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Adjacent Vertex Sum Polynomial of Path Related Graphs

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***Abstract:** Let $G=(V,E)$ be a graph. The adjacent vertex sum polynomial of G is defined as $S(G,x)=\sum_{i=o}^{\Delta(G)} n_{\Delta(G)-i}x^{\alpha_{\Delta(G)-i}}$, where $n_{\Delta(G)-i}$ is the sum of the number of adjacent vertices of all the vertices of degree $\Delta(G)-i$ and $\alpha_{\Delta(G)-i}$ is the sum of the degree of adjacent vertices of all the vertices of degree $\Delta(G)-i$. In this paper I seek to find the Adjacent Vertex Sum Polynomial of some path related Graphs.*
***Keywords:** Adjacent Vertex Sum Polynomial, Splitting graph, Degree splitting graph, Path.*

I. INTRODUCTION

Here I consider simple undirected graphs. The terms not defined here we can refer Frank Harary [3]. The vertex set is denoted by V and the edge set by E. For $v\epsilon V$, $d(v)$ is the number of edges incident with v, the maximum degree of the graph G is defined as $\Delta(G)=\max\{d(v)/v\epsilon V\}$. Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be two graphs, the union $G_1\cup G_2$ is defined to be $G=(V,E)$ where $V=V_1\cup V_2$ and $E=E_1\cup E_2$, the sum G_1+G_2 is defined as $G_1\cup G_2$ together with all the lines joining points of V_1 to V_2. The Cartesian product of two graphs G_1 and G_2 denoted by $G=G_1\times G_2$ is the graph G such that $V(G)=V(G_1)\times V(G_2)$, that is every vertex of $G_1\times G_2$ is an ordered pair (u,v), where $u\in V(G_1)$ and $v\in V(G_2)$ and two distinct vertices (u,v) and (x,y) are adjacent in $G_1\times G_2$ if either $u=x$ and $vy\in E(G_2)$ or $v=y$ and $ux\in E(G_1)$. The graph G with $V=S_1\cup S_2\cup\dots\cup S_i\cup T$, where each S_i is a set of vertices having at least two vertices and having the same degree and $T=V\backslash\cup S_i$. The degree splitting graph of G denoted by $DS(G)$ and is obtained from G by adding the vertices $w_1,w_2,\dots,w_t$ and joining w_i to each vertex of S_i, $1\le i\le t$ [5]. For each vertex v of a graph G, take a new vertex v′, join v′ to all the vertices of G which are adjacent to v. The graph $s(G)$ thus obtained is called splitting graph of G [1]. The Path consisting of length n is denoted by P_n. The graph $G=(V,E)$ is simply denoted by G. Number of vertices in G is called order of G.

II. MAIN RESULTS

A. Theorem 2.1

Let P_m and P_n be paths with order m and n respectively. Then $S(P_m\cup P_n,x)=[2(m+n)-8]x^{4(m+n)-20}+4x^8$.

1) *Proof:* Let P_m and P_n be paths with order m and n respectively. In P_m, $m-2$ vertices have degree 2 and 2 vertices have degree 1. In P_n, $n-2$ vertices have degree 2 and 2 vertices have degree 1. Therefore in this graph $P_m\cup P_n$, $m-2$ vertices have degree 2; $n-2$ vertices have degree 2 and 4 vertices have degree 1. Hence, sum of the number of adjacent vertices of all the vertices of degree 2 is $2(m+n)-8$, sum of the degree of adjacent vertices of all the vertices of degree 2 is $4(m+n)-20$, sum of the number of adjacent vertices of all the vertices of degree 1 is 4, sum of the degree of adjacent vertices of all the vertices of degree 1 is8. This gives, $S(P_m\cup P_n,x)=[2(m+n)-8]x^{4(m+n)-20}+4x^8$.

B. Example 2.2

Consider the graph $P_3\cup P_4$, then $S(P_3\cup P_4,x)=10x^8$.

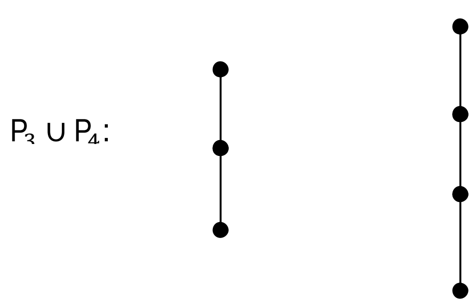

Figure: 2.1

Here, $S(P_3 \cup P_4, x) = [2(3+4)-8]x^{4(3+4)-20} + 4x^8$.

$= 6x^8 + 4x^8$.

$= 10x^8$

C. Theorem 2.3

Let P_m and P_n be paths with order m and n respectively. Then $S(P_m + P_n, x)$

$$= (m-2)(n+2)x^{(m-4)[2(n+2)+(n-2)(m+2)+2(m+1)]+2[(2n+3)+(n-2)(m+2)+2(m+1)]}$$

$$+2(n+1)x^{2[(n+2)+(n-2)(m+2)+2(m+1)]}$$

$$+(n-2)(m+2)x^{(n-4)[2(m+2)+(m-2)(n+2)+2(n+1)]+2[(2m+3)+(m-2)(n+2)+2(n+1)]}$$

$$+2(m+1)x^{2[(m+2)+(m-2)(n+2)+2(n+1)]}$$, where $m, n \geq 4$.

1) *Proof*:Let P_m and P_n be paths with order m and n respectively. In P_m, $m-2$ vertices have degree 2 and 2 vertices have degree 1. In P_n, $n-2$ vertices have degree 2 and 2 vertices have degree 1. Therefore in this graph $P_m + P_n$, $m-2$ vertices have degree $n+2$; $n-2$ vertices have degree $m+2$, 2 vertices have degree $n+1$ and 2 vertices have degree $m+1$. Hence, sum of the number of adjacent vertices of all the vertices of degree $n+2$ is $(m-2)(n+2)$, sum of the degree of adjacent vertices of all the vertices of degree $n+2$ is $(m-4)[2(n+2)+(n-2)(m+2)+2(m+1)] + 2[(2n+3)+(n-2)(m+2)+2(m+1)]$, sum of the number of adjacent vertices of all the vertices of degree $n+1$ is $2(n+1)$, sum of the degree of adjacent vertices of all the vertices of degree $n+1$ is $2[(n+2)+(n-2)(m+2)+2(m+1)]$, sum of the number of adjacent vertices of all the vertices of degree $m+2$ is $(n-2)(m+2)$, sum of the degree of adjacent vertices of all the vertices of degree $m+2$ is $(n-4)[2(m+2)+(m-2)(n+2)+2(n+1)] + 2[(2m+3)+(m-2)(n+2)+2(n+1)]$, sum of the number of adjacent vertices of all the vertices of degree $m+1$ is $2(m+1)$, sum of the degree of adjacent vertices of all the vertices of degree $m+1$ is $2[(m+2)+(m-2)(n+2)+2(n+1)]$.

This gives, $S(P_m + P_n, x)$

$$= (m-2)(n+2)x^{(m-4)[2(n+2)+(n-2)(m+2)+2(m+1)]+2[(2n+3)+(n-2)(m+2)+2(m+1)]}$$

$$+2(n+1)x^{2[(n+2)+(n-2)(m+2)+2(m+1)]}$$

$$+(n-2)(m+2)x^{(n-4)[2(m+2)+(m-2)(n+2)+2(n+1)]+2[(2m+3)+(m-2)(n+2)+2(n+1)]}$$

$$+2(m+1)x^{2[(m+2)+(m-2)(n+2)+2(n+1)]}.$$

D. Example 2.4

Consider the graph $P_4 \cup P_4$, then $S(P_4 + P_4, x) == 24x^{66} + 20x^{56}$.

$P_4 + P_4$:

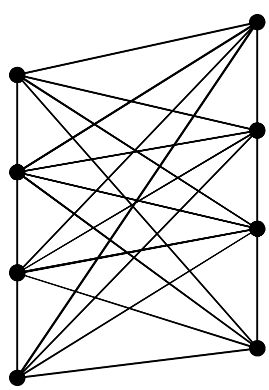

Figure: 2.2

Here, $S(P_4 + P_4, x)$

$$= (m-2)(n+2)x^{(m-4)[2(n+2)+(n-2)(m+2)+2(m+1)]+2[(2n+3)+(n-2)(m+2)+2(m+1)]}$$

$$+2(n+1)x^{2[(n+2)+(n-2)(m+2)+2(m+1)]}$$

$$+(n-2)(m+2)x^{(n-4)[2(m+2)+(m-2)(n+2)+2(n+1)]+2[(2m+3)+(m-2)(n+2)+2(n+1)]}$$

$$+2(m+1)x^{2[(m+2)+(m-2)(n+2)+2(n+1)]}.$$

$$= 12x^{66} + 10x^{56} + 12x^{66} + 10x^{56}.$$

$$= 24x^{66} + 20x^{56}.$$

E. *Theorem 2.5*

Let P_m and P_n be paths with order m and n respectively. Then $S(s(P_m \cup P_n), x) = 4[(m-2)+(n-2)]x^{12[(m-4)+(n-4)]+36} + 2[(m-2)+(n-2)+4]x^{8[(m-4)+(n-4)]+48} + 4x^{16}$, where $m, n \geq 4$.

1) *Proof:* Let P_m and P_n be paths with order m and n respectively. In P_m, $m-2$ vertices have degree 2 and 2 vertices have degree 1. In P_n, $n-2$ vertices have degree 2 and 2 vertices have degree 1. Therefore in this graph $s(P_m \cup P_n)$, $m-2$ vertices have degree 4; $n-2$ vertices have degree 4, m vertices have degree 2; n vertices have degree 2, and 4 vertices have degree 1. Hence, sum of the number of adjacent vertices of all the vertices of degree 4 is $4[(m-2)+(n-2)]$, sum of the degree of adjacent vertices of all the vertices of degree 4 is $4(m+n)-20$, sum of the number of adjacent vertices of all the vertices of degree 2 is $2[(m-2)+(n-2)+4]$, sum of the degree of adjacent vertices of all the vertices of degree 2 is $8[(m-4)+(n-4)]+48$, sum of the number of adjacent vertices of all the vertices of degree 1 is 4, sum of the degree of adjacent vertices of all the vertices of degree 1 is 16. This gives, $S(s(P_m \cup P_n), x) = 4[(m-2)+(n-2)]x^{12[(m-4)+(n-4)]+36}$

$$+2[(m-2)+(n-2)+4]x^{8[(m-4)+(n-4)]+48} + 4x^{16}.$$

F. *Example 2.6*

Consider the graph $s(P_3 \cup P_4)$, then $S(s(P_4 \cup P_4), x) == 16x^{36} + 16x^{48} + 4x^{16}$.

$s(P_4 \cup P_4)$:

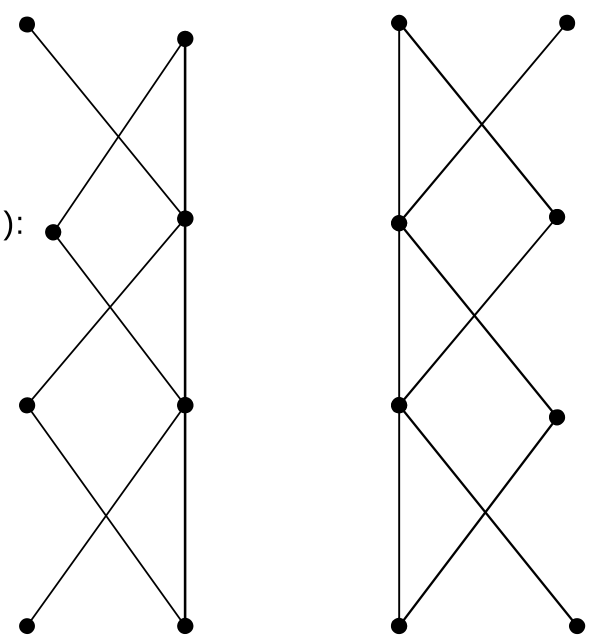

Figure: 2.3

Here, $S(s(P_m \cup P_n), x) = 4[(m-2)+(n-2)]x^{12[(m-4)+(n-4)]+36}$

$$+2[(m-2)+(n-2)+4]x^{8[(m-4)+(n-4)]+48} + 4x^{16}.$$

$$= 16x^{36} + 16x^{48} + 4x^{16}.$$

G. Theorem 2.7

Let P_m and P_n be paths with order m and n respectively.

Then $S(DS(P_m \cup P_n), x) = 3[(m-2)+(n-2)]x^{(n-4)(n+4)+(m-4)(m+4)+2(m+n-4)+20} + (m-2)x^{3(m-2)} + (n-2)x^{3(n-2)} + 12x^{28}$, where $m, n \geq 4$.

1) *Proof:* Let P_m and P_n be paths with order m and n respectively. In P_m, $m-2$ vertices have degree 2 and 2 vertices have degree 1. In P_n, $n-2$ vertices have degree 2 and 2 vertices have degree 1. Therefore in this graph $DS(P_m \cup P_n)$, $m-2$ and $n-2$ vertices have degree 3; 1 vertex has degree $m-2$, 1 vertex has degree $n-2$, 6 vertices have degree 2. Hence, sum of the number of adjacent vertices of all the vertices of degree 3 is $3[(m-2)+(n-2)]$, sum of the degree of adjacent vertices of all the vertices of degree 3 is $(n-4)(n+4)+(m-4)(m+4)+2(m+n-4)+20$, sum of the number of adjacent vertices of all the vertices of degree $m-2$ is $m-2$, sum of the degree of adjacent vertices of all the vertices of degree $m-2$ is $3(m-2)$, sum of the number of adjacent vertices of all the vertices of degree $n-2$ is $n-2$, sum of the degree of adjacent vertices of all the vertices of degree $n-2$ is $3(n-2)$, sum of the number of adjacent vertices of all the vertices of degree 2 is 12, sum of the degree of adjacent vertices of all the vertices of degree 2 is 28. This gives, $S(DS(P_m \cup P_n), x) = 3[(m-2)+(n-2)]x^{(n-4)(n+4)+(m-4)(m+4)+2(m+n-4)+20}$

$$+(m-2)x^{3(m-2)} + (n-2)x^{3(n-2)} + 12x^{28}, \text{ where } m, n \geq 4.$$

H. Example 2.8

Consider the graph $DS(P_4 \cup P_4)$, then $S(DS(P_4 \cup P_4), x) == 12x^{28} + 2x^6 + 2x^6 + 12x^{28}$.

$DS(P_4 \cup P_4)$:

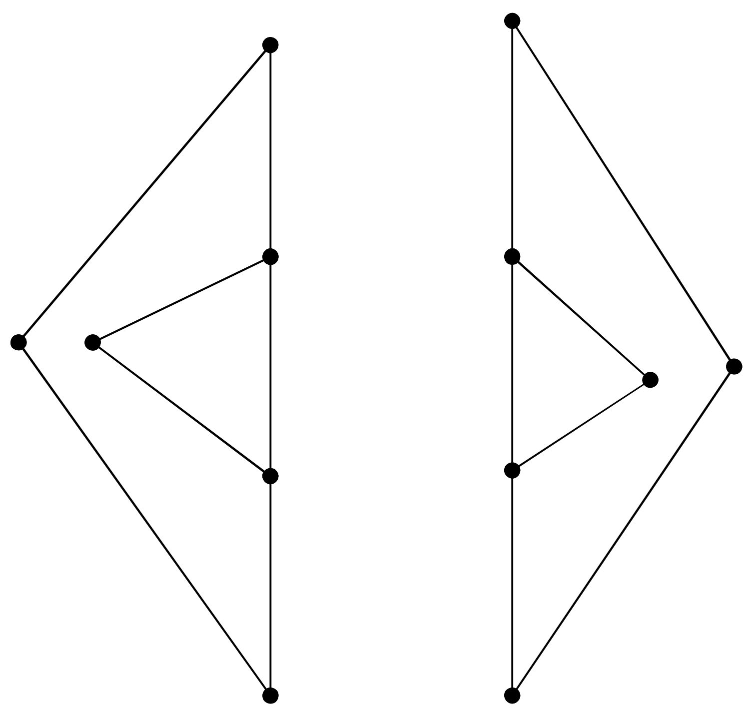

Figure: 2.4

Here, $S(DS(P_m \cup P_n), x) = 3[(4-2)+(4-2)]x^{(4-4)(4+4)+(4-4)(4+4)+2(4+4-4)+20}$

$$+(4-2)x^{3(4-2)} + (4-2)x^{3(4-2)} + 12x^{28}.$$

$$= 12x^{28} + 2x^6 + 2x^6 + 12x^{28}.$$

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