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On Some Properties of Metric F- Structure Satisfying $F^k + (-1)^k F = 0$

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ABSTRACT: The purpose of this paper is to study various properties of the F- structure satisfying $F^k + (-1)^k F = 0$. Where k is positive integer $k \geq 3$ the metric F- structure, kernel and tangent vectors have also been discussed.

Key words: Differentiable manifold, Complementary projection operators, metric, kernel and tangent vectors.

I. INTRODUCTION

Let V_n be a C^∞ differentiable manifold and $F \neq 0$ be a C^∞ (1,1) tensor on V_n such that

$$A. F^k + (-1)^k F = 0$$

we define the the projection operators l and m on V_n by

$$B. l = (-1)^{k-1} F^{2k-1}, \quad m = I - (-1)^{k-1} F^{k-1}$$

Where I denotes the identify operator

From (1.1) and (1.2), we have

$$C. l + m = I, \quad l^2 = l, \quad m^2 = m, \quad lm = ml = 0$$

$$lF = Fl = F, \quad Fm = mF = 0,$$

1) Theorem (1.1): If $rank((F)) = n$ then

$$(1.4) \quad l = I, \quad m = 0$$

Proof: from the fact

$$rank((F)) + nulity((F)) = \dim V_n = n$$

We have

$$nulity((F)) = 0 \Rightarrow \ker((F)) = \{0\}$$

or

$$FX = 0 \Rightarrow X = 0$$

Then $FX_1 = FX_2$

$$F(X_1 - X_2) = 0$$

$$X_1 = X_2 \quad \text{or } F \text{ is 1-1}$$

Also F being an operator on a finite dimensional V_n , F is onto also. thus F invertible. F^{-1} exists. Now operating F^{-1} on $lF = F$ and $mF = 0$, we get (1.4)

2) Theorem (1.2) let us define the (1,1) tensors p, q, α, β by

$$(1.5) \quad p = m + F^{k-1}, \quad q = m - F^{k-1}, \quad \alpha = l + F^{k-1}, \quad \beta = l - F^{k-1}$$

then

$$(1.6) \quad p^2 = q^2 = I, pq = M - l, \alpha\beta = 0$$

Proof: using (1.1), (1.2), (1.5) we have

$$\begin{aligned} p^2 &= (m + F^{k-1})(m + F^{k-1}) \\ &= m^2 + mF^{k-1} + F^{k-1} + F^{2k-2} \\ &= m + 0 + 0 + F^{2k-2} \\ &= m + l^2 \\ &= I \end{aligned}$$

$$\begin{aligned} \alpha\beta &= (l + F^{k-1})(l + F^{k-1}) \\ &= l^2 - lF^{k-1} + F^{k-2} - F^{2k-2} \\ &= l - F^{k-1} + F^{k-1} - l^2 \end{aligned}$$

3) Theorem (1.3) Let $k = 2r$, m and F satisfying (1.7)

$$\begin{aligned} m^2 &= m, \quad Fm = mF = 0, \\ (m + F^r)(m - F^{r-1}) &= I, \text{ then we get (1.1)} \end{aligned}$$

Proof: we have $(m + F^r)(m - F^{r-1}) = I$

$$\begin{aligned} m^2 - mF^{r-1} + F^r m - F^{2r-1} &= I \\ m - 0 + 0 - F^{2r-1} &= I \\ mF - F^{2r} &= F \\ F^{2r} + F &= 0 \text{ which is (1.1)} \end{aligned}$$

4) Theorem (1.4) Let

$$\begin{aligned} k &= 2r + 1, \text{ m and F satisfying (1.8)} \\ m^2 &= m, \quad Fm = mF = 0, \\ (m + F^r)(m - F^r) &= I, \text{ then we get (1.1)} \end{aligned}$$

Proof: we have $(m + F^r)(m - F^r) = I$

$$\begin{aligned} m^2 - mF^r + F^r m - F^{2r} &= I \\ m - 0 + 0 - F^{2r} &= I \\ mF - F^{2r+1} &= -F \\ F^{2r+1} - F &= 0 \text{ which is (1.1)} \end{aligned}$$

II. METRIC F-STRUCTURE

If we define

A. $\forall F(X, Y) = g(FX, Y)$ is skew- symmetric.

Then

B. $g(FX, Y) = -g(X, FY)$, $\{F, g\}$ is called Metric F Structure

1) Theorem (2.1): let us K be the odd, then

C. $g(F^{(k-1)/2}X, F^{(k-1)/2}Y) = (-1)^{(k-1)/2} [g(X, Y) - m(X, Y)]$ where

D. $m(X, Y) = g(mX, Y) = g(X, mY)$.

Proof: we have on using (2.2),(1.2),(2.4) (1.3)

$$\begin{aligned} g(F^{(k-1)/2}X, F^{(k-1)/2}Y) &= (-1)^{(k-1)/2} g(X, F^{k-1}Y) \\ &= (-1)^{(k-1)/2} g(X, lY) \\ &= (-1)^{(k-1)/2} (g, (I - m)Y) \\ &= (-1)^{(k-1)/2} [g(X, Y) - g(X, mY)] \\ &= (-1)^{(k-1)/2} [g(X, Y) - m(X, Y)] \end{aligned}$$

1) Theorem (2.2): $\{F, g\}$ is not unique

Proof: let let us K be the odd, and μ be non zero (1,1) tensor , such that

$$\mu F' = F \mu, \quad /g(X, Y) = g(\mu X, \mu Y) \text{ then}$$

$$\mu F'^k = F^k \mu = F \mu = \mu F' \text{ thus}$$

Or $F'^k - F = 0$

Also $/g(F'^{(k-1)/2}X, F'^{(k-1)/2}Y) = g(\mu F'^{(k-1)/2}X, \mu F'^{(k-1)/2}Y)$

$$\begin{aligned} &= g(F^{(k-1)/2} \mu X, F^{(k-1)/2} \mu Y) \\ &= (-1)^{(k-1)/2} g(\mu X, F^{k-1} \mu Y) \\ &= (-1)^{(k-1)/2} g(\mu X, l \mu Y) \\ &= (-1)^{(k-1)/2} g(\mu X, (I - m) \mu Y) \\ &= (-1)^{(k-1)/2} [g(\mu X, \mu Y) - g(\mu X, m \mu Y)] \\ &= (-1)^{(k-1)/2} [/g(X, Y) - /m(X, Y)] \end{aligned}$$

III. LERNAL AND TANGENT VECTOR:

A. $\ker F = \{ X : FX = 0 \}$

B. $\text{Tan } F = \{ X : FX \parallel X \}$

1) *Theorem (2.3)* For the F-structure satisfying (1.1), we have

C. $\ker F = \ker F^2 = \dots = \ker F^{2k}$

D. $\text{Tan } F = \text{Tan } F^2 = \dots = \text{Tan } F^{2k}$

Proof: Follows easily

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