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MHD Three Dimensional Flow of Jeffrey Fluid over an Exponentially Stretching Sheet

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Abstract: The MHD three dimensional flow of Jeffrey fluid over an exponentially stretching sheet is studied. Using similarity transformations the governing partial differential equations are transformed to ordinary differential equations and are solved using shooting technique. The effects of non dimensional governing parameters on velocity and temperature profiles are discussed through graphs. Also, the coefficient of skin friction and the Nusselt number are presented through tables. It is found that present results have been good agreement with the existed studies under some special cases.
Keywords: MHD, Jeffrey parameter, exponentially stretching sheet.

I. INTRODUCTION

Boundary layer flows having a many applications in engineering and industrial research such as drawing the plates in an electrically conducting fluid subjected to a transverse magnetic field is used to control the cooling rate, extrusions of plastic sheets, paper production, glass blowing, metal and polymer extrusion, crystal glowing and drawing plastic films etc. The electrically conducting fluids are very important in engineering applications such as MHD generators, plasma fluids, geothermal energy extraction and nuclear reactors engineers and designers used the concept of magneto hydrodynamic in the design of pumps, design of heat exchangers, control and reentry space vehicle propulsion and thermal protection. Soret and Dufour effects in three dimensional flow over an exponentially stretching surface with porous medium, chemical reaction and heat source/sink examined Hayat et al. [1]. Kartini Ahmad et al. [2] studied mixed convection Jeffrey fluid flow over an exponentially stretching sheet with magneto hydrodynamic effect. MHD three dimensional flow by an exponentially stretching surface with convective boundary condition reported Hayat et al. [3]. Mahanta et al. [4] presented 3D Casson fluid flow past a porous linearly stretching sheet with convective boundary condition. Flow and heat transfer for three dimensional flow over an exponentially stretching surface discussed Chung Liu et al. [5]. Magyari et al. [6] investigated heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. Hayat et al. [7] discussed three dimensional flow of Jeffrey fluid over a bidirectional stretching surface with heat source/sink. Three dimensional flow of nanofluid induced by an exponentially stretching sheet: An application to solar energy examined Junaid Ahmad Khan et al. [8]. Alsaedi et al. [9] analyzed MHD three dimensional flow of viscoelastic fluid over an exponentially stretching surface with variable thermal conductivity. Hayat et al. [10] found three dimensional flow of a Jeffrey fluid over a linearly stretching sheet. Tasawar Hayat et al. [11] studied on three dimensional flow of Eyring Powell nano fluid over an exponentially stretching sheet. Shehzad et al. [12] carried out on MHD three dimensional flow of Jeffrey fluid with Newtonian heating. Radiative hydromagnetic flow of Jeffrey nano fluid by an exponentially stretching sheet performed by Tariq Hussian et al. [13]. Fazle Mabood et al. [14] studied on MHD flow over exponentially radiating stretching sheet using homotopy analysis method. effects of variable fluid properties on MHD flow and heat transfer over a stretching sheet with variable thickness. The effects of variable fluid properties on the hydromagnetic flow and heat transfer over a non- linearly stretching sheet. Effects of variable fluid properties on the thin film flow of Ostwald de waele fluid over a stretching surface analyzed Prasad et al. [15-17]. MHD boundary layer flow of Casson fluid passing through an exponentially stretching permeable surface with thermal radiation was discussed Swati Mukhopadhyay et al. [18]. Sreenadh et al. [19-20] studied on MHD boundary layer flow of Jeffrey fluid over a stretching/shrinking sheet through porous medium. Flow of a Jeffrey fluid between torsionally oscillating disks. The present study reveals that the steady MHD three dimensional flow of Jeffrey fluid over an exponentially stretching sheet. Using self similarity transformations the governing partial differential equations are transformed to nonlinear differential equations and are using solved shooting technique. The effect of pertinent parameters on velocity, temperature profiles, the skin friction coefficient and the rate of heat transfer are discussed in detailed.

II. MATHEMATICAL FORMULATION

Consider, the MHD three dimensional flow of Jeffry fluid over an exponentially stretching sheet. It is consider that sheet is stretched along the xy plane while fluid is placed along the z-axis. Moreover it is consider that constant magnetic field is applying normal to the fluid flow and the induced magnetic field assumed to be negligible. The sheet at z = 0 is stretched in the x and y directions with velocities U_w and V_w respectively. The continuity, momentum and energy equations for the present boundary layer flow are to the following equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{\upsilon}{1+\lambda_1}\frac{\partial^2 u}{\partial z^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{\upsilon}{1+\lambda_1}\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B_0^2}{\rho}v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2} \tag{4}$$

The associated boundary conditions of equations (2) – (4) are

$$u=U_w=U_0 e^{\frac{x+y}{L}},\ v=V_w=V_0 e^{\frac{x+y}{L}},\ w=0,\ T=T_w \text{ at } z=0 \tag{5}$$

$$u=0,\ v=0,\ T=T_\infty \text{ as } z\to\infty \tag{6}$$

where u, v and w are velocity components in the x, y and z - directions respectively. υ is the kinematics viscosity, λ_1 is the Jeffrey parameter, B_0 is the magnetic induction, σ is the electrical conductivity, k is the thermal conductivity, c_p is the specific heat at constant pressure, the subscript w denotes the wall conduction, L is the reference length and T_∞ is the ambient temperature.

The similarity transformations of equations (1)-(4) are introduced as

$$\left.\begin{aligned} & u=U_0 e^{\frac{x+y}{L}} f',v=V_0 e^{\frac{x+y}{L}} g',w=\left(\frac{\upsilon U_0}{2L}\right)^{\frac{1}{2}} e^{\frac{x+y}{L}}\left(f+\eta f'+g+\eta g'\right) \\ & T=T_\infty+T_0 e^{\frac{A(x+y)}{2L}}\theta,\eta=\left(\frac{U_0}{2\upsilon L}\right)e^{\frac{x+y}{L}}z \end{aligned}\right\} \tag{7}$$

Where A is the temperature exponent.

Using the dimensionless variables and similarity transformations in equations (1)-(4) are reduces to the given below

$$\frac{1}{1+\lambda_1}f'''+\left(f+g\right)f''-2\left(f'+g'\right)f'-Mf'=0 \tag{8}$$

$$\frac{1}{1+\lambda_1}g'''+\left(f+g\right)g''-2\left(f'+g'\right)g'-Mg'=0 \tag{9}$$

$$\theta''+\Pr\left(f+g\right)\theta'+\Pr A\left(f'+g'\right)\theta=0 \tag{10}$$

The corresponding boundary conditions (5) and (6) becomes

$$f=g=0,\ f'=1,\ g'=\alpha,\ \theta=1 \text{ at } \eta=0 \tag{11}$$

$$f'=0=g',\ \theta=0 \text{ as } \eta\to\infty \tag{12}$$

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where $\alpha = \frac{V_0}{U_0}$ is the stretching ratio, $\Pr = \frac{\mu c_p}{k}$ is the Prandtl number, $M = \sqrt{\frac{2\sigma B_0^2 L}{\rho U_w}}$ is the magnetic parameter and primes denotes with respect to η .

The physical quantities of interest are the skin friction coefficients along the x - and y -directions are given by

$$C_{fx} = -\frac{\frac{\mu}{1+\lambda_1}\left(\frac{\partial u}{\partial z}\right)_{z=0}}{\frac{1}{2}\rho U_w^2} \text{ and } C_{fy} = -\frac{\frac{\mu}{1+\lambda_1}\left(\frac{\partial v}{\partial z}\right)_{z=0}}{\frac{1}{2}\rho U_w^2} \tag{13}$$

the skin friction coefficients in dimensionless form are

$$C_{fx} = -\left(\frac{\mathrm{Re}}{2}\right)^{-1/2} e^{\frac{3(x+y)}{2L}} \frac{f''(0)}{1+\lambda_1} \text{ and } C_{fy} = -\left(\frac{\mathrm{Re}}{2}\right)^{-1/2} e^{\frac{3(x+y)}{2L}} \frac{g''(0)}{1+\lambda_1} \tag{14}$$

The local Nusselt number Nu_x is defined as

$$Nu_x = -\frac{x}{T_w - T_\infty} \frac{\partial T}{\partial z}\bigg|_{z=0} = -\frac{x}{L}\left(\frac{\mathrm{Re}}{2}\right)^{1/2} e^{\frac{(x+y)}{2L}} \theta'(0) \tag{15}$$

where Re is the Reynolds number defined by $\mathrm{Re} = \frac{U_w L}{\upsilon}$.

III. SOLUTION OF THE PROBLEM

The present study on MHD three dimensional flow of Jeffrey fluid over an exponentially stretching sheet for the both Newtonian and Non Newtonian cases. The transformed governing equations are solved numerically using shooting technique with fourth order Runge-Kutta method. The numerical computations are performed for several values of dimensionless parameters involved in the equations, viz. M is the magnetic parameter, λ_1 is the Jeffrey parameter, α is the stretching ratio parameter, $\Pr$ is the Prandtl number and A is the temperature exponent. The numerical computations have been carried out for various values of the parameters on velocity and temperature are depicts in figures 1- 8.

In order to assure the accuracy of the applied numerical scheme the computed values of Skin friction coefficient $-\frac{f''(0)}{1+\lambda_1}$ is compared with the available results of Liu et *al.* [5] in Table-1 and Table-2 depicts the local Nusselt number $\theta'(0)$ compared the results of Magyari and Keller [6] for the two dimensional case $\alpha = 0$ are have been found in good agreement.

The variations of the stretching ratio parameter α , the Jeffrey parameter λ_1 and the Magnetic parameter M on the velocity $f'(\eta)$ are depicts in figures 1-3 for both Newtonian and Non Newtonian cases. From figure 1 we examine that the velocity and momentum boundary layer thickness decreases with increasing α which is natural result due to the bi stretching sheet effect. From figure 2, it is noticed that for higher values of Jeffrey parameter λ_1 it reduces both boundary layer thickness and the Jeffrey parameter λ_1. From figure 3, we observe that the magnitude of velocity that for higher values of magnitude parameter M it reduces both boundary layer thickness and the magnitude of the velocity. It causes for large values of M the Lorentz force enhances, leading to more resistance to the motion of fluid and hence a reduction of the velocity distribution.

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Figures 4 and 5 illustrates the temperature $\theta(\eta)$ profiles for several values of Prandtl number $\Pr$ with $A=-2$ and $A=2$. We report that for increasing values $\Pr$ of thermal boundary layer thickness reduces, which owes that the smaller thermal diffusive effect.

The effects of the temperature exponent A on temperature $\theta(\eta)$ is depict in figure 6. We reveals that the temperature augment and the enlarging effect becomes more pronounced for small values of A,correspondingly the shortening of thermal boundary layer thickness becomes thickened for decreasing A.

The variation on temperature $\theta(\eta)$ in different values of stretching ratio parameter α are display in figures 7 and 8. We examine that the temperature profile decreases from unity to zero as the distance η from 0 to infinity for $A \geq -1$. The slopes at the surface for these curves corresponding to $A=-1$ are zero, indicates that there is no heat transfer between the wall and the fluid. We examine shortening of thermal boundary layer thickness for enhancing α is a natural consequence of increased entrainment intensity of the cooler fluid from the ambient to the hotter fluid near the surface.

The skin friction coefficient $-\dfrac{f''(0)}{1+\lambda_1}$ and $-\dfrac{g''(0)}{1+\lambda_1}$ for various values of stretching ratio parameter α are shown in Table-1. When the absence of Jeffrey parameter and magnetic parameter (i.e. $M=0,\ \lambda_1=0$) we examine that α increases the values of $-f''(0)$ and $-g''(0)$ enhances accordingly. This reveals that the shear stress acting along the respective directions are both enhanced when the surface is stretched in the orthogonal directions for $0 \leq \alpha \leq 1$. We conclude that the flow far from the surface compensates towards the stretching surface due to the bidirectional driving effect.

The temperature gradient $\theta'(0)$ for different values of stretching ratio parameter α is depicts in Table-2. We observe that the different values of $\Pr$ and A the temperature gradient $\theta'(0)$ increases negatively although this conclusion applies for the two dimensional case $\alpha=0$ the results of Magyari and Keller [6] are exist in good agreement.

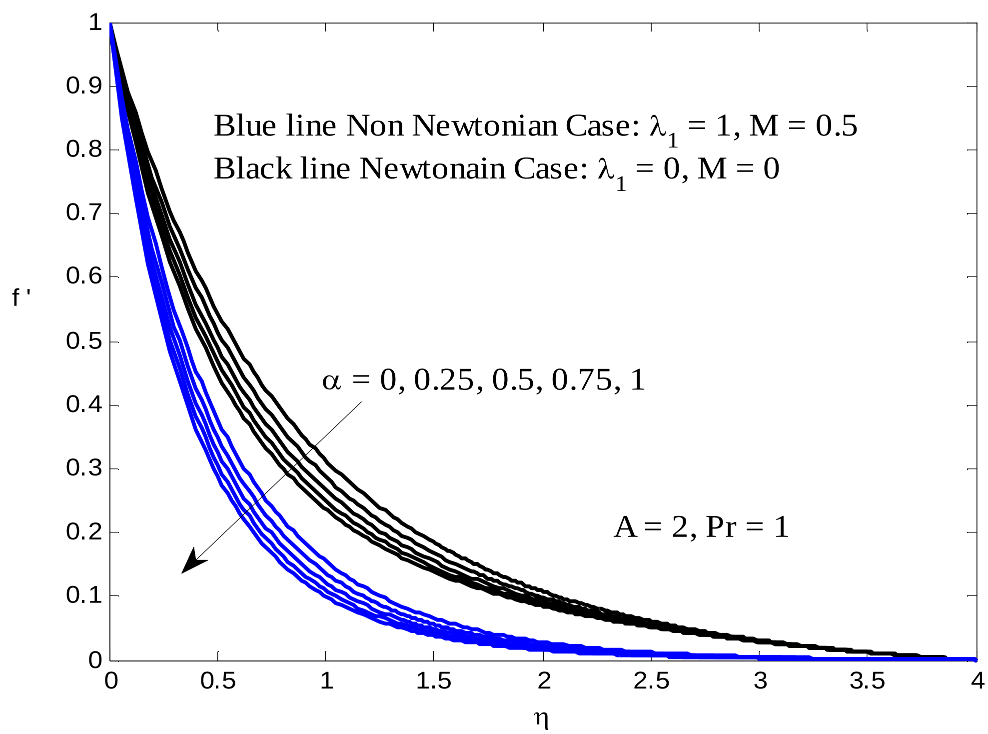

Fig 1. Velocity profile for different values of alpha

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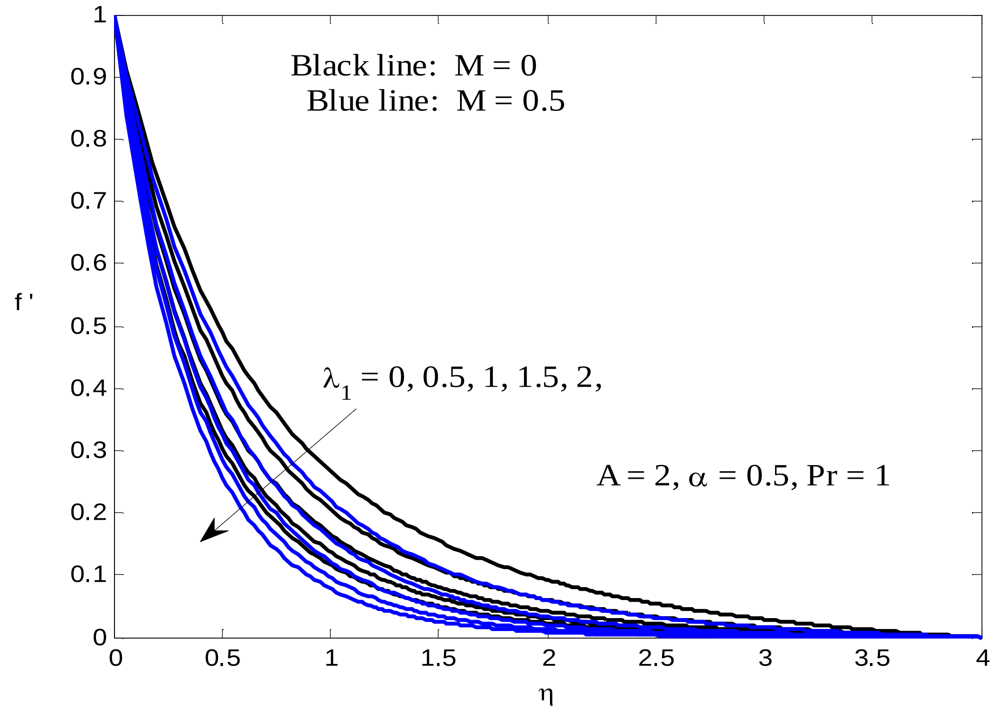

Fig 2. Velocity profile for different values of lambda

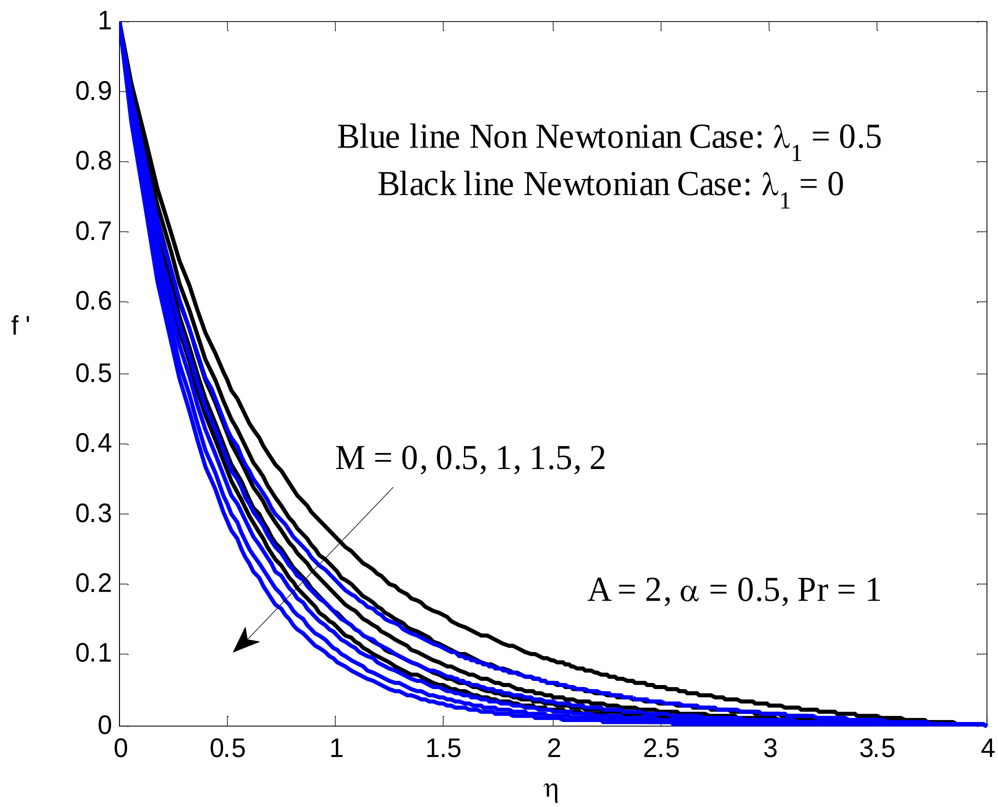

Fig 3. Velocity profile for different values of M

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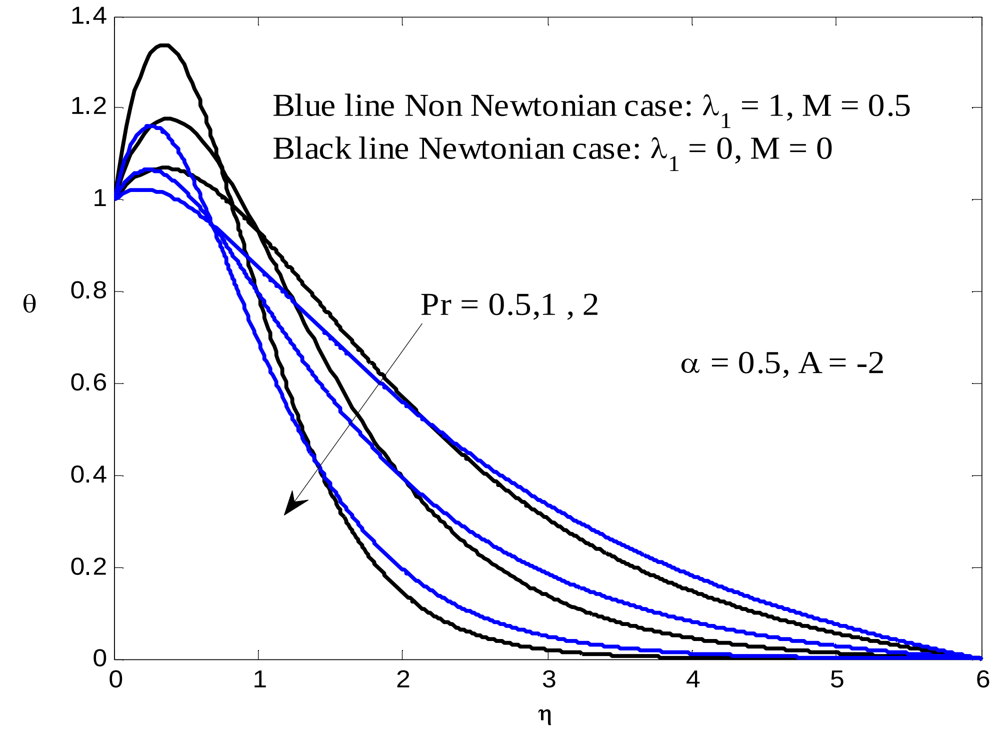

Fig 4. Temperature profile for different values of Pr

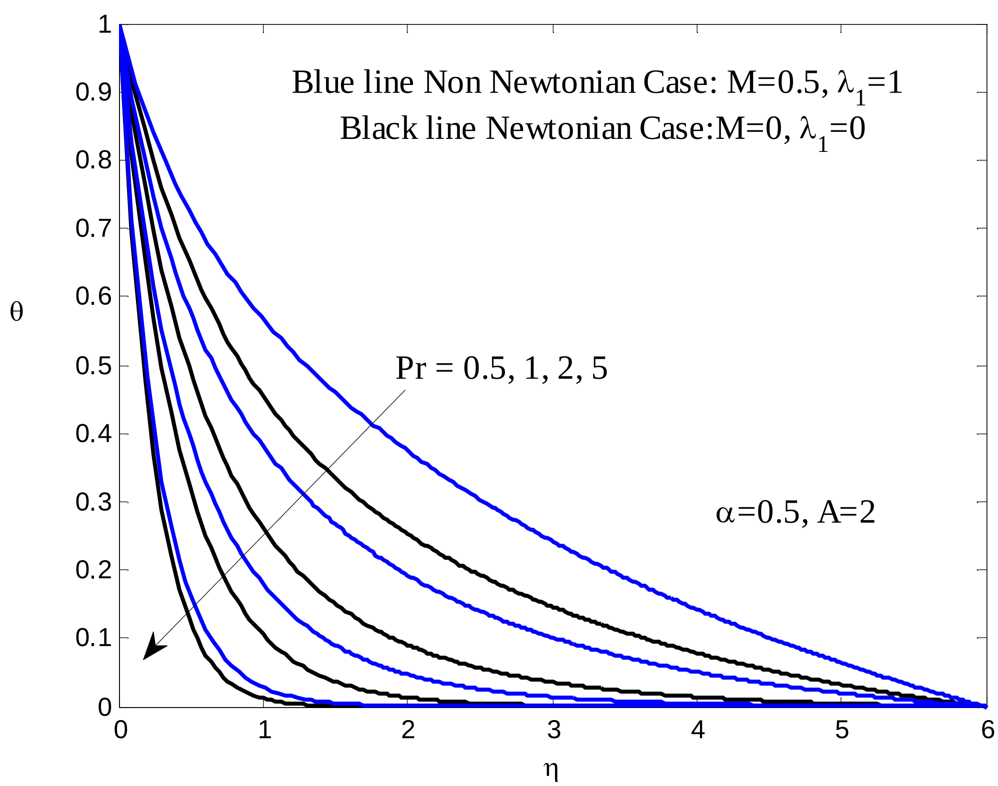

Fig 5. Temperature profile for different values of Pr

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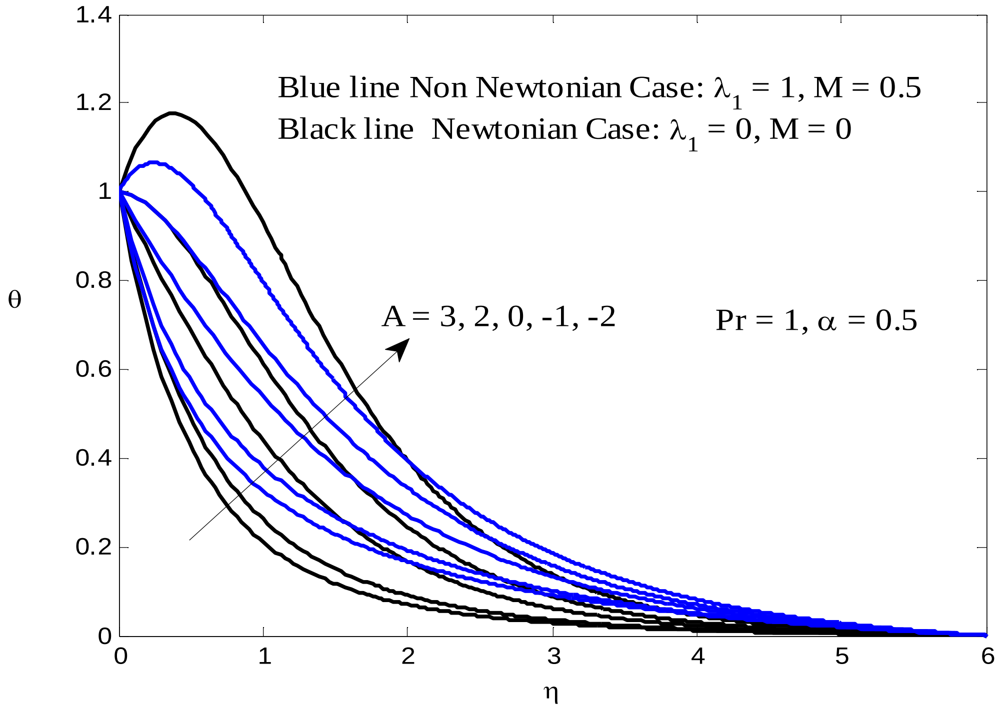

Fig 6. Temperature profile for different values of A

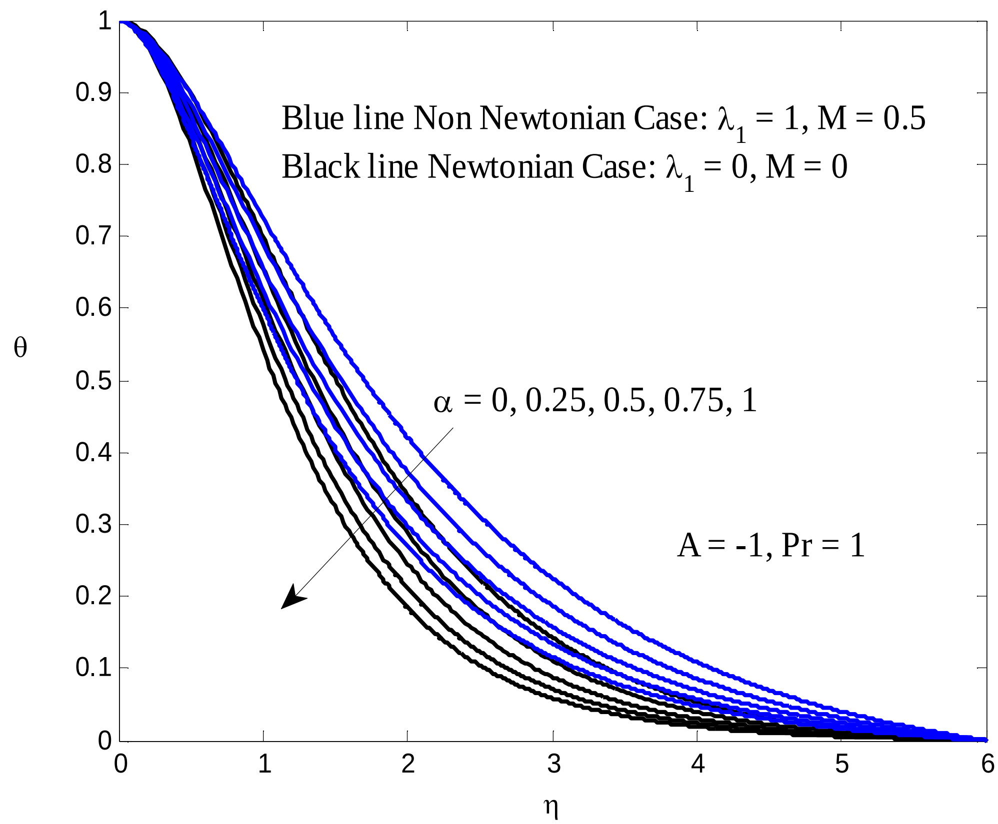

Fig 7. Temperature profile for different values of alpha

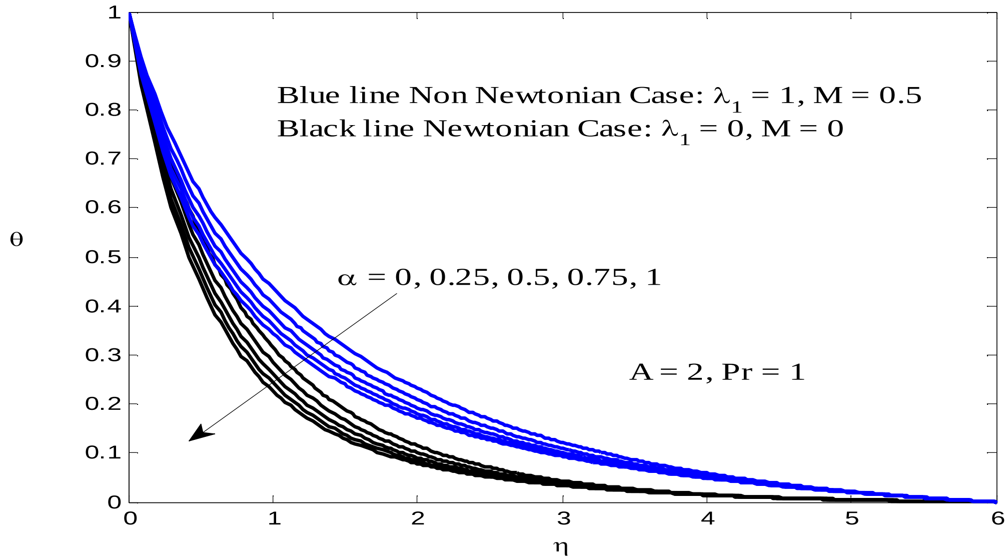

Fig 8. Temperature profile for different values of alpha

Table 1: comparative table values of $-\frac{f''(0)}{1+\lambda_1}$ and $-\frac{g''(0)}{1+\lambda_1}$ for various values of α when $M=0,\ \lambda_1=0$

α	Liu *et al* [5]		Present results ($M=0,\ \lambda_1=0$)	
	$-f''(0)$	$-g''(0)$	$-f''(0)$	$-g''(0)$
0	1.28180856	0	1.2818114	0
0.5	1.56988846	0.78494423	1.5699104	0.7849552
1.0	1.81275105	1.81275105	1.8127654	1.8127654

Table 2: comparison of $\theta'(0)$ of Magyari and Keller [6] and Liu *et al.* [5] with the present study for $\alpha=0, M=0$ and $\lambda_1=0$

Pr	A	$\theta'(0)$		
		Magyari and Keller [6]	Liu *et al.* [5]	Present study $\alpha=0, M=0$ and $\lambda_1=0$
1	-1.5	0.377413	0.37741256	0.3774106
	0	-0.549643	-0.54964375	-0.5496460
	1	-0.954782	-0.95478270	-0.9547852
	3	-1.560294	-1.56029540	-1.5603044
5	-1.5	1.353240	1.35324050	1.3532396
	0	-1.521243	-1.52123900	-1.5212381
	1	-2.500135	-2.50013157	-2.500704
	3	-3.886555	-3.88655510	-3.8865583
10	-1.5	2.200000	2.20002816	2.2000298
	0	-2.257429	-2.25742372	-2.2574288
	1	-3.660379	-3.66037218	-3.6603259
	3	-5.635369	-5.62819631	-5.6281349

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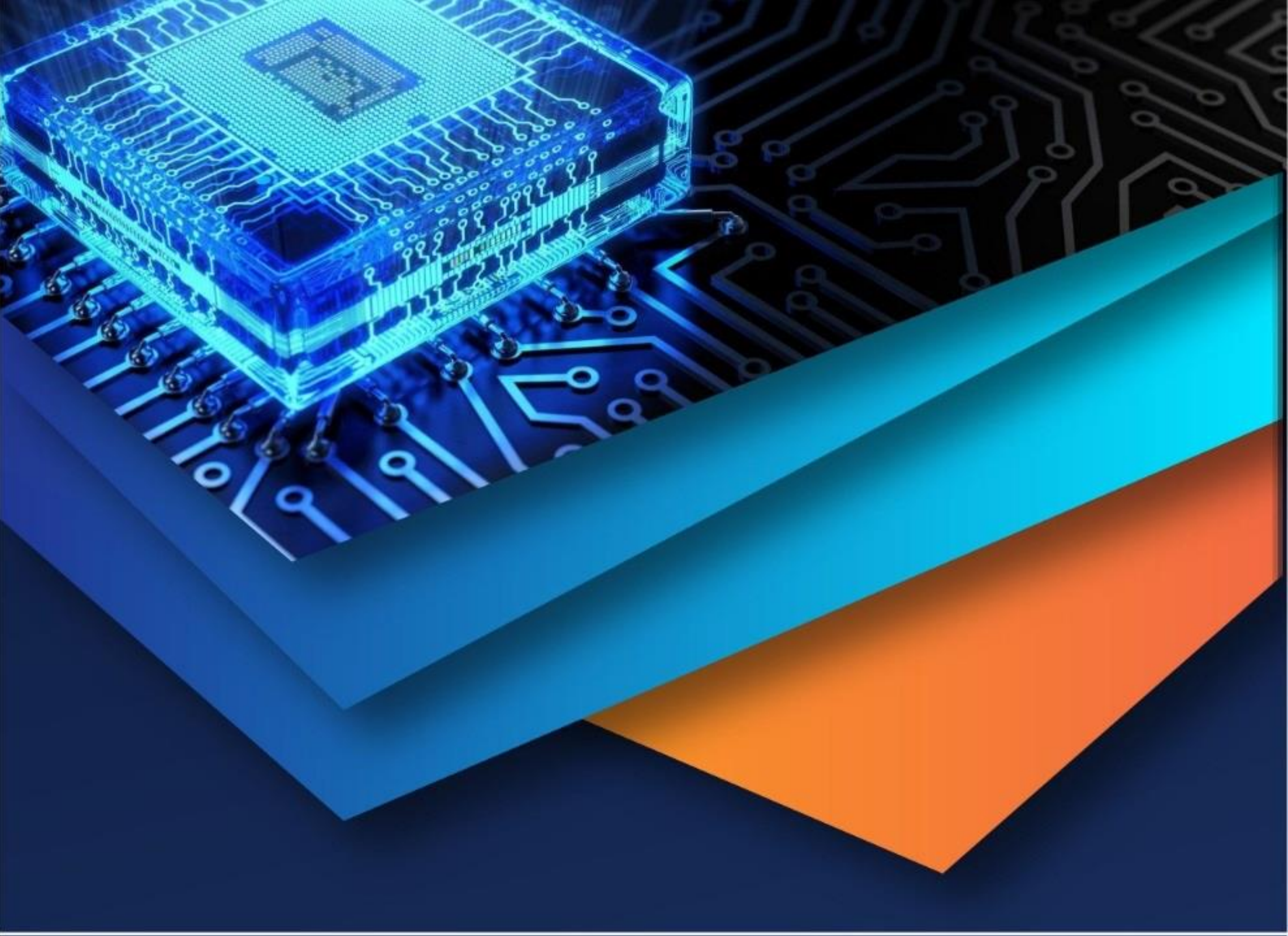

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