



IJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 2017 **Issue:** conference **Month of publication:** December 2017

DOI:

www.ijraset.com

Call:  08813907089

E-mail ID: ijraset@gmail.com

Lehmer-3 Mean Number of Graphs

S. Somasundaram¹, T.S. Pavithra²

¹Professor in Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627012

²Research Scholar, Manonmaniam Sundaranar University, Tirunelveli-627012.

I. INTRODUCTION

A graph considered here are finite, undirected and simple. The vertex set and edge set of a graph are denoted by $V(G)$ and $E(G)$ respectively. A path of length n is denoted by P_n . For standard terminology and notations we follow Harary[1]. S Somasundaram & S.S Sandhya introduced the concept of Harmonic Mean Labeling of Graphs in [2]. We will provide a brief summary of other information's which are necessary for our present investigation.

A. Definition 1.1

A graph $G=(V,E)$ with P vertices and q edges is called Lehmer -3 mean graph. If it is possible to label vertices $x \in V$ with distinct labels $f(x)$ from $1,2,3,\dots,q+1$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv)=\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$ (or) $\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$, then the edge labels are distinct. In this case "f" is called Lehmer -3 mean labeling of G .

B. Definition 1.2

Let G be a graph and $f:V(G) \rightarrow \{1,2,\dots,n\}$ be a function such that the label of the edge $f(e=uv)$ is Labeled with $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$ (or) $\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$, where $\{f(e); e \in E(G)\} \subseteq \{1,2,\dots,n\}$. If n is the smallest positive integer satisfying this condition together with the condition that there is no edges in common. Then n is called the Lehmer-3 mean number of a graph G and is denoted as $L_{3m}(G)$.

II. MAIN RESULTS

A. Theorem :2.1

$L_{3m}(P_n) = n$.

1) Proof : Let u_1, u_2, \dots, u_n be the vertices of the path P_n . Define a function $f:V(P_n) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)=i+1; 2 \leq i \leq n$. Then the edge labels are $f(u_i u_{i+1}) = i; 1 \leq i \leq n-1, f(u_{n-1} u_n) = n$. Thus $L_{3m}(P_n) = n$.

B. Example 2.2

Lehmer-3 mean number of P_7 is

$L_{3m}(P_n) = n = 7$ is given below.

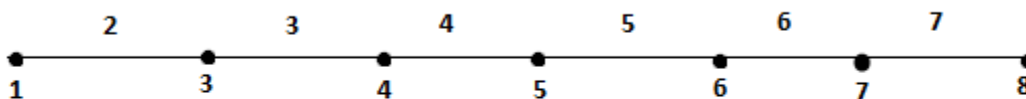


Figure-1

C. Theorem :2.3

$L_{3m}(C_n) = n + 1$

1) Proof : Let C_n be a cycle of n vertices $u_1, u_2, \dots, u_n, u_1$. We define a function $f:V(C_n) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)=i+1; 2 \leq i \leq n$. Then the distinct edge labels are $f(u_i u_{i+1}) = i+1; 1 \leq i \leq n-1, f(u_n u_1) = n+1$. Thus $L_{3m}(C_n) = n+1$.

D. Example 2.4

$L_{3m}(C_8) = n+1 = 8+1 = 9$ is displayed below.

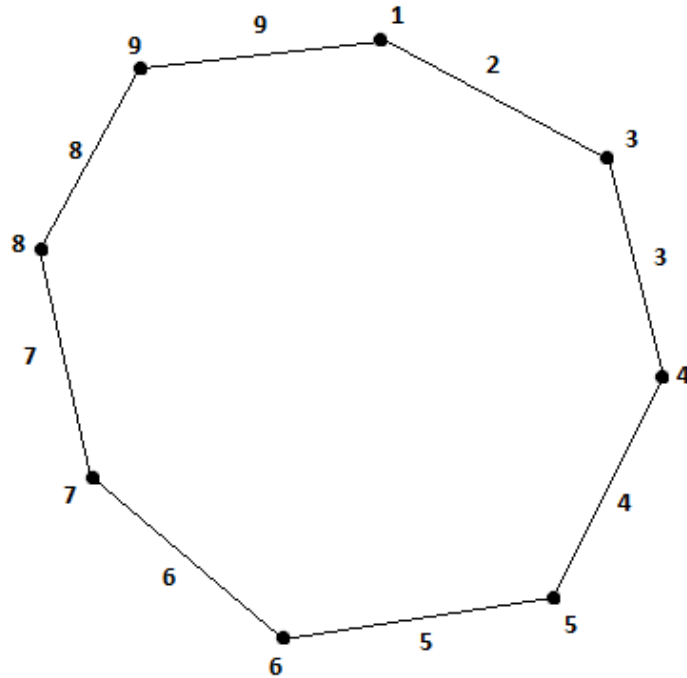


Figure-2

E. Theorem :2.5

$$L_{3m}(P_n \circ K_1) = 2n.$$

1) Proof : Let G be a $P_n \circ K_1$ graph and its vertices $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$. respectively. Let us define a function $f:V(G) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)= 2i ; 2 \leq i \leq n, f(v_i)= 2i+1 ; 1 \leq i \leq n$. and the obtained distinct edge labels are $f(u_i, u_{i+1})=2i+1 ; 1 \leq i \leq n-1, f(u_i, v_i)= 2i ; 1 \leq i \leq n-1, f(u_n, v_n)= 2n$

Hence $L_{3m}(P_n \circ K_1) = 2n$

F. Example 2.6

$L_{3m}(P_6 \circ K_1) = 2n = 2 \times 6 = 12$ is given below

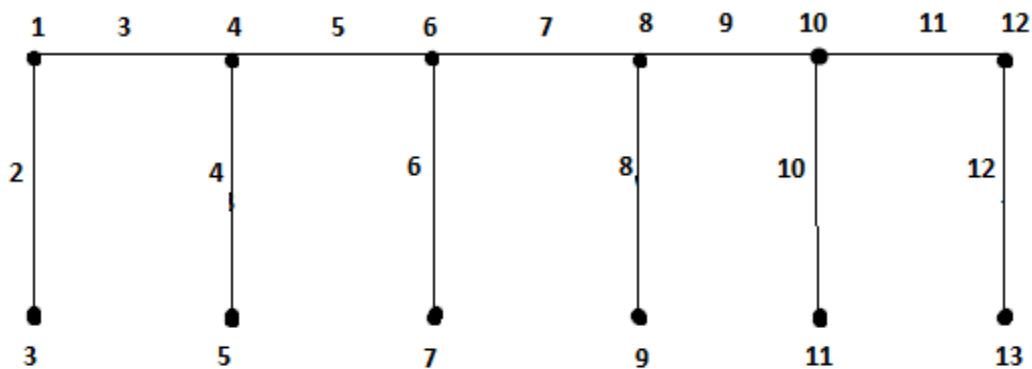


Figure-3

G. Theorem :2.7

$$L_{3m}(P_n \circ K_1) \circ K_1 = 3n.$$

1) Proof : Let G be a graph of n vertices $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$ respectively. Define a function $f:V(G) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)=3i-1; 2 \leq i \leq n, f(v_i)=3i; 1 \leq i \leq n, f(w_i)=3i+1; 1 \leq i \leq n$. then the distinct edge labels are $f(u_i, u_{i+1})=3i+1; 1 \leq i \leq n-1, f(u_i, v_i)=3i-1; 1 \leq i \leq n, f(v_i, w_i)=3i; 1 \leq i \leq n-1$ and $f(v_n, w_n)=3n$. Therefore $L_{3m}((P_n \circ K_1) \circ K_1) = 3n$

H. Example 2.8

Lehmer-3 mean number of graph with n=5 is given as

$$L_{3m}(G) = 3n = 3 \times 5 = 15$$

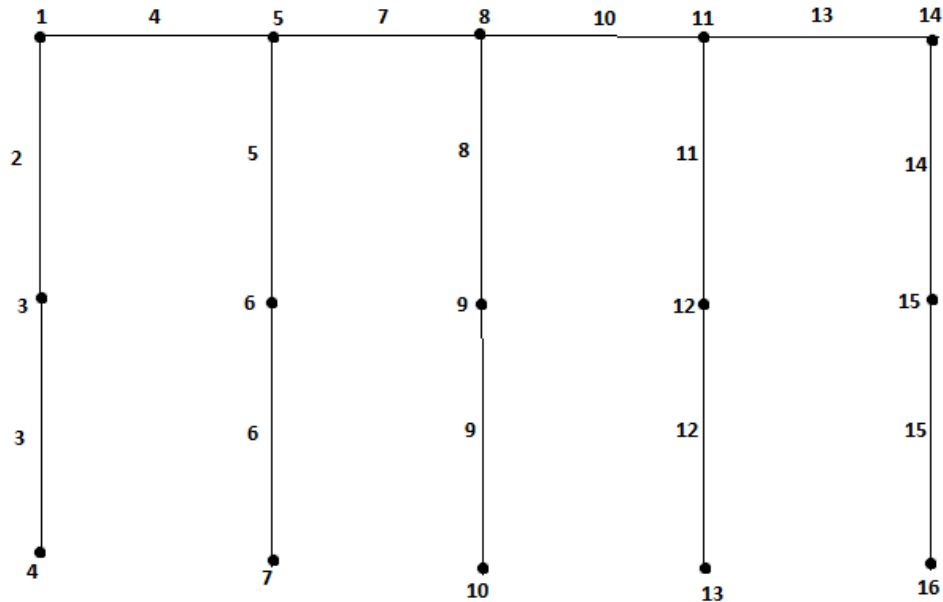


Figure-4

I. Theorem:2.9

$$L_{3m}(P_n \circ K_{1,2}) = 3n.$$

1) Proof : Let P_n be a path of n vertices u_1, u_2, \dots, u_n , and the vertices of $K_{1,2}$ is denoted as $v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$. Define a function $f:V(G) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)=3i-1; 2 \leq i \leq n, f(v_i)=2, f(v_i)=3i; 2 \leq i \leq n, f(w_1)=3; f(w_i)=3i+1; 2 \leq i \leq n$. The distinct edge labels are $f(u_i, u_{i+1})=3i+1; 1 \leq i \leq n-1, f(u_i, v_i)=1, f(u_i, v_i)=3i-1; 2 \leq i \leq n, f(u_i, w_i)=2, f(u_i, w_i)=3i, 2 \leq i \leq n-1, f(u_n, w_n)=3n$ Thus $L_{3m}(P_n \circ K_{1,2}) = 3n$

J. Example 2.10

Lehmer-3 mean number of $(P_5 \circ K_{1,2})$ is

$$L_{3m}(P_5 \circ K_{1,2}) = 3n = 3 \times 5 = 15$$

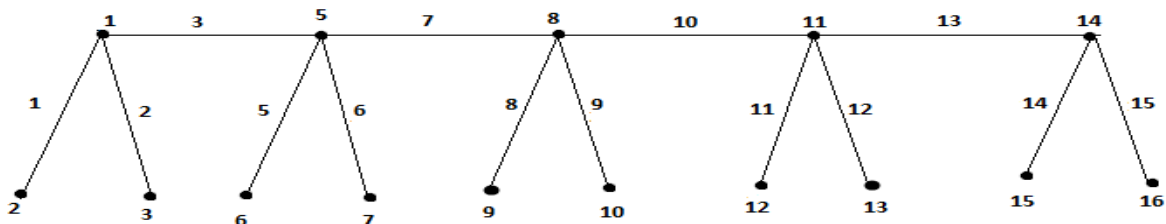


Figure-5

K. Theorem :2.11

$$L_{3m}(P_n \circ K_{1,3}) = 4n.$$

1) *Proof* : Let P_n be a path of n vertices u_1, u_2, \dots, u_n , Let $K_{1,3}$ be a complete graph with vertices v_i, w_i, x_i , such that $1 \leq i \leq n$ attached to the vertices of the path respectively. Define a function $f:V(G) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)= 4i-2 ; 2 \leq i \leq n, f(v_1)= 2, f(v_i)= 4i-1 ; 2 \leq i \leq n, f(w_i)= 4n ; 1 \leq i \leq n, f(x_i)= 4i + 1 ; 1 \leq i \leq n$, Then we obtain distinct edge labels as $f(u_i, u_{i+1})=4i+1 ; 1 \leq i \leq n-1, f(u_1v_1)= 1, f(u_iv_i)= 4i-2 ; 2 \leq i \leq n, f(u_iw_i)= 4i-1 ; 1 \leq i \leq n, f(u_ix_i)= 4i, 1 \leq i \leq n-1$ and $f(u_nx_n)= 4n$ Thus $L_{3m}(P_n \circ K_{1,3}) = 4n$

L. Example 2.12

$L_{3m}(P_6 \circ K_{1,3}) = 4n = 4 \times 6 = 24$ is given below

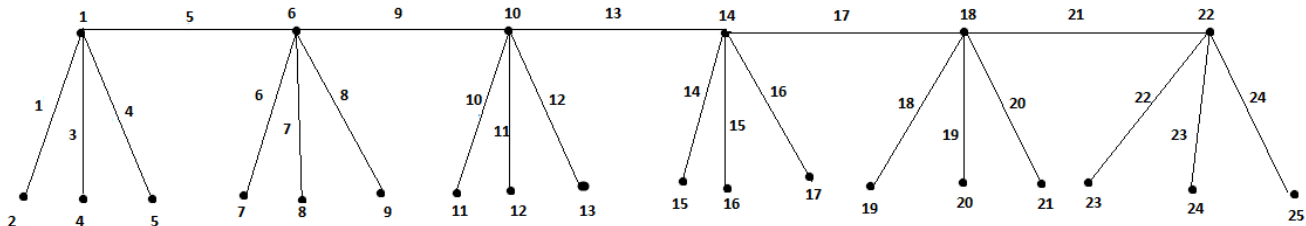


Figure-6

M. Theorem:2.13

$$L_{3m}(P_n \circ K_3) = 4n.$$

1) *Proof* : Let P_n be a path of n vertices and Let $K_{1,3}$ be a complete graph of n vertices attached to each vertices of the path. We define a function $f:V(G) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)= 4i-2 ; 2 \leq i \leq n, f(v_1)= 2, f(v_i)= 4i-1 ; 2 \leq i \leq n, f(w_i)= 4i ; 1 \leq i \leq n$. Then the edge labels as $f(u_i, u_{i+1})=4i+1 ; 1 \leq i \leq n-1, f(u_1v_1)= 1, f(u_iv_i)= 4i-2 ; 2 \leq i \leq n, f(v_1w_1)= 3; f(v_iw_i)= 4i - 1 ; 2 \leq i \leq n$ and $f(u_iw_i)= 4i ; 1 \leq i \leq n-1$ and $f(u_nw_n)= 4n$ Hence $L_{3m}(P_n \circ K_3) = 4n$.

N. Example 2.14

Lehmer-3 mean number of $(P_5 \circ K_3)$ is given below

$$L_{3m}(P_5 \circ K_3) = 4n = 4 \times 5 = 20$$

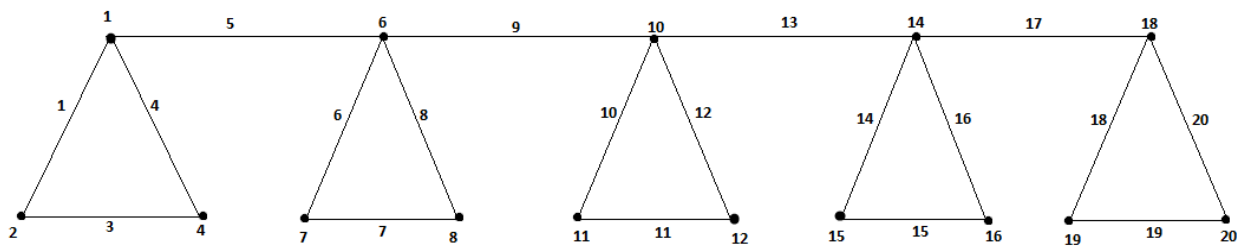


Figure-7

O. Theorem:2.15

$$L_{3m}(P_n \circ K_1) \circ K_{1,2} = 4n$$

1) *Proof* : Let $(P_n \circ K_1) \circ K_{1,2}$ be a graph with vertices $u_i, v_i, w_i, x_i ; 1 \leq i \leq n$, where $P_n \circ K_1$ be a comb graph in which $K_{1,2}$ is attached to each vertex of the comb. A function defined on G by $f:V(G) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)= 4i-2 ; 2 \leq i \leq n, f(v_i)= 2 ; f(v_i)= 4i - 1 ; 2 \leq i \leq n, f(w_i)=4i, 1 \leq i \leq n, f(x_i) = 4i+1 ; 1 \leq i \leq n$ then the distinct edge labels are $f(u_i, u_{i+1})=4i+1 ; 1 \leq i \leq n-1, f(u_1v_1)= 1; f(u_iv_i)= 4i-2 ; 2 \leq i \leq n, f(v_iw_i)= 4i-1 ; 1 \leq i \leq n, f(v_ix_i)= 4i ; 1 \leq i \leq n-$, $f(v_nx_n)= 4n$. Thus the Lehmer -3 mean number of $(P_n \circ K_1) \circ K_{1,2}$ is $4n$

P. Example 2.16

$L_{3m}(P_5 \circ K_1) \circ K_{1,2} = 4n = 4 \times 5 = 20$ is given below

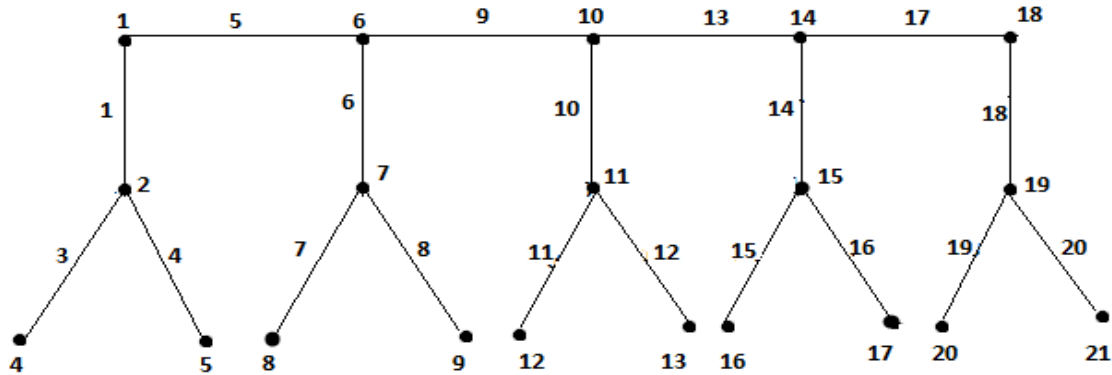


Figure-8

Q. Theorem:2.17

$$L_{3m}(C_n \circledast K_1) = 2n+1$$

1) Proof : Let C_n be a cycle with vertices u_1, u_2, \dots, u_n , Let K_1 be a graph attached to each vertex of the cycle. Such that its vertices be v_1, v_2, \dots, v_n , Define a function $f:V(G) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)= 2i ; 2 \leq i \leq n, f(v_i)= 2i+1, 1 \leq i \leq n$. The edge labels as $f(u_i, u_{i+1})=2i+1 ; 1 \leq i \leq n-1, f(u_n u_1)= 2n, f(u_i v_i)= 2i ; 1 \leq i \leq n-1, f(u_n v_n)= 2n+1$. Thus $L_{3m}(C_n \circledast K_1) = 2n+1$

R. Example 2.18

Lehmer -3 mean number of $C_6 \circledast K_1 = 2n+1 = 2 \times 6+1 = 13$ is given below

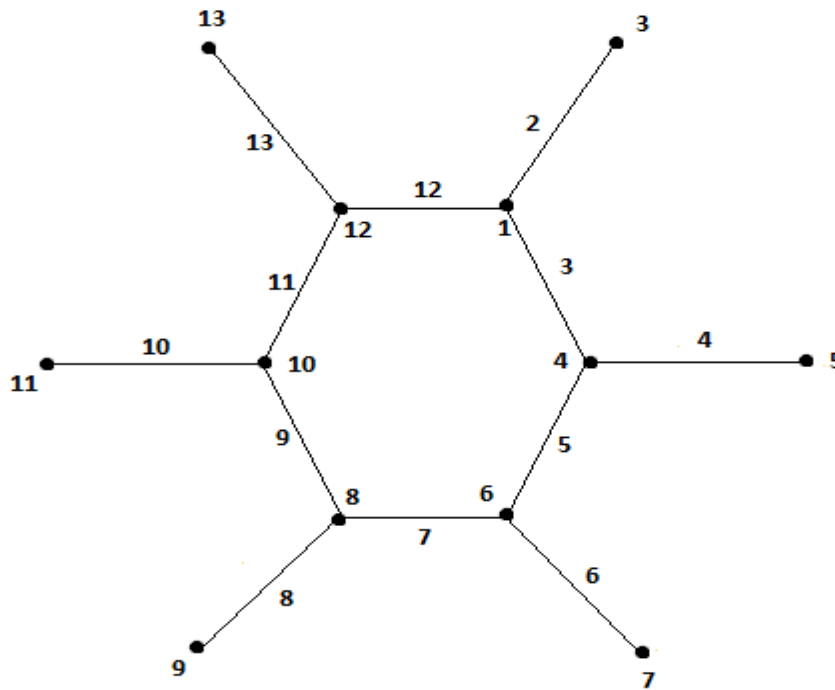


Figure-9

S. *Theorem :2.19*

$$L_{3m}(C_n \circ K_{1,2}) = 3n+1$$

1) *Proof* : Let $C_n \circ K_{1,2}$ be a graph of n vertices $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$ respectively. A function defined on $(C_n \circ K_{1,2})$ as $f:V(C_n \circ K_{1,2}) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)= 3i-1 ; 2 \leq i \leq n, f(v_i)= 3i, 1 \leq i \leq n, f(w_i)= 3i+1 ; 1 \leq i \leq n$. The edge labels as $f(u_i, u_{i+1})=3i+1 ; 1 \leq i \leq n-1, f(u_n u_1)= 3n-1, f(u_i v_i)= 3i-1 ; 1 \leq i \leq n-1, f(u_n v_n)= 3n, f(u_i w_i)= 3i ; 1 \leq i \leq n-1$ and $f(u_n w_n)= 3n+1n$ Therefore $L_{3m}(C_n \circ K_{1,2}) = 3n+1$ Hence the proof

T. *Example 2.20*

$L_{3m}(C_5 \circ K_{1,2}) = 3n+1 = 3 \times 5 + 1 = 16$ is shown below

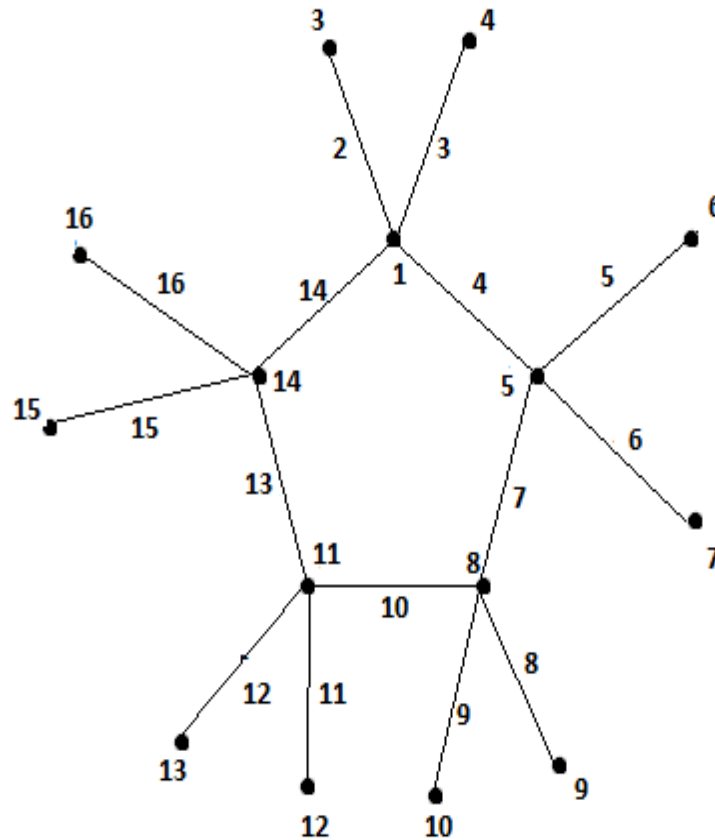


Figure-10

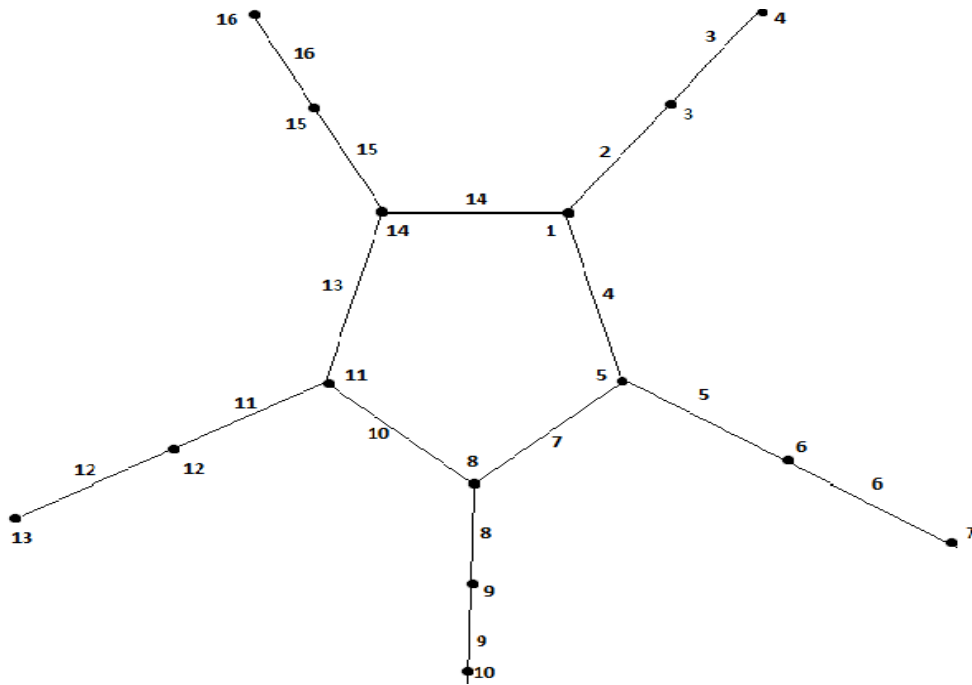
U. *Theorem :2.21*

$$L_{3m}(C_n \circ K_1) \circ K_1 = 3n+1$$

1) *Proof* : Let G be a $(C_n \circ K_1) \circ K_1$ graph the vertices be denoted u_i, v_i, w_i , where $1 \leq i \leq n$ respectively, w_i be the vertices attached to each pendant vertices of the crown. Define a function $f:V(G) \rightarrow \{1,2,\dots,n\}$ by $f(u_1)=1, f(u_i)= 3i-1 ; 2 \leq i \leq n, f(v_i)= 3i ; i=1,2,\dots,n, f(w_i)= 3i+1 ; 1 \leq i \leq n$, The edge labels are $f(u_i, u_{i+1})=3i+1 ; 1 \leq i \leq n-1, f(u_n u_1)= 3n-1 ; f(u_i v_i)= 3i-1 ; 2 \leq i \leq n-1, f(u_n v_n)= 3n, f(v_i w_i)= 3i ; 1 \leq i \leq n-1, f(v_n w_n)= 3n+1$. Thus $L_{3m}(G) = 3n+1$ hence the proof

V. *Example 2.22*

$L_{3m}(C_5 \circ K_1) \circ K_1 = 3n+1 = 3 \times 5 + 1 = 16$ diagram pattern is given below



W. Theorem :2.23

$$L_{3m}(C_n \circledast K_1) \circledast K_{1,2} = 4n+1$$

1) Proof : Let G be a graph of vertices $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n, x_1, x_2, \dots, x_n$ respectively. This is a crown attached with $K_{1,2}$ at each pendant vertex of a crown. Define a function $f:V(G) \rightarrow \{1,2,3,\dots,n\}$ by $f(u_1)=1, f(u_i)= 4i-2 ; 2 \leq i \leq n, f(v_i)= 4i-1 ; 1 \leq i \leq n, f(w_i)= 4i ; 1 \leq i \leq n, f(x_i)= 4i+1 ; 1 \leq i \leq n$ The distinct edge labels are $f(u_i, u_{i+1})=4i+1 ; 1 \leq i \leq n-1, f(u_n u_1)= 4n-2; f(u_i v_i)= 4i-1 ; 1 \leq i \leq n-1, f(u_n v_n)= 4n-1, f(v_i w_i)= 4i-1 ; 1 \leq i \leq n-1, f(v_n w_n)= 4n, f(v_i x_i)= 4i ; 1 \leq i \leq n-1, f(v_n x_n)= 4n+1$ Thus $L_{3m}(C_n \circledast K_1) \circledast K_{1,2} = 4n+1$

X. Example 2.24

Lehmer -3 mean number of $(C_4 \circledast K_1) \circledast K_{1,2}$ is $4n+1 = 4 \times 4+1 = 16+1 = 17$ is given below

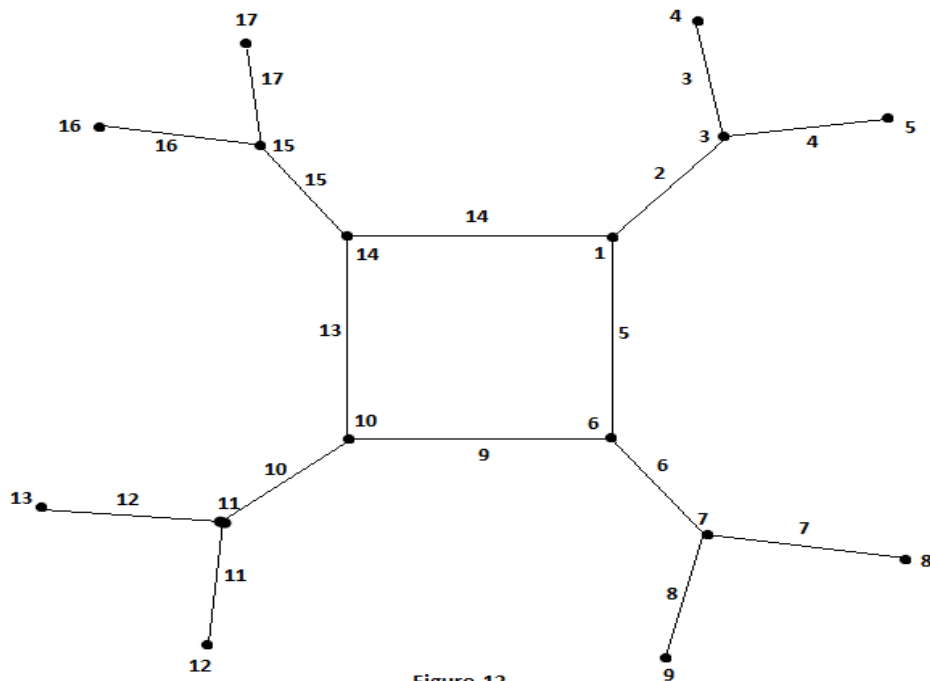


Figure-12

Y. Theorem :2.25

$$L_{3m}(L_n) = 3n - 1$$

1) Proof : Let L_n be a ladder graph of n vertices $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$, respectively. Define a function $f:V(L_n) \rightarrow \{1,2,\dots,n\}$ by $f(u_i)=3i-2 ; 1 \leq i \leq n, f(v_i)= 3i-1 ; 1 \leq i \leq n$. The edge are labeled as $f(u_i u_{i+1}) = 3i ; 1 \leq i \leq n-1, f(v_i v_{i+1}) = 3i+1 ; 1 \leq i \leq n-1, f(u_i v_i) = 1, f(u_i v_i) = 3i-1 ; 2 \leq i \leq n-1, f(u_n v_n) = 3n-1$. Hence $L_{3m}(L_n) = 3n-1$. Hence ladder satisfies lehmer -3 mean number

Z. Example 2.26

Lehmer -3 mean number of L_7 is given as $L_{3m}(L_7) = 3 \times 7 - 1 = 21 - 1 = 20$.

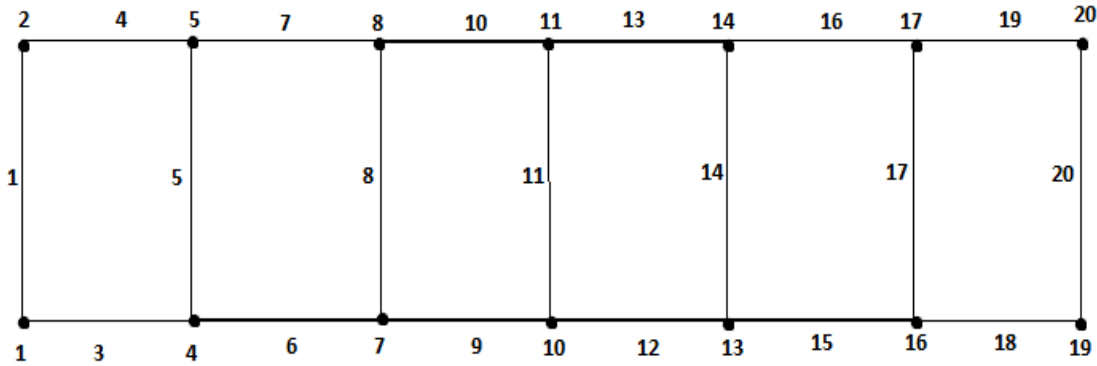


Figure-13

AA. Theorem :2.27

$$L_{3m}(D_n) = n + m$$

1) Proof : Let D_n be a dragon graph where the head of the dragon has n vertices u_1, u_2, \dots, u_n , and the path attached to it be v_1, v_2, \dots, v_m . Define a function $f:V(D_n) \rightarrow \{1,2,\dots,n\}$ by $f(u_1=v_1) = 1, f(u_i) = i+1 ; 2 \leq i \leq n, f(v_i) = n+j ; 2 \leq j \leq m$. The distinct edge labels are $f(u_i u_{i+1}) = i+1 ; 1 \leq i \leq n, f(v_j v_{j+1}) = (n+1)+j ; 1 \leq j \leq m-2, f(v_{n-1} v_n) = n + m$.

Thus $L_{3m}(D_n) = n+m$.

BB. Example 2.28

Lehmer -3 mean number of D_n is given below. $L_{3m}(D_n) = m+n$

Hence $m=5$ and $n=6; n+m = 5+6 = 11$ is drawn as

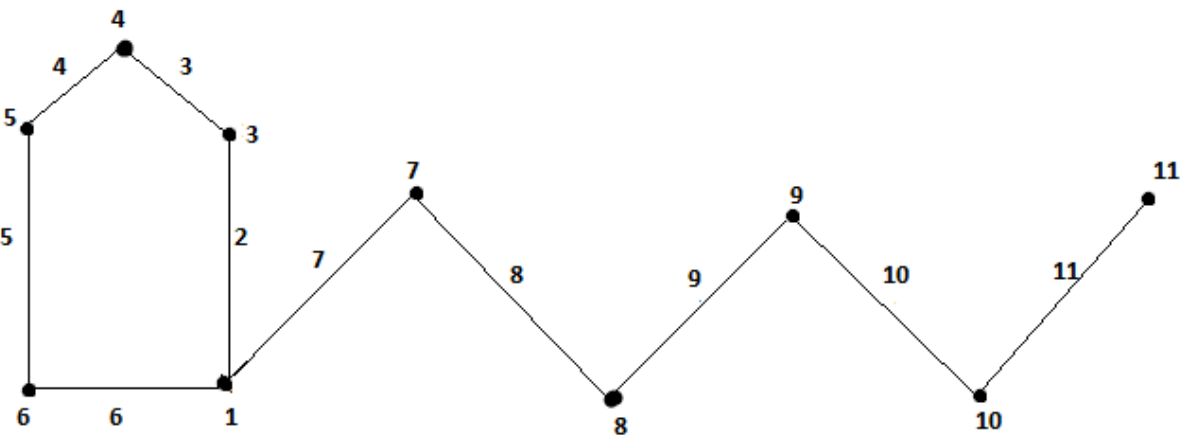


Figure-15

CC. Theorem :2.29

Lehmer -3 mean number of caterpillar is $3n$

1) *Proof* : Let G be a graph of n vertices where each vertex of path is attached to pendant vertex on its both side. Let the vertices be denoted as $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$. Let $f:V(G) \rightarrow \{1,2,\dots,n\}$ be a function defined by $f(u_1) = 1, f(u_i) = 3i-1; 2 \leq i \leq n, f(v_i) = 3i; 1 \leq i \leq n, f(w_i) = 3i+1; 1 \leq i \leq n$ and The edges are labeled as $f(u_i u_{i+1}) = 3i+1; 1 \leq i \leq n-1, f(u_i v_i) = 3i-1; 1 \leq i \leq n, f(u_i w_i) = 3i; 1 \leq i \leq n-1,$ and $f(u_n w_n) = 3n$. hence the Lehmer -3 mean number of a caterpillar is $3n$

DD. Example 2.30

Lehmer -3 mean number of a caterpillar of 5 vertices is $3n = 3 \times 5$ is given below

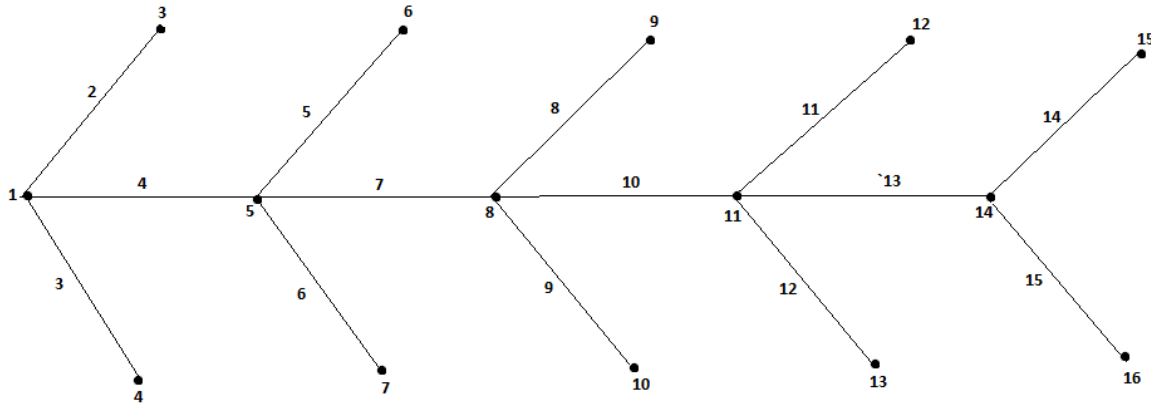


Figure-16

REFERENCES

- [1] Harary.F 1988 Graph theory, Narosa Publication House reading, New Delhi
- [2] S Somasundaram and R Ponraj and S S Sandhya ‘Harmonic mean labeling of graphs’ ommunicated to journal of combinatorial mathematics and combinatorial computing.
- [3] S.Somasundaram, S.S.Sandhya and T.S.Pavithra, “Lehmer-3 Mean Labeling of graphs” “International Journal of Mathematical Forum”
- [4] S.Somasundaram, S.S.Sandhya and T.S.Pavithra, “Lehmer -3 Mean Labeling of Disconnected Graphs” in “Asia Pacific Journal of Research ”ISSN 2320-5504 ,vol:I. issue XL, June 2016.
- [5] S.Somasundaram, S.S.Sandhya and T.S.Pavithra, “Lehmer -3 Mean Labeling of Some New Disconnected Graphs” in “International Journal of Mathematics Trends and Technology ” ISSN:2231-5373 ,vol:35. Number 1, July 2016.
- [6] S.Somasundaram, S.S.Sandhya and T.S.Pavithra, “Lehmer -3 Mean Labeling of Some Disconnected Graphs” in “International Journal of Mathematics Research” ISSN:0976- 5840, vol 8 ,Number 2(2016) .



10.22214/IJRASET



45.98



IMPACT FACTOR:
7.129



IMPACT FACTOR:
7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24*7 Support on Whatsapp)