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Common Fixed-Point Theorems for Compatible Mappings of Type (E) in \mathcal{M} - Fuzzy Metric Spaces

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Abstract: The aim of this paper is to obtain common fixed point theorems for self mappings in complete \mathcal{M} - fuzzy metric spaces by compatible of type (E). Our result generalizes and improves other similar results in Manandhar, Jha and Pathak [6].

Keywords: Common fixed point theorem, Generalized fuzzy metric spaces, Compatible mappings.

AMS Mathematics Subject Classification (2010): 47H10, 54H25

I. INTRODUCTION

In 1965, the concept of fuzzy set was introduced by Zadeh [15]. Then in 1975, Kramosil and Michalek [5] introduced the fuzzy metric space as a generalization of a metric space. In 1994, George and Veeramani [1] modified the notion of fuzzy metric spaces with the help of continuous t-norms. In 1986, Jungck [2] introduced notion of compatible mappings in metric spaces. In 2000, Singh and Chouhan [11] introduced the concept of compatible mappings in fuzzy metric spaces. In 1993, Jungck, Murthy and Cho [3] gave a generalization of compatible mappings called compatible mappings of type (A) which is equivalent to the concept of compatible mappings under some conditions. In 1994, Pathak, Cho, Chang and Kang [8] introduced the concept of compatible mappings of type (P) and compared with compatible mappings of type (A) and compatible mappings. In 1998, Pant [7] introduced the notion of reciprocal continuity of mappings in metric spaces. In 1999, Vasuki [13] introduced the notion point wise R - weakly commuting mappings in \mathcal{M} - fuzzy metric spaces. Recently, in 2007, Singh and Singh [10] introduced the concept of compatible mappings of type (E) in metric spaces. Since then, many authors have obtained fixed point theorems in \mathcal{M} - fuzzy metric spaces using these compatible notions. The purpose of this paper is to establish common fixed point theorems for compatible mappings of type (E) in \mathcal{M} - fuzzy metric spaces with an example.

II. PRELIMINARIES

A. Definition 2.1.

A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t - norm if it satisfies the following conditions

- 1) $*$ is associative and commutative
- 2) $*$ is continuous
- 3) $a * 1 = a$, for all $a \in [0, 1]$
- 4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$

Examples for continuous t - norm are $a * b = \min \{a, b\}$ and $a * b = ab$

B. Definition 2.2.

A 3 - tuple $(X, \mathcal{M}, *)$ is called \mathcal{M} fuzzy metric space if X is an arbitrary non - empty set, $*$ is a continuous t - norm, and \mathcal{M} is a fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$

(FM - 1) $\mathcal{M}(x, y, z, t) > 0$,

(FM - 2) $\mathcal{M}(x, y, z, t) = 1$ if $x = y = z$,

(FM - 3) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, where p is a permutation function,

(FM - 4) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, s) \leq \mathcal{M}(x, y, z, t + s)$,

(FM - 5) $\mathcal{M}(x, y, z, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous,

(FM - 6) $\lim_{t \rightarrow \infty} \mathcal{M}(x, y, z, t) = 1$.

C. Example 2.3.

Let X be a non- empty set and D^* is the D^* - metric on X . Denote $a * b = a.b$ for all $a, b \in [0, 1]$. For each $t \in (0, \infty)$, define $\mathcal{M}(x, y, z, t) = \frac{t}{t+D^*(x,y,z)}$ for all $x, y, z \in X$, then $(X, \mathcal{M}, *)$ is a \mathcal{M} - fuzzy metric space. We call this \mathcal{M} - fuzzy metric induced by D^* - metric space. Thus every D^* - metric induces a \mathcal{M} - fuzzy metric.

D. *Lemma 2.4.*

Let $(X, \mathcal{M}, *)$ be a \mathcal{M} - fuzzy metric space. Then for every $t > 0$ and for every $x, y \in X$. We have $\mathcal{M}(x, x, y, t) = \mathcal{M}(x, y, y, t)$.

E. *Lemma 2.5.*

Let $(X, \mathcal{M}, *)$ be a \mathcal{M} - fuzzy metric space. Then $\mathcal{M}(x, y, z, t)$ is non - decreasing with respect to t , for all x, y, z in X .

F. *Definition 2.6.*

Let $(X, \mathcal{M}, *)$ be a \mathcal{M} - fuzzy metric space and $\{x_n\}$ be a sequence in X .

- 1) $\{x_n\}$ is said to be converges to a point $x \in X$ if $\lim_{n \rightarrow \infty} \mathcal{M}(x, x, x_n, t) = 1$ for all $t > 0$
- 2) $\{x_n\}$ is called Cauchy sequence, if $\lim_{n \rightarrow \infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t) = 1$ for all $t > 0$ and $p > 0$.
- 3) A \mathcal{M} - fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

G. *Definition 2.7.*

Let S and T be two self mappings of a \mathcal{M} – fuzzy metric space $(X, \mathcal{M}, *)$.

Then the mappings S and T are said to be weakly compatible if they commute at their coincidence points, that is, $Sx = Tx$ for some $x \in X$, then $STx = TSx$.

H. *Definition 2.8.*

The self mappings A and S of a \mathcal{M} -fuzzy metric space $(X, \mathcal{M}, *)$ are called point wise R - weakly commuting if there exists $R > 0$ such that

$$\mathcal{M}(ASx, SAx, SAx, t) \geq \mathcal{M}(Ax, Sx, Sx, t/R) \text{ for all } x \text{ in } X \text{ and } t > 0.$$

I. *Definition 2.9.*

The self mappings A and S of a metric space (X, d) are said to be compatible of type (E), if $\lim_{n \rightarrow \infty} AAx_n = \lim_{n \rightarrow \infty} ASx_n = S(t)$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = t$ and $\lim_{n \rightarrow \infty} Sx_n = t$, for some $t \in X$.

J. *Definition 2.10.*

A self mappings A and S of a \mathcal{M} - fuzzy metric space $(X, \mathcal{M}, *)$ are said to be compatible of type (E) iff $\lim_{n \rightarrow \infty} \mathcal{M}(AAx_n, ASx_n,$

$$ASx_n, t) = 1,$$

$$\lim_{n \rightarrow \infty} \mathcal{M}(AAx_n, Sx_n, Sx_n, t) = 1, \lim_{n \rightarrow \infty} \mathcal{M}(ASx_n, Sx_n, Sx_n, t) = 1 \text{ and}$$

$$\lim_{n \rightarrow \infty} \mathcal{M}(SSx_n, SAx_n, SAx_n, t) = 1, \lim_{n \rightarrow \infty} \mathcal{M}(SSx_n, Ax_n, Ax_n, t) = 1,$$

$$\lim_{n \rightarrow \infty} \mathcal{M}(Sx_n, Ax_n, Ax_n, t) = 1, \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that}$$

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x \text{ for some } x \text{ in } X \text{ and } t > 0.$$

K. *Lemma 2.11.*

Let $\{y_n\}$ be a sequence in a \mathcal{M} - fuzzy metric space $(X, \mathcal{M}, *)$ with the condition (FM- 6). If there exists $k \in (0, 1)$ such that $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t)$ for all $t > 0$ and $n \in \mathbb{N}$, then $\{y_n\}$ is a Cauchy sequence in X .

We need the following proposition for the proof of our main result.

L. *Proposition 2.12*

If A and S are compatible mappings of type (E) on a \mathcal{M} - fuzzy metric space $(X, \mathcal{M}, *)$ and if one of the function is continuous. Then, we have

$$A(x) = S(x) \text{ and } \lim_{n \rightarrow \infty} AAx_n = \lim_{n \rightarrow \infty} SSx_n = \lim_{n \rightarrow \infty} ASx_n = \lim_{n \rightarrow \infty} SAx_n$$

If these exist $u \in X$ such that $Au = Su = x$ then $ASu = SAu$.

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x$ for some x in X .

1) *Proof:* Let $\{x_n\}$ be a sequence of X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x$ for some x in X .

Then by definition of compatible of type (E), we have $\lim_{n \rightarrow \infty} AAx_n = ASx_n = S(x)$.

If A is a continuous mapping, then we get

$$\lim_{n \rightarrow \infty} AAx_n = A(\lim_{n \rightarrow \infty} Ax_n) = A(x). \text{ This implies } A(x) = S(x). \text{ Also}$$

$$\lim_{n \rightarrow \infty} AAx_n = \lim_{n \rightarrow \infty} SSx_n = \lim_{n \rightarrow \infty} ASx_n = \lim_{n \rightarrow \infty} SAx_n.$$

Similarly, if S is continuous then, we get the same result. This is the proof of part (i). Again, suppose $Au = Su = x$ for some $u \in X$. Then, $ASu = A(Su) = Ax$ and

$$SAu = S(Au) = Sx. \text{ From (i), we have } Ax = Sx. \text{ Hence, } ASu = SAu.$$

This is the proof of part (ii).

III. MAIN RESULTS

If P, Q, S, T, A and B are self mappings in \mathcal{M} -fuzzy metric space $(X, \mathcal{M}, *)$. We denote

$$\begin{aligned} \mathcal{M}_\alpha(x, y, t) &= \mathcal{M}(STx, Px, Px, t) * \mathcal{M}(ABx, Qy, Qy, t) * \mathcal{M}(STx, ABx, ABx, t) \\ &* \mathcal{M}(ABx, Px, Px, \alpha t) * \mathcal{M}(STx, Qy, Qy, (2-\alpha)t), \end{aligned}$$

for all $x, y \in X, \alpha \in (0, 2)$ and $t > 0$.

A. Theorem 3.1

Let $(X, \mathcal{M}, *)$ be a complete \mathcal{M} -fuzzy metric space with a $* a \geq a$ for all $a \in [0, 1]$ and with the condition (FM-6). Let one of the mapping of self mappings (P, ST) and (Q, AB) of X be continuous such that

$$PX \subset ABX, QX \subset STX;$$

There exists $k \in (0, 1)$ such that $\mathcal{M}(Px, Qy, Qy, kt) \geq \mathcal{M}_\alpha(x, y, y, t)$ for all $x, y \in X, \alpha \in (0, 2)$ and $t > 0$.

If (P, ST) and (Q, AB) compatible of type of (E) then P, Q, ST and AB have a unique common fixed point. If the pair $(A, B), (S, T), (Q, B)$ and (T, P) are commuting mappings then A, B, S, T, P and Q have a unique common fixed point.

1) *Proof:* Let x_0 be any point in X . From the condition (i) there exists $x_1, x_2 \in X$ such that $Px_0 = ABx_1 = y_0$ and $Qx_1 = STx_2 = y_1$.

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Px_{2n} = ABx_{2n+1} = y_{2n}$ and $Qx_{2n+1} = STx_{2n+2} = y_{2n+1}$ for $n = 0, 1, 2, \dots$ for $t > 0$ and $\alpha = 1 - q$ with $q \in (0, 1)$ in (ii) then, we have

$$\begin{aligned} \mathcal{M}(Px_{2n}, Qx_{2n+1}, Qx_{2n+1}, kt) &\geq \{ \mathcal{M}((STx_{2n}, Px_{2n}, Px_{2n}, t) \\ &* \mathcal{M}(ABx_{2n+1}, Qx_{2n+1}, Qx_{2n+1}, t) \\ &* \mathcal{M}(STx_{2n}, ABx_{2n+1}, ABx_{2n+1}, t) \\ &* \mathcal{M}(ABx_{2n+1}, Px_{2n}, Px_{2n}, (1-q)t) \\ &* \mathcal{M}(STx_{2n}, Qx_{2n+1}, Qx_{2n+1}, (1+q)t) \} \\ \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) &\geq \{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &* \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) \\ &* \mathcal{M}(y_{2n}, y_{2n}, y_{2n}, (1-q)t) * \mathcal{M}(y_{2n-1}, y_{2n+1}, y_{2n+1}, (1+q)t) \} \\ &\geq \{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &* \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt) \}, \\ &\geq \{ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &* \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt) \}. \end{aligned}$$

Since t -norm $*$ is continuous, letting $q \rightarrow 1$, we have

$$\begin{aligned} \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) &\geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &* \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) \\ &\geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t). \end{aligned}$$

It follows that $\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, kt) \geq \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t) * \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$.

Similarly, $\mathcal{M}(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt) \geq \mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t) * \mathcal{M}(y_{2n+1}, y_{2n+2}, y_{2n+2}, t)$.

Therefore, for all n even or odd, we have $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, t)$.

Consequently, $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}(y_{n-1}, y_n, y_n, k^{-1} t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-1} t)$ and hence $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t) * \mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-1} t)$.

Since $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, k^{-m} t) \rightarrow 1$ as $k \rightarrow 0$, it follows that

$\mathcal{M}(y_n, y_{n+1}, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, y_n, t)$ for all $n \in \mathbb{N}$ and $t > 0$.

Therefore, by lemma (2.11), $\{y_n\}$ is a Cauchy sequence.

Since X is complete, then there exists a point z in X such that $y_n \rightarrow z$ as $n \rightarrow \infty$.

Moreover, we have $y_{2n} = Px_{2n} = ABx_{2n+1} \rightarrow z$ and $y_{2n+1} = Qx_{2n+1} = STx_{2n+2} \rightarrow z$.

If P and ST are compatible of type(E) and one of mapping of the pair (P, ST) is continuous then by proposition (2.12), we have $Pz = STz$.

Since $P(X) \subset AB(X)$, there exists a point w in X such that $Pz = ABw$.

Using condition (ii), with $\alpha = 1$, we have

$$\begin{aligned} \mathcal{M}(Pz, Qw, Qw, kt) &\geq \{ \mathcal{M}(STz, Pz, Pz, t) * \mathcal{M}(ABw, Qw, Qw, t) \\ &\quad * \mathcal{M}(STz, ABw, ABw, t) \\ &\quad * \mathcal{M}(ABw, Pz, Pz, t) * \mathcal{M}(STz, Qw, Qw, t) \}. \\ &= \{ \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Qw, Qw, t) \\ &\quad * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, Qw, Qw, t) \} \\ &\geq \mathcal{M}(Pz, Qw, Qw, t). \end{aligned}$$

This implies $Pz = Qw$. Thus, we have $Pz = STz = Qw = ABw$. Also, we get

$$\begin{aligned} \mathcal{M}(Pz, Qx_{2n+1}, Qx_{2n+1}, kt) &\geq \{ \mathcal{M}(STz, Pz, Pz, t) * \mathcal{M}(ABx_{2n+1}, Qx_{2n+1}, Qx_{2n+1}, t) \\ &\quad * \mathcal{M}(STz, ABx_{2n+1}, ABx_{2n+1}, t) * \mathcal{M}(ABx_{2n+1}, Pz, Pz, t) \\ &\quad * \mathcal{M}(STz, Qx_{2n+1}, Qx_{2n+1}, t) \}. \end{aligned}$$

Letting $n \rightarrow \infty$, we get

$$\begin{aligned} \mathcal{M}(Pz, z, z, kt) &\geq \{ \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, z, z, t) * \mathcal{M}(Pz, Pz, Pz, t) \\ &\quad * \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pz, z, z, t) \} \\ &\geq \mathcal{M}(Pz, z, z, t). \end{aligned}$$

Hence, we get $STz = Pz = z$. Therefore, z is a common fixed point of P and ST .

Again, if Q and AB are compatible with the type (E) and one of the mapping of (Q, AB) is continuous. So, we get $Qw = ABw = Pz = z$.

By using proposition (2.12), we get $QQw = QABw = ABQw = ABABw$.

Thus, we get $Qz = ABz$. Also, using condition (ii) with $\alpha = 1$. We have,

$$\begin{aligned} \mathcal{M}(Px_{2n}, Qz, Qz, kt) &\geq \{ \mathcal{M}(STx_{2n}, Px_{2n}, Px_{2n}, t) * \mathcal{M}(ABz, Qz, Qz, t) \\ &\quad * \mathcal{M}(STx_{2n}, ABz, ABz, t) * \mathcal{M}(ABz, Px_{2n}, Px_{2n}, t) \\ &\quad * \mathcal{M}(STx_{2n}, Qz, Qz, t) \}. \end{aligned}$$

Letting $n \rightarrow \infty$, we get

$$\begin{aligned} \mathcal{M}(z, Qz, Qz, kt) &\geq \{ \mathcal{M}(z, z, z, t) * \mathcal{M}(ABz, Qz, Qz, t) * \mathcal{M}(z, ABz, ABz, t) \\ &\quad * \mathcal{M}(ABz, z, z, t) * \mathcal{M}(z, Qz, Qz, t) \} \\ &\geq \mathcal{M}(z, Qz, Qz, t). \end{aligned}$$

Hence, we have $Qz = ABz = z$. Therefore z is a common fixed point of Q and AB .

Hence z is a common fixed point of P, Q, ST and AB .

For uniqueness, suppose that $(Pw \neq Pz = z)$ is another common fixed point of P, Q, ST and AB . Then, using condition (ii) with $\alpha = 1$. We have,

$$\begin{aligned} \mathcal{M}(PPz, QPw, QPw, kt) &= \mathcal{M}(Pz, Pw, Pw, kt) \\ &\geq \{ \mathcal{M}(STPz, PPz, PPz, t) * \mathcal{M}(ABPw, QPw, QPw, t) \\ &\quad * \mathcal{M}(STPz, ABPw, ABPw, t) * \mathcal{M}(ABPw, PPz, PPz, t) \\ &\quad * \mathcal{M}(STPz, QPw, QPw, t) \} \\ &= \mathcal{M}(Pz, Pz, Pz, t) * \mathcal{M}(Pw, Pw, Pw, t) \\ &\quad * \mathcal{M}(Pz, Pw, Pw, t) * \mathcal{M}(Pw, Pz, Pz, t) * \mathcal{M}(Pz, Pw, Pw, t) \end{aligned}$$

$\mathcal{M}(PPz, QPw, QPw, kt) \geq \mathcal{M}(Pz, Pw, Pw, t)$.

That is, $Pw = Pz = z$. Thus, z is a unique common fixed point of P, Q, ST and AB by using the commutativity of the pairs $(A, B), (S, T), (Q, B)$ and (T, P) we can easily prove that z is a unique common fixed point of A, B, S, T, P and Q .

If we take $T = B = I_x$, an identity mapping of X .

B. Corollary 3.2

Let $(X, \mathcal{M}, *)$ be a complete \mathcal{M} -fuzzy metric space with $a * a \geq a$ for all $a \in [0, 1]$ and with the condition (FM-6). If one of the mapping of self mappings (P, Q) and (Q, A) of X is continuous such that for $k \in (0, 1)$ we have

$$\mathcal{M}(Px, Qy, Qy, kt) \geq \mathcal{M}_\alpha(x, y, y, t) \text{ for all } x, y \in X, \alpha \in (0, 2) \text{ and } t > 0, \text{ and if } (P, S) \text{ and } (Q, A) \text{ are compatible of type of (E)}$$

$$\mathcal{M}((x, y, y, t) = \mathcal{M}(Sx, Px, Px, t) * \mathcal{M}(Ay, Qy, Qy, t) * \mathcal{M}(Sx, Ay, Ay, t)$$

$$* \mathcal{M}(Ay, Px, Px, \alpha t) * \mathcal{M}(Sx, Qy, Qy, (2 - \alpha) t),$$

for all $x, y \in X, \alpha \in (0, 2)$ and $t > 0$.

Similarly if we get the result for three self maps by taking $S = A, T = B = I_x$ in the Theorem (3.1) and also by taking $P = Q, T = B = I_x$ in Theorem (3.1) and obtain for two self maps by taking $P = Q, A = S, B = T = I_x$ in Theorem (3.1) then P, A, S, Q have a unique common fixed point.

C. Example 3.3

Let $X = [2, 10]$ with the metric d defined by

$$D^*(x, y, z) = |x - y| + |y - z| + |z - x| \text{ and define } \mathcal{M}(x, y, z, t) = \frac{t}{t + D^*(x, y, z)}$$

for all $x, y, z \in X, t > 0$. Clearly $(X, \mathcal{M}, *)$ is a complete \mathcal{M} -fuzzy metric space. Define P, Q, S, T, A and $B: X \rightarrow X$ as follows;

$$Px = 2 \text{ for all } x, Qx = 2 \text{ if } x < 4 \text{ and } x \geq 5, Qx = 3+x \text{ if } 4 \leq x < 5$$

$$Sx = 2 \text{ if } x \leq 8, Sx = 8 \text{ if } x > 8;$$

$$Ax = 2 \text{ if } x < 4 \text{ or } x \geq 5, Ax = 5 + x \text{ if } 4 \leq x < 5$$

$$Bx = Tx = x, \text{ for all } x \in [2, 10].$$

Then P, Q, S, T, A and B satisfy all the conditions of the above theorem and have a unique common fixed point $x = 2$.

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