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On Some Properties of Metric F- Structure Satisfying $F^k + (-1)^k F = 0$

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***ABSTRACT:* The purpose of this paper is to study various properties of the F- sturcture satisfying $F^k + (-1)^k F = 0$. Where k is positive integer $k \geq 3$ the metric F- structure, kernel and tangent vectors have also been discussed.**

***Key words:* Differentiable manifold, Complementary projection operators, metric, kernel and tangent vectors.**

I. INTRODUCTION

Let V_n be a C^∞ differentiable manifold and $F \neq 0$ be a C^∞ (1,1) tensor on V_n such that

A. $F^k + (-1)^k F = 0$

we define the the projection operators l and m on V_n by

B. $l = (-1)^{k-1} F^{2k-1}, \qquad m = I - (-1)^{k-1} F^{k-1}$

Where I denotes the identify operator

From (1.1) and (1.2), we have

C. $l + m = I, \qquad l^2 = l,\ m^2 = m, \qquad lm = ml = 0$

$$lF = Fl = F, \quad Fm = mF = 0,$$

1) Theorem (1.1): If $rank((F)) = n$ then

(1.4) $l = I$, $m = 0$

Proof: from the fact

$$rank((F)) + nulity((F)) = \dim V_n = n$$

We have

$$nulity((F)) = 0 \Rightarrow \ker((F)) = \{0\}$$

or

$$FX = 0 \Rightarrow X = 0$$

Then $FX_1 = FX_2$

$$F(X_1 - X_2) = 0$$

$X_1 = X_2$ or F is 1-1

Also F being an operator on a finite dimensional V_n, F is onto also. thusF invertible. F^{-1} exists. Now operating F^{-1} on $Fl = F$ and $mF = 0$, we get (1.4)

2) Theorem (1.2) let us define the (1,1) tensors $p,\ q,\ \alpha,\ \beta$ by

(1.5) $p = m + F^{k-1},\ q = m - F^{k-1},\ \alpha = l + F^{k-1},\ \beta = l - F^{k-1}$

then

(1.6) $p^2 = q^2 = I, pq = M - l, \alpha\beta = 0$

Proof: using (1.1), (1.2), (1.5) we have

$$
\begin{aligned}
p^2 &= (m + F^{k-1})(m + F^{k-1}) \\
&= m^2 + mF^{k-1} + F^{k-1} + F^{2k-2} \\
&= m + 0 + 0 + F^{2k-2} \\
&= m + l^2 \\
&= I
\end{aligned}
$$

$$
\begin{aligned}
\alpha\beta &= (l + F^{k-1})(l + F^{k-1}) \\
&= l^2 - lF^{k-1} + F^{k-2} - F^{2k-2} \\
&= l - F^{k-1} + F^{k-1} - l^2
\end{aligned}
$$

3) *Theorem (1.3) Let* $k = 2r$, m and F satisfying (1.7)

$$m^2 = m, \quad Fm = mF = 0,$$

$(m + F^r)(m - F^{r-1}) = I$, then we get (1.1)

Proof: we have $(m + F^r)(m - F^{r-1}) = I$

$$
\begin{aligned}
& m^2 - mF^{r-1} + F^r m - F^{2r-1} = I \\
& m - 0 + 0 - F^{2r-1} = I \\
& mF - F^{2r} = F \\
& F^{2r} + F = 0
\end{aligned}
$$

which is (1.1)

4) *Theorem (1.4) Let*

$k = 2r + 1$, m and F satisfying (1.8)

$$m^2 = m, \quad Fm = mF = 0,$$

$(m + F^r)(m - F^r) = I$, then we get (1.1)

Proof: we have $(m + F^r)(m - F) = I$

$$
\begin{aligned}
& m^2 - mF^r + F^r m - F^{2r} = I \\
& m - 0 + 0 - F^{2r} = I \\
& mF - F^{2r+1} = -F \\
& F^{2r+1} - F = 0
\end{aligned}
$$

which is (1.1)

II. METRIC F-STRUCTURE

If we define

A. $`F(X, Y) = g(FX, Y)$ is skew- symmetric.

Then

B. $g\left(FX,Y\right)=-g\left(X,FY\right)$, $\{F,g\}$ is called Metric F Structure

1) Theorem (2.1): let us K be the odd, then

C. $g\left(F^{(k-1)/2}X,F^{(k-1)/2}Y\right)=(-1)^{(k-1)/2}\left[g\left(X,Y\right)-\grave{m}\left(X,Y\right)\right]$ where

D. $\grave{m}\left(X,Y\right)=g\left(mX,Y\right)=g\left(X,mY\right).$

Proof: we have on using (2.2),(1.2),(2.4) (1.3)

$$g\left(F^{(k-1)/2}X,F^{(k-1)/2}Y\right)=(-1)^{(k-1)/2}g\left(X,F^{k-1}Y\right)$$

$$=(-1)^{(k-1)/2}g\left(X,lY\right)$$

$$=(-1)^{(k-1)/2}\left(g,(I-m)Y\right)$$

$$=(-1)^{(k-1)/2}\left[g\left(X,Y\right)-g\left(X,mY\right)\right]$$

$$=(-1)^{(k-1)/2}\left[g\left(X,Y\right)-\grave{m}\left(X,Y\right)\right]$$

1) *Theorem (2.2):* $\{F,g\}$ is not unique

Proof: let let us K be the odd, and μ be non zero (1,1) tensor , such that

$$\mu F'=F\mu,\quad {}^{/}g(X,Y)=g(\mu X,\mu Y) \text{ then}$$

$$\mu F'^{k}=F^{k}\mu=F\mu=\mu F' \quad \text{thus}$$

Or $F'^{k}-F=0$

Also ${}^{/}g(F'^{(k-1)/2}X,F'^{(k-1)/2}Y)=g(\mu F'^{(k-1)/2}X,\mu F'^{(k-1)/2}Y)$

$$=g(F^{(k-1)/2}\mu X,F^{(k-1)/2}\mu Y)$$

$$=(-1)^{(k-1)/2}g(\mu X,F^{k-1}\mu Y)$$

$$=(-1)^{(k-1)/2}g(\mu X,l\mu Y)$$

$$=(-1)^{(k-1)/2}g(\mu X,(I-m)\mu Y)$$

$$=(-1)^{(k-1)/2}\left[g(\mu X,\mu Y)-g(\mu X,m\mu Y)\right]$$

$$=(-1)^{(k-1)/2}\left[{}^{/}g(X,Y)-{}^{/}m(X,Y)\right]$$

III. LERNAL AND TANGENT VECTOR:

A. $\ker F = \{X : FX = 0\}$

B. $\operatorname{Tan} F = \{X : FX \text{ II } X\}$

1) *Theorem (2.3)* For the F-structure satisfying (1.1), we have

C. $\ker F = \ker F^2 = ------ = \ker F^{2k}$

D. $\operatorname{Tan} F = \operatorname{Tan} F^2 = ------ = \operatorname{Tan} F^{2k}$

Proof: Follows easily

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