_i)$ $\forall v_i\in\{F_1\cup C_1\}$, $\forall u_i\in V[n(G)]-\{F_1\cup C_1\}$. Then $\{F_1\cup C_1\}$ is a strong dominating set of $n(G)$. Hence $|F_1\cup C_1|\leq|S''|+|D'\cup\{w\}|-1$ gives $\gamma_{Sn}(G)\leq\gamma(G)+\gamma_l(G)-1$.

The following theorem relates cutvertices of tree T and $\gamma_{Sn}(T)$.

Theorem 12.For any tree T with K number of cutvertices, then $\gamma_{Sn}(T)\leq K$. Further equality holds if $T=K_{1,p}$, $p\geq 3$.

Proof. Let $H=\{v_1,v_2,...,v_n\}\subseteq V(T)$ be the set of all cutvertices in T with $|H|=K$. Further, let $E=\{e_1,e_2,...,e_k\}$ be the set of edges which are incident with the vertices of H. Now by the definition of lict graph, suppose $D'=\{u_1,u_2,...,u_n\}\subseteq E(T)$ be the set of vertices which covers all the vertices in $n(T)$, $\deg(u_k)\geq\deg(u_n)$ where $\forall u_k\in D'$ and $u_n\in V[n(T)-D']$. ClearlyD' forms a minimal strong lict dominating set of $n(T)$, which gives $|D'|\leq|H|$. Hence $\gamma_{Sn}(T)\leq K$.

Theorem 13. For any non trivial (p,q) tree T,every cutvertex of $n(T)$ which is incident to end blocks is in every $\gamma_{Sn}(T)-set$

.Proof. Let $C=\{v_1,v_2,...,v_n\}\subseteq V[n(T)]$ be the set of all cutvertices which are incident to the end blocks. Suppose $D\subseteq C$ be the set of cut vertices with $N[D]=V[n(T)]$ and $\deg(u)\geq\deg(v)$, $\forall u\in D$, $v\in V[n(T)-D$. Then D forms a strong minimal dominating set of $n(T)$. Suppose $B=\{B_1,B_2,B_3,.........,B_m\}$ be the set of blocks in $n(T)$. $D'\subset D$ and $\forall v_i\in D'$ is adjacent to $\forall v_j\in B$. Then D' is a proper subset of cut vertices of D. Hence D' is in every $\gamma_{Sn}(T)-set$ of $n(T)$.

In the following theorem,we establish the relation between $\gamma_{Sn}(G)$ and $\gamma_{Snn}(G)$.

Theorem 14. For any (p,q) graph G, $\gamma_{Sn}(G)\leq\gamma_{Snn}(G)$.

Proof. Let $E=\{e_1,e_2,e_3,........,e_m\}$ and $C(G)=\{c_1,c_2,c_3,.......,c_n\}$ be the edge set and cutvertexset of G. In $n(G)$, $V[n(G)]=\{E\cup C\}$. Now $E_1=\{e_1,e_2,e_3,........,e_i\}\subseteq E(G)$ with $\deg(e_j)\geq 3$ $\forall e_j\in E_1$ and $E_2=E(G)-E_1(G)$. Let V,V_1 and V_2 are the corresponding vertex set of E,E_1 and E_2 in $n(G)$. Suppose $D_1\subseteq V_2$ and $D=\{D_1\}\cup\{V_1\}$ is a dominating set of $n(G)$. Then $\deg(v)\geq\deg(u),\forall v\in D$ and $u\in V[n(G)]-D$ forms a strong dominating set of $n(G)$. If $\langle V[n(G)]-D\rangle$ is connected, then D itself is a strong non split lict dominating set. Otherwise, if $\langle V[n(G)]-D\rangle$ has at least two components let the components be $\{f_1,f_2,f_3,........,f_k\}$. Suppose $K>2$. Then $\{f_1,f_2,f_3,........,f_k\}\cup\langle V[n(G)]-D\rangle$ forms $\gamma_{Snn}-set$. If $K=2$, then consider $v\in V[n(G)]-D$ such that $\{V[n(G)]-D\}\cup\{v\}$ forms $\gamma_{Snn}-set$. Hence $|D|\leq|\{V[n(G)]-D\}\cup\{v\}|$ gives $\gamma_{Sn}(G)\leq\gamma_{Snn}(G)$.

The following theorems relates restrained lict domination number of G and $\gamma_{Sn}(G)$.

Theorem 15.For any non-trivial connected (p,q) graph G with $G\neq C_5$, then

$\gamma_{Sn}(G) \le \gamma_{rn}(G) - 1$.

Proof.Since $G = C_5$,then $\gamma_{Sn}(C_5) = \gamma_{rn}(C_5)$. Hence $G \neq C_5$ .Suppose $E' = \{e_1, e_2, e_3, \ldots\ldots, e_i\}$ and $C' = \{c_1, c_2, c_3, \ldots\ldots, c_j\}$ be the set of edges and cutvertices in G. In $n(G)$, $V[n(G)] = E'(G) \cup C'(G)$ and in G $\forall e_i$ incident with C_j, $1 \le j \le i$ forms an edge disjoint induced subgraph which is complete in $n(G)$, such that the number of blocks in $n(G) = |C'|$. Let $\{e_1, e_2, e_3, \ldots\ldots, e_j\} \in E'(G)$, are non end edges of G which forms cutvertices $C''(G) = \{c_1, c_2, c_3, \ldots\ldots, c_j\}$ in $n(G)$. Let $C_1'' \le C''$ be a restrained dominating set in $n(G)$, such that $|C_1''| = \gamma_{rn}(G)$. Otherwise, let $D' = \{v_1, v_2, v_3, \ldots\ldots\ldots, v_n\} \subseteq V[n(G)]$ such that $\deg(u,v) \ge 2$, $\forall u \in V[n(G)] - D'$ and $\forall v \in D'$ and D covers all the vertices of $n(G)$. Then D' forms a minimal restrained dominating set of $n(G)$. Hence $|D'| = \gamma_{rn}(G)$.

let$H' = \{u_1, u_2, u_3, \ldots\ldots\ldots, u_n\} \subseteq V[n(G)]$ be the set of vertices such that $\{u_i\} = \{e_i\} \in E'(G)$, $1 \le i \le n$ where $\{e_i\}$ are incident with the vertices of $E'(G)$. Suppose $D \subseteq H'$ be the set of vertices with $\deg(w) \ge 3$ for every $w \in D$ such that $N[D] = V[n(G)]$ and if $\forall v_i \in V[n(G)] - D$ with $\deg(v_i) > 3$. Then $\{D\} \cup \{v_i\}$ forms a strong lict dominating set. It follows that $|\{D\} \cup \{v_i\}| \le |C_1''| - 1$ which gives $\gamma_{Sn}(G) \le \gamma_{rn}(G) - 1$ or$|\{D\} \cup \{v_i\}| \le |D'| - 1$ gives $\gamma_{Sn}(G) \le \gamma_{rn}(G) - 1$.

The following theorem relates $\gamma_{Sn}(G)$ with $\gamma_{Snsb}(G)$.

Theorem 16. For any acyclic connected (p,q) graph G, $\gamma_{Sn}(G) \le \gamma_{Snsb}(G)$.

Proof. Suppose $G = K_{1,n}$ $n \ge 2$. Then $\gamma_{Sn}(G) = 1 = \gamma_{Snsb}(G)$. Now assume G is a path P_n $n \ge 2$, hence $\gamma_{Snsb} - set$ consists of $\{E(G) - 1\} + \{C(G) - 1\}$ elements and $\gamma_{Sn} - set$ consists of either $\frac{P}{2} - 1$ or $\left\lfloor \frac{p}{2} \right\rfloor$ elements. Clearly $\gamma_{Sn}(G) \le \gamma_{Snsb}(G)$.

Further, we consider a tree which is neither a star nor a path. Assume $\Delta(T) \ge 3$, in $n(T)$ each block is complete and every cutvertex of $n(G)$ lies on exactly two blocks which are complete. Let $K = \{v_1, v_2, v_3 \ldots, v_n\}$ be a set of cutvertices with a maximum degree and $\forall v_i \in K$ are incident with $B_1, B_2, \ldots\ldots, B_m$ blocks which are complete. Suppose $M \subseteq K$ such that $\forall v_j \in V[n(T)] - M$ is adjacent to at least one vertex of M and $\deg(v_j) \le \deg(v_k)$ $\forall v_k \in M$. Then M is a $\gamma_{Sn} - set$ of T. But in case of $\gamma_{Snsb} - set$, let $N = \{v_1, v_2, v_3 \ldots, v_p\}$ be the set of cutvertices of $n(T)$, $S = \{v_1, v_2, v_3 \ldots, v_j\}$ be the set of vertices lie on the corresponding blocks $B_1, B_2, \ldots\ldots, B_m$. Consider $S_1 \subset S$ such that $V[n(T)] - \{N \cup S\} = J$ and its induced graph is complete. Hence $|M| < |J|$ which gives $\gamma_{Sn}(T) \le \gamma_{Snsb}(T)$.

By considering these cases, we have $|M| \le |J|$ which gives $\gamma_{Sn}(T) \le \gamma_{Snsb}(T)$.

Corollary.For any graph G with exactly one cutvertex incident with at least two blocks and each vertex of each block is adjacent to a cutvertex, then that cutvertexis in $\gamma_{Sn} - set$ of G.

In the following result we prove the Nordhaus-Gaddum type results.

Theorem 17. Let G be any (p,q) graph G. If G and its complement $\overline{G}$ are connected, then

$$\gamma_{Sn}(G) + \gamma_{Sn}(\overline{G}) \le (P-1).$$

$$\gamma_{Sn}(G).\gamma_{Sn}(\overline{G}) \le (P-1)^2.$$

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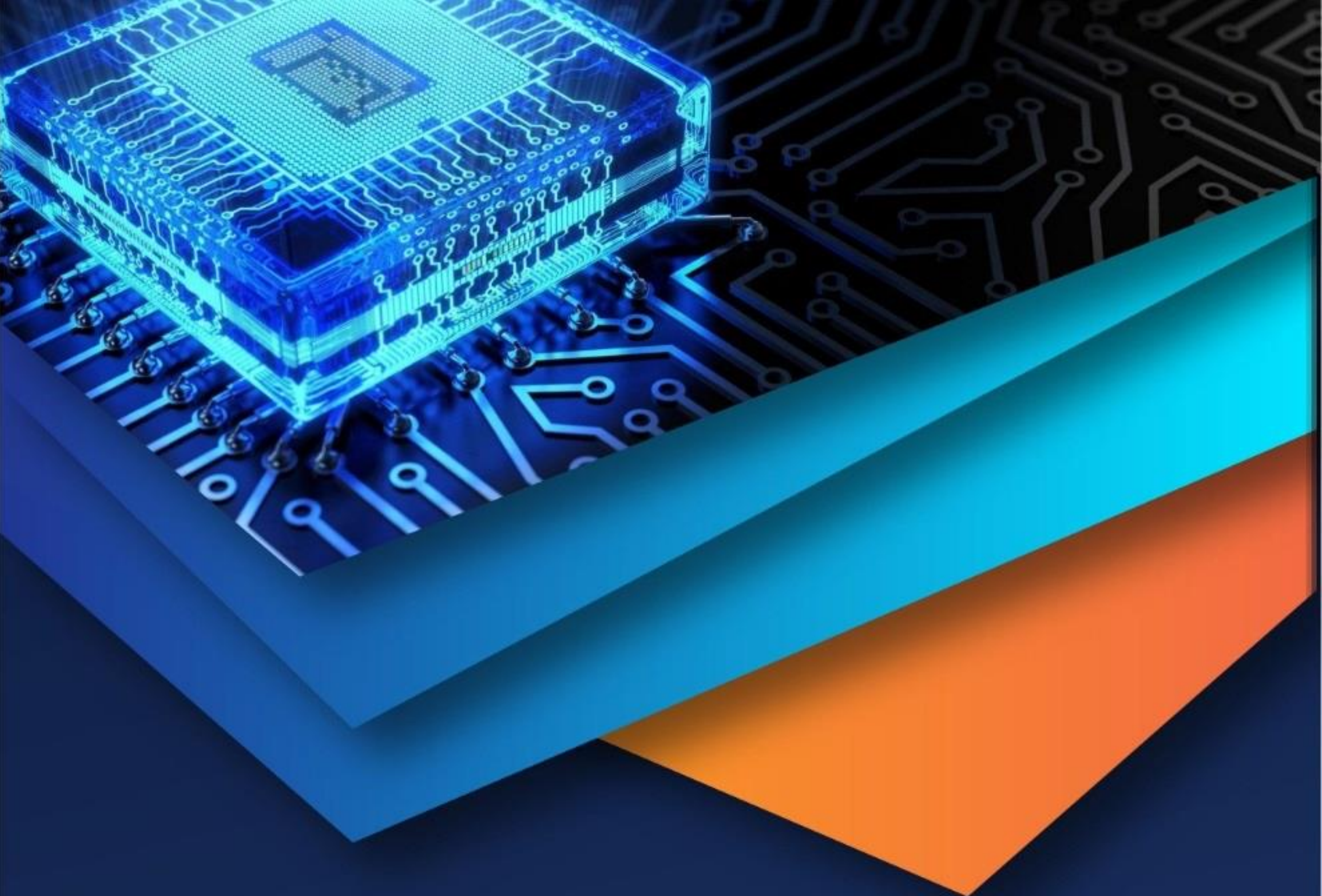

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