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Finite element Analysis of thermo mechanical stresses of two layered composite cylindrical pressure vessel

Arnab Choudhury[*1], Samar Chandra Mondol[#2], Susenjit Sarkar[#3]
*Faculty, Mechanical Engineering Department, Dr. B.C Roy Engineering College, Durgapur, W.B, India
[#]Faculty, Mechanical Engineering Department, Jadavpur University, Kolkata-700032, India

Abstract- The study of thermo mechanical stresses on pressure vessels has a great significance from the theoretical and practical point of view due to their importance in many applications. This paper presents a complete analysis of thermo mechanical stresses within a thick-walled hollow two layered composite cylinder subjected to internal fluid pressure and thermal load under steady state condition. A complete evaluation of temperature and stress distributions, in a steady state taking into consideration of the effect of centripetal and centrifugal heat flow, is obtained using finite element method.
Keywords- thermo mechanical, steady state, composite, centripetal, centrifugal.

I. INTRODUCTION

The most important structures in various industries such as chemical, petroleum industries etc. are thick hollow cylinder that is used as pressure vessel and cylindrical shells. In modern industries the use of new material such as composite and functionally graded materials are increasing due to their wide applications in controlling stresses due to thermal and mechanical loads. Conventional pressure vessels made up of single layer (single material) may very often satisfy the service requirement under high temperature, high fluid pressure and corrosive environment. Therefore, a multilayered composite pressure vessel is widely used to meet the requirement under different industrial service condition by using different layers of different material. Generally the inner layer is made up of material of high performance alloy because the inner layer is subjected to high fluid pressure and high temperature. While the outer layer is made up of steel or fiber, All layers are firmly bonded together. They are used widely in various industries such as high pressure reactor.

There has been a lot of paper in this field. The existence of any temperature gradient across the wall of a thick-walled vessel induces a thermal stress. Often, thermal stresses are greater than those generated by application of either internal and/or external pressure. From an economical point of view, the thermo elasto-plastic method is used for design of such vessels. Detailed analyses of thermal stress in spherical and cylindrical vessels in the elastic range are given in [2–6]. In [7] the behavior of thick-walled spherical and cylindrical vessels under thermal and mechanical stresses is considered .Reactor pressure vessels or boiler often have a combination of high temperatures together with high pressures. The pressure vessels have to be designed carefully in order to deal with the operating pressure and temperature. Thick-walled cylindrical pressure vessels have a wide spectrum of applications in various industries, likewise high pressure reactor in extreme conditions, due to their high corrosion resistance and high safety performance. Zhang et al. [8] derived an analytical solution for determining the stress distribution of a multilayered composite pressure vessel subjected to an internal fluid pressure and a thermal load. The stress distribution of the pressure vessel was computed using FE method. Ali, Ghosh, and Alam [9] investigated the effect of autofrettage process in strain hardened thick-walled pressure vessels theoretically by FE modeling. Wang and Ding [10] obtained the thermoelastic dynamic solution of a multilayered orthotropic hollow cylinder in the state of axisymmetric plane strain. Atefi and Mahmoudi [11] offered an analytical solution for obtaining thermal stresses in a pipe caused by periodic time varying of temperature of medium fluid. Jabbari, Sohrabpour, and Eslami [12] developed a general analysis of one-dimensional steady-state thermal stresses in a hollow thick cylinder made of functionally graded material. Shao, Wang, and Ang [13] carried out thermo-mechanical analysis of functionally graded hollow cylinder subjected to axisymmetric mechanical and transient thermal loads.

Thick-walled cylinders subjected to internal heat flow are used in many engineering applications. Typical examples are nuclear engineering structures, nozzle sections of rockets, gun tubes, and dies of hot forming tools. The study of thick-walled cylinders subjected to internal heat flow and/or internal pressure is a problem of great practical interest. Industrial demands for such applications have focused the attention of the investigators on this point of research. However, most investigators have only dealt with the analysis of thermal stresses of thick-walled cylinders under steady-state conditions [14-19].

Stasynk *et al.* [14] have studied the steady-state thermal stress of hollow cylinders considering the effect of variation of thermal conductivity as a function of temperature. They concluded that the effect of thermal conductivity on the temperature and stresses is slight for small values of internal heat flow. However, for large heat flow, the difference in temperature and stresses between temperature-dependent and -independent thermal conductivity can be as much as 20%. Vollbrecht [15] has analysed the stresses in both cylindrical and spherical walls subjected to internal pressure and stationary heat flow. Kandil [16] has studied the effect of steady-state temperature and pressure gradient on compound cylinders fitted together by shrink fit. The finite element method has been used by Sinha [17] to analyse the thermal stresses and temperature distribution in a hollow thick cylinder subjected to a steady-state heat load in the radial direction.

Naga [18] has presented the stress analysis and the optimization of both thick-walled impermeable and permeable cylinders under the combined effect of steady-state temperature and pressure gradient. Zukhova and Pimshtein [19] have studied the one-dimensional, steady-state thermal problem for a laminated cylinder consisting of concentric layers and subjected to internal pressure and external heating. Their calculations show that the radial compressive stress due to the internal pressure can permit external heating without layer separation. They found that the distribution of temperatures and stresses depends on the manner of stress application and heating.

Despite the fact that the theory of thermo elasticity has been widely used to solve the problem related to the pressure vessel [20-23], there is not enough literature available to determine the thermo mechanical stresses in pressure vessel using finite element approach. In this paper, thermo mechanical stresses was computed in a two layered composite hollow thick cylindrical pressure vessel taking into the effect of centrifugal and centripetal heat flow by using finite element approach. The proposed finite element solution may be used to design multilayered composite pressure vessel under steady state condition.

II. PROBLEM FORMULATION

Consider an infinitely long thick hollow cylinder composed of multilayered laminate bonded together perfectly. The cylinder is made of a homogeneous isotropic material and is long enough in the axial direction such that the assumption of the plane strain condition satisfies. This work deals with the one-dimensional, coupled, thermo elastic problem i.e. cylindrical vessel is subjected to both thermal load and internal fluid pressure.

A. *Generalized Assumptions*

The following assumptions are considered for solving the problem:

1) The material of each layer is assumed to be homogenous.
2) Deformation and strain satisfy Hooke's law and small strain theory.
3) The composite cylinder is constructed of multilayer laminates bonded perfectly together.
4) All physical variables are assumed to be functions of the radial coordinate, i.e. an axisymmetric problem.
5) Steady-state heat conduction was assumed.
6) Both centrifugal and centripetal heat flow was considered in this problem.
7) The ends of the cylinders are assumed to be unrestrained.
8) The longitudinal strain developed as a result of the stress is uniform and constant, i.e. plane strain with $\varepsilon_z = 0$.

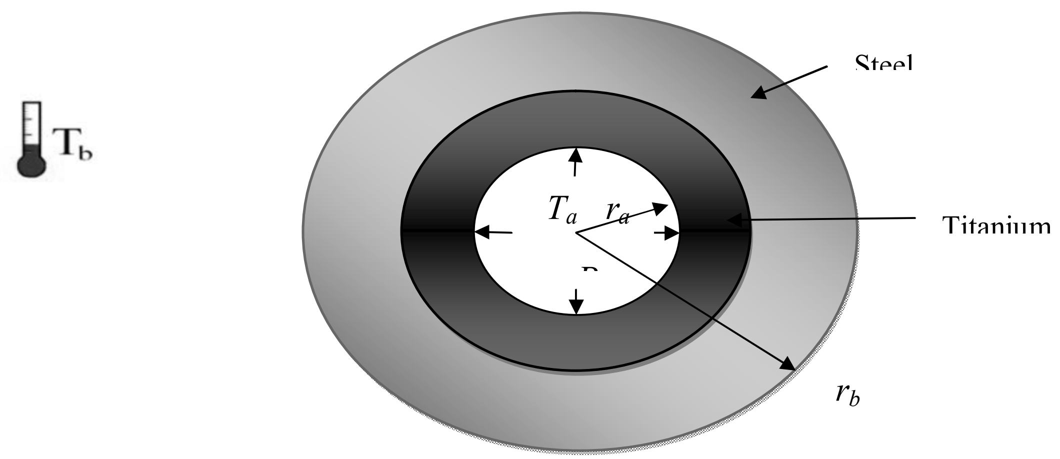

Fig1 Sketch of two layered open ended cylindrical pressure vessel

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B. Steady state heat conduction

The steady state heat conduction equation in cylindrical coordinate is [8]

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)T(r)=0 \tag{1}$$

Where T(r) is the radial distribution function of temperature. Therefore the temperature distribution function of an arbitrary layer is[8]:

$$T_k(r)=\frac{T_k-T_{k-1}}{\ln q_k}ln\frac{r}{r_k}+T_k \tag{2}$$

Where T_k between the kth and k+1 th layer can be determined by [24]

$$T_k=(\lambda_k T_{k-1}lnq_{k+1}+\lambda_{k+1}T_{k+1}lnq_k)/(\lambda_k lnq_{k+1}+\lambda_{k+1}lnq_k) \tag{3}$$

Where λ denotes the thermal conductivity, R denotes the interfacial radius, T denotes the interfacial temperature, and the subscript k (1,2,., n) represents the kth layer .
where q is the radius ratio, and $q_k=\frac{r_k}{r_{k-1}}$

C. Thermo-mechanical Stresses

In order to calculate the stresses due to heat flow and internal pressure the following equilibrium equations are used:

$$\frac{d\sigma_r}{dr}+\frac{\sigma_t-\sigma_r}{r}=0 \tag{4}$$

And the strain displacement equation is as

$$\varepsilon_r=\frac{du}{dr}\ ,\ \varepsilon_t=\frac{u}{r}\ ,\ \varepsilon_z=0 \tag{5}$$

The stresses in the cylindrical pressure vessel can be written as [8]

$$\left.\begin{aligned}\sigma_r&=\frac{E}{(1+\mu)(1-2\mu)}\left[(1-\mu)\varepsilon_r+\mu\varepsilon_t\right]-\frac{E\alpha T}{1-2\mu}\\ \sigma_t&=\frac{E}{(1+\mu)(1-2\mu)}\left[(1-\mu)\varepsilon_t+\mu\varepsilon_r\right]-\frac{E\alpha T}{1-2\mu}\\ \sigma_z&=\frac{E}{(1+\mu)(1-2\mu)}\left[\mu(\varepsilon_t+\varepsilon_r)\right]-\frac{E\alpha T}{1-2\mu}\end{aligned}\right\} \tag{6}$$

Where ε_r, ε_t, and ε_z are the radial strain, hoop strain and axial strain, respectively. u denotes the radial displacement, λ denotes the coefficient of thermal expansion, μ denotes Poisson's ratio, and E denotes Young's modulus. T denotes the temperature change from the reference temperature, and the reference temperature is given as zero in this work.

Boundary conditions:

The pressure vessel subjected to internal fluid pressure and thermal load, the boundary conditions are as:

$$\left.\begin{aligned}\sigma_r(r)&=-P_i\ when\ r=r_a\\ \sigma_r(r)&=0\ when\ r=r_b\\ T(r)&=T_a\ when\ r=r_a\\ T(r)&=T_b\ when\ r=r_b\end{aligned}\right\} \tag{7}$$

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Where T_a and T_b are the uniform temperatures on the inner and outer walls, respectively.
For an infinitely long cylinder, hoop stress is considered as the critical parameter to determine the wall thickness of the pressure vessel and axial stress is neglected. The hoop stress and the radial stress are calculated by introducing radial interface pressure, p between two consecutive layers [25].
The interfacial displacement between two consecutive layers should be

$$u_k \Big|_{r = r_k^{(+)}} = u_{k+1} \Big|_{r = r_k^{(-)}} \tag{8}$$

Combining the equations (4)-(8) and applying the boundary conditions (7), the thermo mechanical stresses in the multilayered composite pressure vessel can be computed. For a multilayered hollow cylindrical pressure vessel, the radial and hoop stresses have been proposed [8]. In this work, these stress formulas for cylindrical vessels were updated as:

$$\sigma_{rk} = \frac{\alpha_k E_k}{1-\mu_k}\left[\frac{\int_{r_{k-1}}^{r_k} T_k(r) r dr}{r_{k-1}^2 - r_k^2}\left(\frac{r_{k-1}^2}{r^2} - 1\right) - \frac{1}{r^2}\int_{r_{k-1}}^{r} T_k(r) r\, dr\right] + \frac{p_k r_k^2 - p_{k=1} r_{k-1}^2}{r_{k-1}^2 - r_k^2} + \frac{r_{k-1}^2 r_k^2 (p_{k-1} - p_k)}{(r_{k-1}^2 - r_k^2) r^2} \tag{9}$$

$$\sigma_{tk} = \frac{\alpha_k E_k}{1-\mu_k}\left[-\frac{\int_{r_{k-1}}^{r_k} T_k(r) r dr}{r_{k-1}^2 - r_k^2}\left(\frac{r_{k-1}^2}{r^2} + 1\right) + \frac{1}{r^2}\int_{r_{k-1}}^{r} T_k(r) r\, dr - T_k(r)\right] + \frac{p_k r_k^2 - p_{k-1} r_{k-1}^2}{r_{k-1}^2 - r_k^2} - \frac{r_{k-1}^2 r_k^2 (p_{k-1} - p_k)}{(r_{k-1}^2 - r_k^2) r^2} \tag{10}$$

Whereas radial interface pressure p_k can be computed as:

$$p_k = \frac{\left(J_k + \left(\frac{z_k + n_k}{1 - q_k^2}\right) p_{k-1} + \left(\frac{z_{k+1} q_{k+1}^2 + n_{k+1} q_{k+1}^2}{1 - q_{k+1}^2}\right) p_{k+1}\right)}{\frac{z_{k+1} + n_{k+1} q_{k+1}^2}{1 - q_{k+1}^2} + \frac{z_k q_k^2 + n_k}{1 - q_k^2}} \tag{11}$$

Where $n_k = \frac{(-1+\mu_k)}{E_k}$; $l_k = \frac{\mu_k}{E_k}$; $z_k = \frac{\mu_k - 1}{E_k}$ and $J_k = \frac{2\alpha_{k+1}\int_{r_k}^{r_{k+1}} T_{k+1}(r) r dr}{r_k^2 - r_{k+1}^2}$ - $\frac{2\alpha_k \int_{r_{k-1}}^{r_k} T_{k+1}(r) r dr}{r_{k-1}^2 - r_k^2}$

III. FINITE ELEMENT MODELING OF TWO LAYERED COMPOSITE PRESSURE VESSEL

Finite element model of a two layered composite cylindrical pressure vessel of infinite length was modeled. Due to the axis-symmetry of the pressure vessel and boundary conditions, a quarter of the geometrical model (Fig.2) was constructed with the axisymmetrical 4-node thermal elements Plane55 and the axisymmetrical 4-node plane elements Plane182 using the finite element tool: ANSYS. The symmetrical boundary condition was applied at the bottom and top of the cylinder, and the numbers of nodes and elements were 26466 and 25632, respectively.
Two layers are bonded together perfectly. The inside layer is made up of titanium and outside laminate is made up of Steel. The inner radius of the cylinder is 1m and outer radius is 4.5 m. inside layer has a thickness of 2m and that of outside is 2.5m. The internal fluid pressure was 22 MPa, and the outer pressure 0 MPa. The inner temperature was 200 °C, and the outer temperature 190 °C. The thermo mechanical properties and the wall thicknesses of each layer are listed in Table 1.

An indirect approach [26] was used to calculate the thermo mechanical coupling stresses of the two-layered composite pressure vessel. The solution procedure was as follows: Firstly, the thermal model was created and the temperature boundary condition was given, and the temperature distribution was calculated. Secondly, the element type was converted from the thermal element PLANE55 to the mechanical element PLANE182. Finally, the internal fluid pressure was imposed on the inner layer of the FE model of the pressure vessel, and the two stresses (σ_t, σ_r) were determined.

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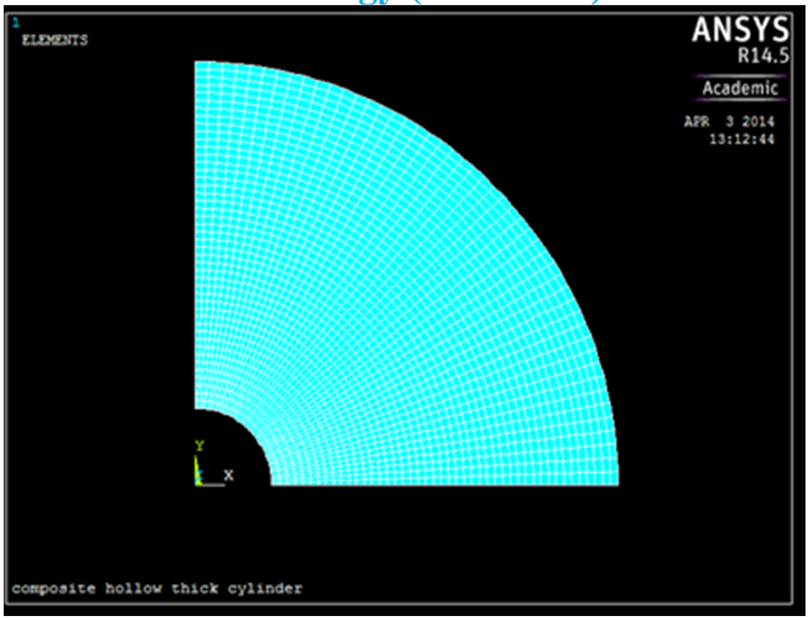

Figure 2: FEA model of two layered pressure vessel

Table 1

Material and geometric Properties of each layer.

	Layer-1(Titanium)	Layer-2 (Steel-1025)
Thickness	2 m	2.5 m
Young's Modulus	108e9 kJ	207e9
Poisson's Ratio	0.3	0.3
Thermal conductivity	20 kW/m k	17
Thermal expansion coefficient	11e-6	11e-6
density	4 kg/m^3	7.8
Specific heat	0.4 kJ/kg k	0.48

IV. RESULT AND DISCUSSION

The pressure vessel has multilayered properties, so there is step changes in the distribution of radial and hoop stress. In this work two cases are considered:

Case-I: Inner temperature is greater than the outer temperature; heat will flow from inner to outer surface i.e. centrifugal flux. The temperature distribution is shown in the fig3. The distribution of hoop stress and radial stress is shown in the fig5 and fig7 respectively. The maximum hoop stress exists at the external surface since heat will flow from internal to external surface. There is sharp change in pressure at the interface of two layers.

Case-II: Outer temperature is greater than the inner temperature; heat will flow from outer to inner surface i.e. centripetal flux. The temperature distribution is shown in the fig4.The distribution of hoop stress and radial stress is shown in the fig6 and fig8 respectively. The maximum hoop stress exists at the internal surface since heat will flow from external to internal surface. There is sharp change in pressure at the interface of two layers.

It is found that maximum thermo mechanical stress due to centripetal thermal flux at the inner fiber is greater than that of centrifugal flux. Thus centripetal thermal flux is more dangerous than centrifugal thermal flux.

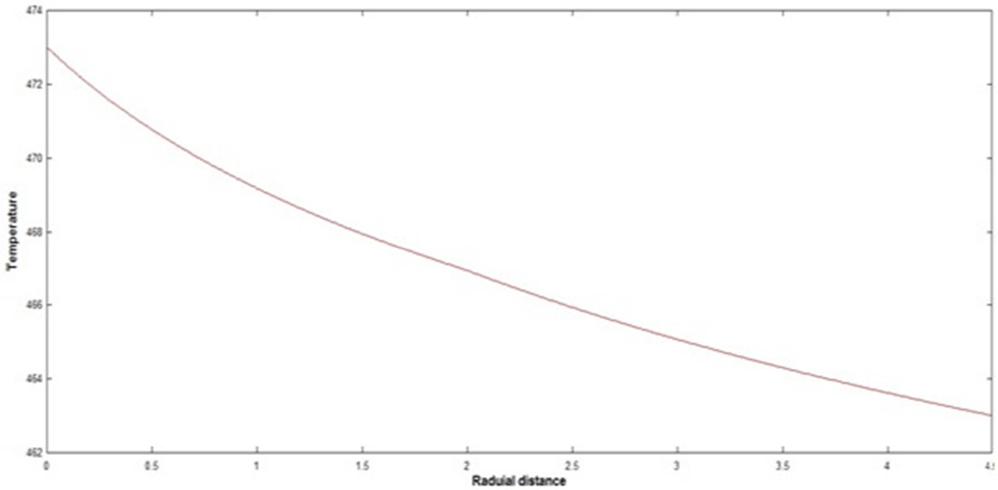

Fig 3: Temperature distribution for centrifugal heat flow.

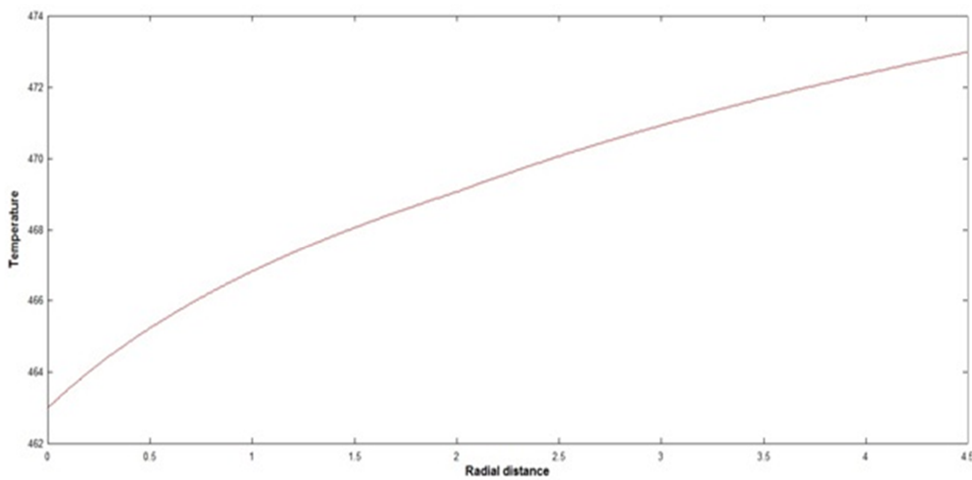

Figure 4 : Temperature distribution for centripetal heat flow.

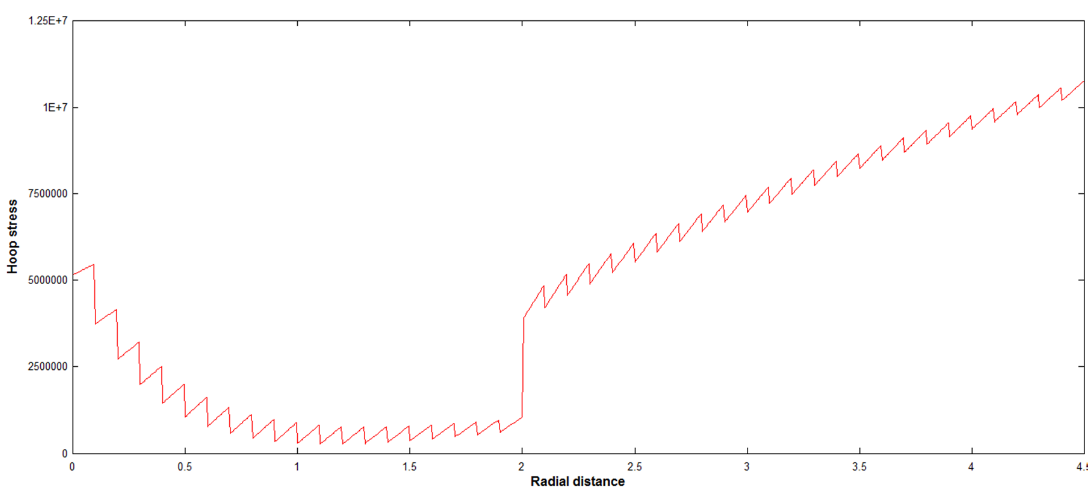

Fig 5: Hoop stress distribution for centrifugal heat flow.

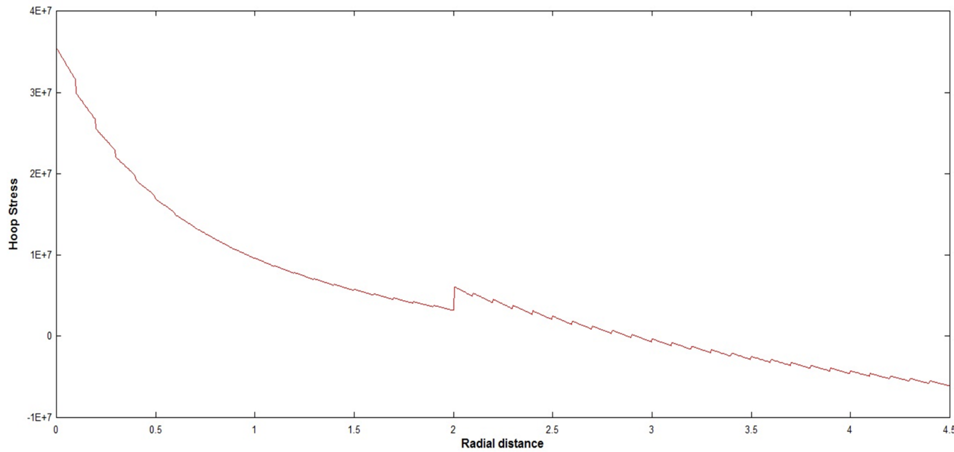

Fig 6: Hoop stress distribution for centripetal heat flow.

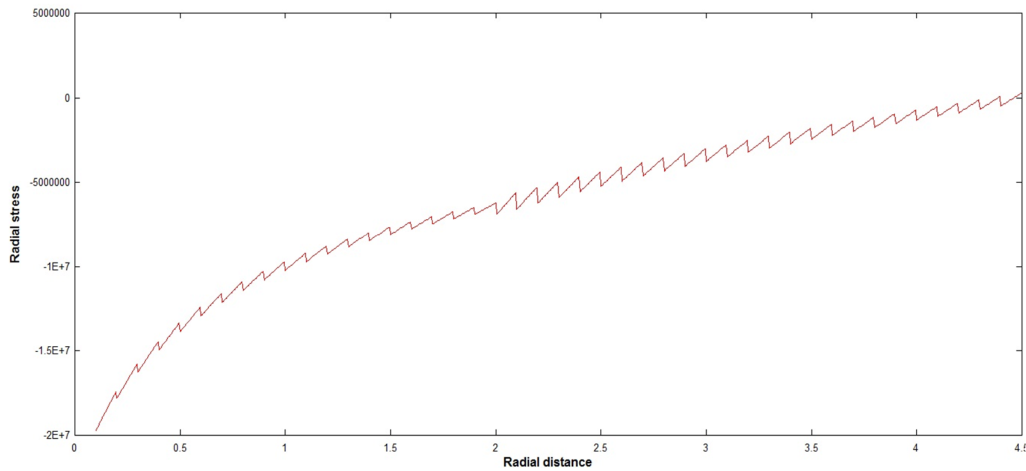

Fig 7: Radial stress distribution for centrifugal heat flow.

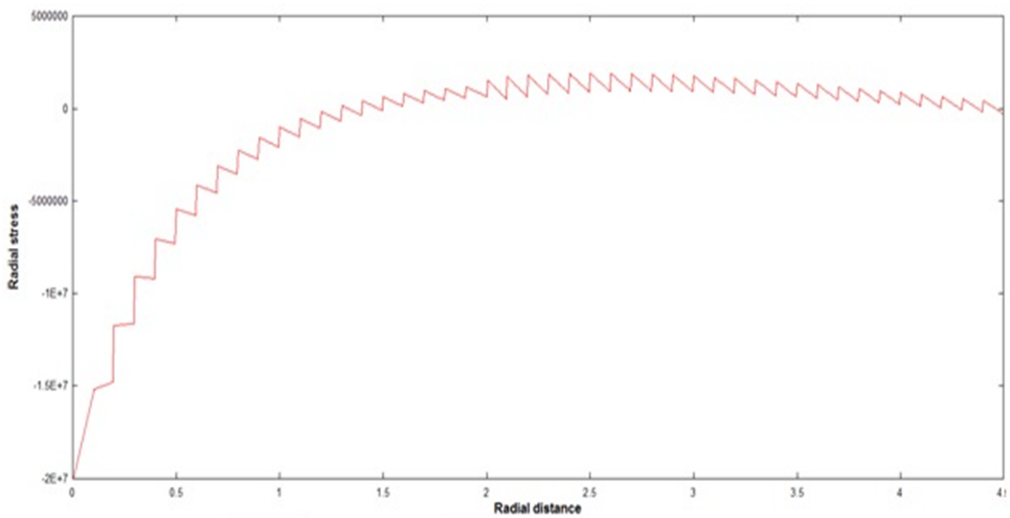

Fig 8: Radial stress distribution for centripetal heat flow.

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V. CONCLUSION

In this study, we attempted to investigate the problem for the combined mechanical and thermal stresses in a multilayered composite hollow cylinder. It is found that maximum thermo mechanical stress due to centripetal thermal flux at the inner fiber is greater than that of centrifugal flux. Thus centripetal thermal flux is more dangerous than centrifugal thermal flux. So Finite element method is easier technique to deal with such type of complex problems.

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Notation:

T Temperature (K)

ε_z longitudinal strain

ε_t Tangential strain

ε_r Radial strain

r Radius (m)

u radial displacement (m)

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σ_t Tangential stress (Pa)

σ_r Radial stress (Pa)

σ_z Axial stress (Pa)

μ Poisson's ratio

E Young's modulus (Pa)

K Thermal conductivity (W/m K)

h convective heat transfer coefficient (W/m^2 K)

T_α Ambient temperature (K)

T_a Fluid temperature (K)

P_i Fluid pressure (N/m^2)

r_a Inside radius of cylinder (m)

r_b outside radius of cylinder (m)