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Adjacent Vertex Sum Polynomial of Path Related Graphs

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Abstract: Let $G = (V, E)$ be a graph. The adjacent vertex sum polynomial of G is defined as $S(G, x) = \sum_{i=0}^{\Delta(G)} n_{\Delta(G)-i} x^{\alpha_{\Delta(G)-i}}$, where $n_{\Delta(G)-i}$ is the sum of the number of adjacent vertices of all the vertices of degree $\Delta(G) - i$ and $\alpha_{\Delta(G)-i}$ is the sum of the degree of adjacent vertices of all the vertices of degree $\Delta(G) - i$. In this paper I seek to find the Adjacent Vertex Sum Polynomial of some path related Graphs.

Keywords: Adjacent Vertex Sum Polynomial, Splitting graph, Degree splitting graph, Path.

I. INTRODUCTION

Here I consider simple undirected graphs. The terms not defined here we can refer Frank Harary [3]. The vertex set is denoted by V and the edge set by E . For $v \in V$, $d(v)$ is the number of edges incident with v , the maximum degree of the graph G is defined as $\Delta(G) = \max\{d(v)/v \in V\}$. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs, the union $G_1 \cup G_2$ is defined to be $G = (V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$, the sum $G_1 + G_2$ is defined as $G_1 \cup G_2$ together with all the lines joining points of V_1 to V_2 . The Cartesian product of two graphs G_1 and G_2 denoted by $G = G_1 \times G_2$ is the graph G such that $V(G) = V(G_1) \times V(G_2)$, that is every vertex of $G_1 \times G_2$ is an ordered pair (u, v) , where $u \in V(G_1)$ and $v \in V(G_2)$ and two distinct vertices (u, v) and (x, y) are adjacent in $G_1 \times G_2$ if either $u = x$ and $vy \in E(G_2)$ or $v = y$ and $ux \in E(G_1)$. The graph G with $V = S_1 \cup S_2 \cup \dots \cup S_i \cup T$, where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V \setminus \cup S_i$. The degree splitting graph of G denoted by $DS(G)$ and is obtained from G by adding the vertices w_1, w_2, \dots, w_t and joining w_i to each vertex of S_i , $1 \leq i \leq t$ [5]. For each vertex v of a graph G , take a new vertex v' , join v' to all the vertices of G which are adjacent to v . The graph $S(G)$ thus obtained is called splitting graph of G [1]. The Path consisting of length n is denoted by P_n . The graph $G = (V, E)$ is simply denoted by G . Number of vertices in G is called order of G .

II. MAIN RESULTS

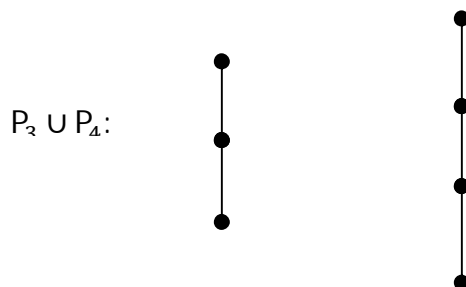
A. Theorem 2.1

Let P_m and P_n be paths with order m and n respectively. Then $S(P_m \cup P_n, x) = [2(m+n) - 8]x^{4(m+n)-20} + 4x^8$.

1) *Proof:* Let P_m and P_n be paths with order m and n respectively. In P_m , $m - 2$ vertices have degree 2 and 2 vertices have degree 1. In P_n , $n - 2$ vertices have degree 2 and 2 vertices have degree 1. Therefore in this graph $P_m \cup P_n$, $m - 2$ vertices have degree 2; $n - 2$ vertices have degree 2 and 4 vertices have degree 1. Hence, sum of the number of adjacent vertices of all the vertices of degree 2 is $2(m+n) - 8$, sum of the degree of adjacent vertices of all the vertices of degree 2 is $4(m+n) - 20$, sum of the number of adjacent vertices of all the vertices of degree 1 is 4, sum of the degree of adjacent vertices of all the vertices of degree 1 is 8. This gives, $S(P_m \cup P_n, x) = [2(m+n) - 8]x^{4(m+n)-20} + 4x^8$.

B. Example 2.2

Consider the graph $P_3 \cup P_4$, then $S(P_3 \cup P_4, x) = 10x^8$.



$$\begin{aligned} \text{Here, } S(P_3 \cup P_4, x) &= [2(3 + 4) - 8]x^{4(3+4)-20} + 4x^8. \\ &= 6x^8 + 4x^8. \\ &= 10x^8 \end{aligned}$$

C. Theorem 2.3

Let P_m and P_n be paths with order m and n respectively. Then $S(P_m + P_n, x)$

$$\begin{aligned} &= (m - 2)(n + 2)x^{(m-4)[2(n+2)+(n-2)(m+2)+2(m+1)]+2[(2n+3)+(n-2)(m+2)+2(m+1)]} \\ &+ 2(n + 1)x^{2[(n+2)+(n-2)(m+2)+2(m+1)]} \\ &+ (n - 2)(m + 2)x^{(n-4)[2(m+2)+(m-2)(n+2)+2(n+1)]+2[(2m+3)+(m-2)(n+2)+2(n+1)]} \\ &+ 2(m + 1)x^{2[(m+2)+(m-2)(n+2)+2(n+1)]}, \text{ where } m, n \geq 4. \end{aligned}$$

1) *Proof:* Let P_m and P_n be paths with order m and n respectively. In P_m , $m - 2$ vertices have degree 2 and 2 vertices have degree 1. In P_n , $n - 2$ vertices have degree 2 and 2 vertices have degree 1. Therefore in this graph $P_m + P_n$, $m - 2$ vertices have degree $n + 2$; $n - 2$ vertices have degree $m + 2$, 2 vertices have degree $n + 1$ and 2 vertices have degree $m + 1$. Hence, sum of the number of adjacent vertices of all the vertices of degree $n + 2$ is $(m - 2)(n + 2)$, sum of the degree of adjacent vertices of all the vertices of degree $n + 2$ is $(m - 4)[2(n + 2) + (n - 2)(m + 2) + 2(m + 1)] + 2[(2n + 3) + (n - 2)(m + 2) + 2(m + 1)]$, sum of the number of adjacent vertices of all the vertices of degree $n + 1$ is $2(n + 1)$, sum of the degree of adjacent vertices of all the vertices of degree $n + 1$ is $2[(n + 2) + (n - 2)(m + 2) + 2(m + 1)]$, sum of the number of adjacent vertices of all the vertices of degree $m + 2$ is $(n - 2)(m + 2)$, sum of the degree of adjacent vertices of all the vertices of degree $m + 2$ is $(n - 4)[2(m + 2) + (m - 2)(n + 2) + 2(n + 1)] + 2[(2m + 3) + (m - 2)(n + 2) + 2(n + 1)]$, sum of the number of adjacent vertices of all the vertices of degree $m + 1$ is $2(m + 1)$, sum of the degree of adjacent vertices of all the vertices of degree $m + 1$ is $2[(m + 2) + (m - 2)(n + 2) + 2(n + 1)]$.

This gives, $S(P_m + P_n, x)$

$$\begin{aligned} &= (m - 2)(n + 2)x^{(m-4)[2(n+2)+(n-2)(m+2)+2(m+1)]+2[(2n+3)+(n-2)(m+2)+2(m+1)]} \\ &+ 2(n + 1)x^{2[(n+2)+(n-2)(m+2)+2(m+1)]} \\ &+ (n - 2)(m + 2)x^{(n-4)[2(m+2)+(m-2)(n+2)+2(n+1)]+2[(2m+3)+(m-2)(n+2)+2(n+1)]} \\ &+ 2(m + 1)x^{2[(m+2)+(m-2)(n+2)+2(n+1)]}. \end{aligned}$$

D. Example 2.4

Consider the graph $P_4 \cup P_4$, then $S(P_4 + P_4, x) = 24x^{66} + 20x^{56}$.

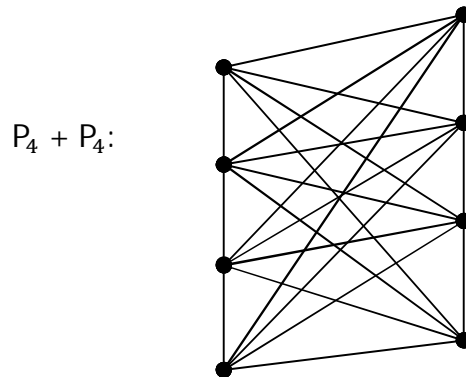


Figure: 2.2

Here, $S(P_4 + P_4, x)$

$$\begin{aligned}
 &= (m - 2)(n + 2)x^{(m-4)[2(n+2)+(n-2)(m+2)+2(m+1)]+2[(2n+3)+(n-2)(m+2)+2(m+1)]} \\
 &+ 2(n + 1)x^{2[(n+2)+(n-2)(m+2)+2(m+1)]} \\
 &+ (n - 2)(m + 2)x^{(n-4)[2(m+2)+(m-2)(n+2)+2(n+1)]+2[(2m+3)+(m-2)(n+2)+2(n+1)]} \\
 &+ 2(m + 1)x^{2[(m+2)+(m-2)(n+2)+2(n+1)]}. \\
 &= 12x^{66} + 10x^{56} + 12x^{66} + 10x^{56}. \\
 &= 24x^{66} + 20x^{56}.
 \end{aligned}$$

E. Theorem 2.5

Let P_m and P_n be paths with order m and n respectively. Then $S(s(P_m \cup P_n), x) = 4[(m - 2) + (n - 2)]x^{12[(m-4)+(n-4)]+36} + 2[(m - 2) + (n - 2) + 4]x^{8[(m-4)+(n-4)]+48} + 4x^{16}$, where $m, n \geq 4$.

1) *Proof:* Let P_m and P_n be paths with order m and n respectively. In P_m , $m - 2$ vertices have degree 2 and 2 vertices have degree 1. In P_n , $n - 2$ vertices have degree 2 and 2 vertices have degree 1. Therefore in this graph $s(P_m \cup P_n)$, $m - 2$ vertices have degree 4; $n - 2$ vertices have degree 4, m vertices have degree 2; n vertices have degree 2, and 4 vertices have degree 1. Hence, sum of the number of adjacent vertices of all the vertices of degree 4 is $4[(m - 2) + (n - 2)]$, sum of the degree of adjacent vertices of all the vertices of degree 4 is $4(m + n) - 20$, sum of the number of adjacent vertices of all the vertices of degree 2 is $2[(m - 2) + (n - 2) + 4]$, sum of the degree of adjacent vertices of all the vertices of degree 2 is $8[(m - 4) + (n - 4)] + 48$, sum of the number of adjacent vertices of all the vertices of degree 1 is 4, sum of the degree of adjacent vertices of all the vertices of degree 1 is 16. This gives, $S(s(P_m \cup P_n), x) = 4[(m - 2) + (n - 2)]x^{12[(m-4)+(n-4)]+36}$

$$+ 2[(m - 2) + (n - 2) + 4]x^{8[(m-4)+(n-4)]+48} + 4x^{16}.$$

F. Example 2.6

Consider the graph $s(P_3 \cup P_4)$, then $S(s(P_4 \cup P_4), x) = 16x^{36} + 16x^{48} + 4x^{16}$.

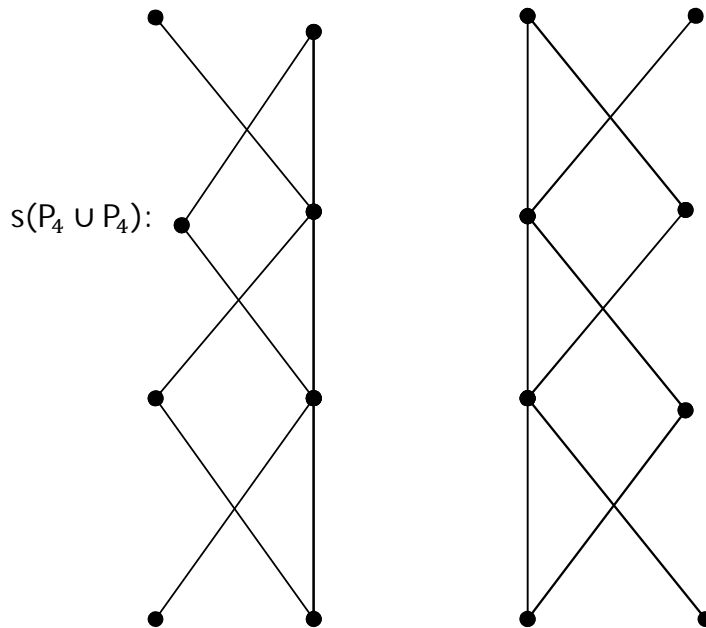


Figure: 2.3

Here, $S(s(P_m \cup P_n), x) = 4[(m - 2) + (n - 2)]x^{12[(m-4)+(n-4)]+36}$
 $+ 2[(m - 2) + (n - 2) + 4]x^{8[(m-4)+(n-4)]+48} + 4x^{16}.$
 $= 16x^{36} + 16x^{48} + 4x^{16}.$

G. Theorem 2.7

Let P_m and P_n be paths with order m and n respectively.

Then $S(DS(P_m \cup P_n), x) = 3[(m - 2) + (n - 2)]x^{(n-4)(n+4)+(m-4)(m+4)+2(m+n-4)+20} + (m - 2)x^{3(m-2)} + (n - 2)x^{3(n-2)} + 12x^{28}$, where $m, n \geq 4$.

1) Proof: Let P_m and P_n be paths with order m and n respectively. In P_m , $m - 2$ vertices have degree 2 and 2 vertices have degree 1. In P_n , $n - 2$ vertices have degree 2 and 2 vertices have degree 1. Therefore in this graph $DS(P_m \cup P_n)$, $m - 2$ and $n - 2$ vertices have degree 3; 1 vertex has degree $m - 2$, 1 vertex has degree $n - 2$, 6 vertices have degree 2. Hence, sum of the number of adjacent vertices of all the vertices of degree 3 is $3[(m - 2) + (n - 2)]$, sum of the degree of adjacent vertices of all the vertices of degree 3 is $(n - 4)(n + 4) + (m - 4)(m + 4) + 2(m + n - 4) + 20$, sum of the number of adjacent vertices of all the vertices of degree $m - 2$ is $m - 2$, sum of the degree of adjacent vertices of all the vertices of degree $m - 2$ is $3(m - 2)$, sum of the number of adjacent vertices of all the vertices of degree $n - 2$ is $n - 2$, sum of the degree of adjacent vertices of all the vertices of degree $n - 2$ is $3(n - 2)$, sum of the number of adjacent vertices of all the vertices of degree 2 is 12, sum of the degree of adjacent vertices of all the vertices of degree 2 is 28. This gives, $S(DS(P_m \cup P_n), x) = 3[(m - 2) + (n - 2)]x^{(n-4)(n+4)+(m-4)(m+4)+2(m+n-4)+20} + (m - 2)x^{3(m-2)} + (n - 2)x^{3(n-2)} + 12x^{28}$, where $m, n \geq 4$.

H. Example 2.8

Consider the graph $DS(P_4 \cup P_4)$, then $S(DS(P_4 \cup P_4), x) = 12x^{28} + 2x^6 + 2x^6 + 12x^{28}$.

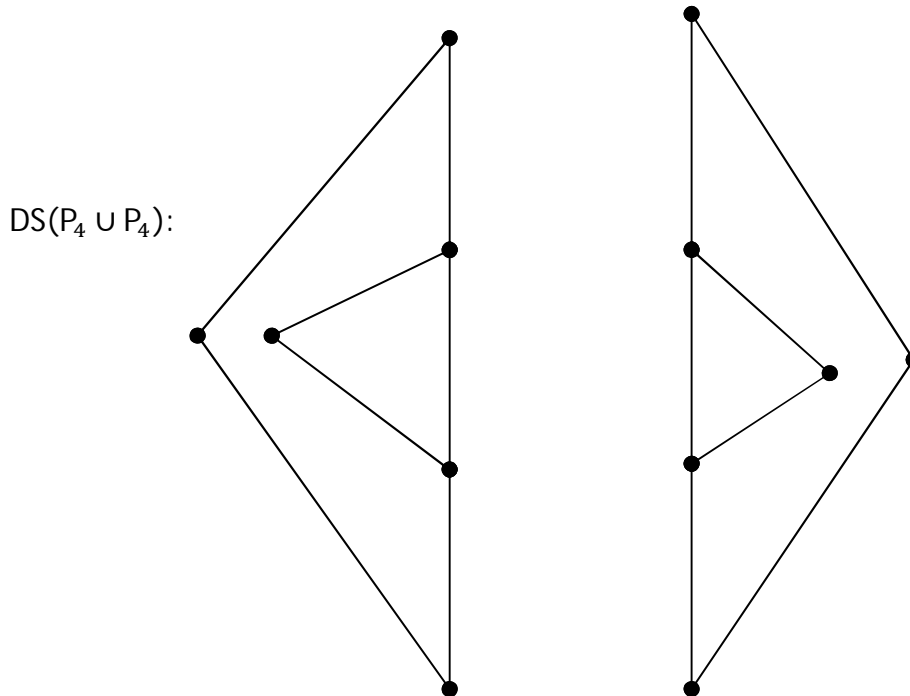


Figure: 2.4

Here, $S(DS(P_m \cup P_n), x) = 3[(4 - 2) + (4 - 2)]x^{(4-4)(4+4)+(4-4)(4+4)+2(4+4-4)+20}$
 $+ (4 - 2)x^{3(4-2)} + (4 - 2)x^{3(4-2)} + 12x^{28}.$
 $= 12x^{28} + 2x^6 + 2x^6 + 12x^{28}.$

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