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Lower level subsets of an anti-fuzzy HX ideal of a HX ring

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Abstract: In this paper, we introduce the concept of lower level subsets of an anti-fuzzy HX ideal of a HX ring. We also discuss the relation between a given anti-fuzzy HX ideals of a HX ring and its lower level HX ideals and investigate the conditions under which a given HX ring has a properly inclusive chain of HX ideals. We introduce the concept of homomorphism and anti homomorphism of lower level subsets of an anti-fuzzy HX ideal and discuss some of its properties.
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I. INTRODUCTION

In 1965,Lotfi.A.Zadeh [9] introduced the concept of fuzzy set. Fuzzy sets attracted many mathematicians and grew enormously by finding applications in many areas. We introduce a notion of anti fuzzy HX ideal of a HX ring and some of its properties are discussed. We prove that a fuzzy subset of a HX ring is an anti fuzzy HX ideal if and only if the lower level subsets are HX ideals of a HX ring. In 1982 Wang-jin Liu [6] introduced the concept of fuzzy subring and fuzzy ideal. In 1988, Professor Li Hong Xing [5] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [10] gave the structures of HX ring on a class of ring.

II. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, $R = (R ,+, \cdot)$ is a Ring, e is the additive identity element of R and xy, we mean $x \cdot y$.

III. LOWER LEVEL SUBSETS OF AN ANTI-FUZZY HX IDEAL

A. Definition

Let λ_μ be an anti-fuzzy HX ideal of a HX ring $\Re$. For any $t \in [0,1]$, we define the set $L(\lambda_\mu ; t) = \{ A \in \Re / \lambda_\mu (A) \le t \}$ is called a lower level subset of λ_μ.

B. Theorem

Let λ_μ be an anti-fuzzy HX right ideal of a HX ring $\Re$ and $L(\lambda_\mu ; t)$ is non-empty, then for $t \in [0,1]$, $L(\lambda_\mu ; t)$ is a HX right ideal of $\Re$.

1) Proof: Let λ_μ be an anti-fuzzy HX right ideal of a HX ring $\Re$.

For any $A , B \in L(\lambda_\mu; t)$, we have , $\lambda_\mu (A) \le t$ and $\lambda_\mu (B) \le t$.

Now, $\lambda_\mu (A - B) \le \max \{ \lambda_\mu (A) , \lambda_\mu (B) \}$
$\le \max \{ t , t \} = t$, for some $t \in [0,1]$.
$\lambda_\mu (A - B) \le t$.

For any $A \in L(\lambda_\mu; t)$ and $B \in \Re$, we have , $\lambda_\mu (A) \le t$.

Now, $\lambda_\mu (AB) \le \lambda_\mu (A) \le t$.
$\lambda_\mu (AB) \le t$.

Hence, $A - B , AB \in L(\lambda_\mu; t)$.

Hence, $L(\lambda_\mu; t)$ is a HX right ideal of a HX ring $\Re$.

C. Theorem

Let λ_μ be an anti-fuzzy HX left ideal of a HX ring $\Re$ and L (λ_μ ; t) is non-empty, then for any $t \in [0,1]$, L(λ_μ ; t) is a HX left ideal of $\Re$.

1) *Proof:* Let λ_μ be an anti-fuzzy HX left ideal of a HX ring $\Re$.

For any A , B $\in$ L(λ_μ; t), we have , λ_μ (A) $\leq$ t and λ_μ (B) $\leq$ t.

Now, λ_μ (A – B) $\leq$ max { λ_μ (A) , λ_μ (B) }

$\leq$ max { t , t } = t , for some t$\in$ [0,1].

λ_μ (A – B) $\leq$ t.

For any A $\in$ L(λ_μ; t) and B $\in$ $\Re$, we have , λ_μ (A) $\leq$ t.

Now, λ_μ (BA) $\leq$ λ_μ (A) $\leq$ t.

λ_μ (BA) $\leq$ t

Hence, A – B , BA $\in$ L(λ_μ; t) .

Hence , L(λ_μ; t) is a HX left ideal of a HX ring $\Re$.

D. Theorem

Let λ_μ be an anti-fuzzy HX ideal of a HX ring $\Re$ and L(λ_μ ; t) is non-empty, then for $t \in [0,1]$, L(λ_μ ; t) is a HX ideal of $\Re$.

1) *Proof:* It is clear.

E. Theorem

Let $\Re$ be a HX ring and λ_μ be a fuzzy subset of $\Re$ such that L(λ_μ ; t) is a HX right ideal of $\Re$ for all t$\in$ [0,1] then λ_μ is an anti-fuzzy HX right ideal of $\Re$.

1) *Proof:* It is clear.

F. Theorem

Let $\Re$ be a HX ring and λ_μ be a fuzzy subset of $\Re$ such that L(λ_μ ; t) is a HX left ideal of $\Re$ for all t$\in$ [0,1] then λ_μ is an anti-fuzzy HX left ideal of $\Re$.

1) *Proof:* It is clear.

G. Theorem

Let $\Re$ be a HX ring and λ_μ be a fuzzy subset of $\Re$ such that L(λ_μ ; t) is a HX ideal of $\Re$ for all t$\in$ [0,1] then λ_μ is an anti-fuzzy HX ideal of $\Re$.

1) *Proof:* It is clear.

H. Theorem

A fuzzy subset λ_μ of $\Re$ is a fuzzy HX ideal of a HX ring $\Re$ if and only if the level HX subsets L(λ_μ ; t), $t \in$ Image λ_μ, are HX ideals of $\Re$.

1) *Proof:* It is clear.

I. Theorem

Let λ_μ be an anti-fuzzy HX ideal of a HX ring $\Re$. If two lower level HX ideals, L(λ_μ ; t_1), L(λ_μ ; t_2) with $t_1 < t_2$ of λ_μ are equal if and only if there is no A in $\Re$ such that

$t_1 \leq \lambda_\mu (A) < t_2$.

1) *Proof:* It is clear.

J. Theorem

Any HX ideal H of a HX ring $\Re$ can be realized as a lower level HX ideal of some anti-fuzzy HX ideal of $\Re$.

1) *Proof:* It is clear.

K. Remark

As a consequence of the Theorem 3.9 and 3.10, the lower level HX ideals of an anti-fuzzy HX ideal λ^{μ} of a HX ring $\Re$ form a chain. Since $\lambda^{\mu}(Q) \leq \lambda^{\mu}(A)$ for all A in $\Re$ and therefore $L(\lambda^{\mu} ; t_0)$, where $\lambda^{\mu}(Q) = t_0$ is the smallest and we have the chain :

$\{Q\} = L(\lambda_{\mu} ; t_0) \subset L(\lambda_{\mu} ; t_1) \subset L(\lambda_{\mu} ; t_2) \subset \ldots \subset L(\lambda_{\mu} ; t_n) = \Re$,

where $t_0 < t_1 < t_2 < \ldots\ldots < t_n$.

III. HOMOMORPHISM AND ANTI HOMOMORPHISM OF A LOWER LEVEL SUBSETS OF AN ANTI-FUZZY HX IDEAL OF A HX RING

In this section, we introduce the concept of homomorphism and anti homomorphism of lower level subsets of an anti-fuzzy HX ideal and discuss some of its properties. Throughout this section, $t \in [0,1]$.

A. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_{μ} be an anti-fuzzy HX right ideal on $\Re_1$. If f : $\Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the anti-image of a lower level HX right ideal $L(\lambda_{\mu} ; t)$ of an anti-fuzzy HX right ideal λ_{μ} of a HX ring $\Re_1$ is a lower level HX right ideal $L(f(\lambda_{\mu}); t)$ of an anti-fuzzy HX right ideal $f(\lambda_{\mu})$ of a HX ring $\Re_2$.

1) Proof: Let R_1 and R_2 be any two rings and f : $\Re_1 \rightarrow \Re_2$ be a homomorphism.

Let λ_{μ} be an anti-fuzzy HX right ideal of a HX ring $\Re_1$. Clearly, $f(\lambda_{\mu})$ is an anti-fuzzy HX right ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.

Let $L(\lambda_{\mu} ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_{μ} of a HX ring $\Re_1$.

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_{\mu} ; t)$ and hence $X-Y \in L(\lambda_{\mu} ; t)$.

Then, $\lambda_{\mu}(X) \leq t$ and $\lambda_{\mu}(Y) \leq t$ and $\lambda_{\mu}(X-Y) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_{\mu} ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_{\mu} ; t)$, as $L(\lambda_{\mu} ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_{μ} of a HX ring $\Re_1$.

Then, $\lambda_{\mu}(X) \leq t$ and $\lambda_{\mu}(XY) \leq t$.

We have to prove that $L(f(\lambda_{\mu}) ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_{\mu})$ of a HX ring $\Re_2$.

Let $X, Y \in L(\lambda_{\mu} ; t)$ and hence $X-Y \in L(\lambda_{\mu} ; t)$.

For $f(X), f(Y) \in L(f(\lambda^{\mu}) ; t)$,

$$\begin{aligned} (f(\lambda_{\mu}))(f(X) - f(Y)) &= (f(\lambda_{\mu}))(f(X-Y)), \\ &= \lambda_{\mu}(X-Y) \\ &\leq t \\ (f(\lambda_{\mu}))(f(X) - f(Y)) &\leq t . \\ (f(X) - f(Y)) &\in L(f(\lambda_{\mu}); t). \end{aligned}$$

let $X \in L(\lambda_{\mu} ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_{\mu} ; t)$, as $L(\lambda_{\mu} ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_{μ} of a HX ring $\Re_1$.

For, $f(X) \in L(f(\lambda_{\mu}) ; t)$ and $f(Y) \in \Re_2$,

$$\begin{aligned} (f(\lambda_{\mu}))(f(X)\, f(Y)) &\leq (f(\lambda_{\mu}))f(X) \\ &\leq t \\ (f(\lambda_{\mu}))(f(X)(f(Y)) &\leq t. \\ (f(X)\, f(Y)) &\in L(f(\lambda_{\mu}); t). \end{aligned}$$

Hence, $L(f(\lambda_{\mu}) ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_{\mu})$ of a HX ring $\Re_2$.

B. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_{μ} be an anti-fuzzy HX left ideal on $\Re_1$. If f : $\Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the anti-image of a lower level HX left ideal $L(\lambda_{\mu} ; t)$ of an anti-fuzzy HX left ideal λ_{μ} of a HX ring $\Re_1$ is a lower level HX left ideal $L(f(\lambda_{\mu}); t)$ of an anti-fuzzy HX left ideal $f(\lambda_{\mu})$ of a HX ring $\Re_2$.

1) Proof: Let R_1 and R_2 be any two rings and f : $\Re_1 \rightarrow \Re_2$ be a homomorphism.

Let λ_{μ} be an anti-fuzzy HX left ideal of a HX ring $\Re_1$. Clearly, $f(\lambda_{\mu})$ is an anti-fuzzy HX left ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.

Let $L(\lambda_{\mu} ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_{μ} of a HX ring $\Re_1$.

Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_{\mu} ; t)$ and hence $X-Y \in L(\lambda_{\mu} ; t)$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(X-Y) \leq t$.

For this $t \in [0,1]$, let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.

Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.

We have to prove that $L(f(\lambda_\mu) ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

Let $X, Y \in L(\lambda_\mu ; t)$ and hence $X-Y \in L(\lambda_\mu ; t)$.

For $f(X), f(Y) \in L(f(\lambda^\mu) ; t)$,

$$\begin{aligned}
(f(\lambda_\mu))(f(X) - f(Y)) &= (f(\lambda_\mu))(f(X-Y)), \\
&= \lambda_\mu(X-Y) \\
&\leq t \\
(f(\lambda_\mu))(f(X) - f(Y)) &\leq t. \\
(f(X) - f(Y)) &\in L(f(\lambda_\mu); t).
\end{aligned}$$

let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $YX \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.

For, $f(X) \in L(f(\lambda_\mu) ; t)$ and $f(Y) \in \Re_2$,

$$\begin{aligned}
(f(\lambda_\mu))(f(Y) f(X)) &\leq (f(\lambda_\mu))(f(X)) \\
&\leq t \\
(f(\lambda_\mu))(f(Y)(f(X)) &\leq t. \\
(f(Y) f(X)) &\in L(f(\lambda_\mu); t).
\end{aligned}$$

Hence, $L(f(\lambda_\mu) ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

C. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism and onto, then the anti-image of a lower level HX ideal $L(\lambda_\mu ; t)$ of an anti-fuzzy HX ideal λ_μ of a HX ring $\Re_1$ is a lower level HX ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

1) Prof: It is clear.

D. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX right ideal on $\Re_2$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be a homomorphism.

Let η_α be an anti-fuzzy HX right ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX right ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.Let $X, Y \in \Re_1$ then $f(X), f(Y) \in \Re_2$.

Choose $t \in [0, 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence $,f(X)-f(Y) \in L(\eta_\alpha ; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(X)-f(Y)) \leq t$.

For this $t \in [0, 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$. Now, Let $X, Y \in L(f^{-1}(\eta_\alpha) ; t)$.

$$\begin{aligned}
(f^{-1}(\eta_\alpha))(X-Y) &= \eta_\alpha(f(X-Y)) \\
&= \eta_\alpha(f(X) - f(Y)) \\
&\leq t \\
(f^{-1}(\eta_\alpha))(X-Y) &\leq t \\
X-Y &\in L(f^{-1}(\eta_\alpha); t).
\end{aligned}$$

Let $f(X) \in L(\eta_\alpha ;\ t)$ and $f(Y)\in\Re_2$ then $f(X)f(Y)\in L(\eta_\alpha ;\ t)$, as $L(\eta_\alpha ;\ t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.

$$\begin{aligned}(f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(X)\\ &= \eta_\alpha(f(X))\\ &\leq t\\ (f^{-1}(\eta_\alpha))(XY) &\leq t\\ XY &\in L(f^{-1}(\eta_\alpha);\ t).\end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha);\ t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal of a HX ring $\Re_1$.

E. Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX left ideal on $\Re_2$. If $f : \Re_1\rightarrow \Re_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha ;\ t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha);\ t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1\rightarrow \Re_2$ be a homomorphism.

Let η_α be an anti-fuzzy HX left ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX left ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $L(\eta_\alpha ;\ t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$. Let $X ,Y \in\Re_1$ then $f(X) ,f(Y) \in\Re_2$.

Choose $t \in [\,0 , 1\,]$ in such a way that $f(X), f(Y)\in L(\eta_\alpha;\ t)$ and hence ,$f(X)-f(Y) \in L(\eta_\alpha ;\ t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(X)-f(Y)) \leq t$.

For this $t \in [\,0 , 1\,]$, $f(X) \in L(\eta_\alpha;\ t)$ and $f(Y)\in\Re_2$ then $f(X)f(Y) \in L(\eta_\alpha;\ t)$, as $L(\eta_\alpha ;\ t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha);\ t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$. Now, Let $X , Y \in L(f^{-1}(\eta_\alpha) ;\ t)$.

$$\begin{aligned}(f^{-1}(\eta_\alpha))(X-Y) &= \eta_\alpha(f(X-Y))\\ &= \eta_\alpha(f(X) - f(Y))\\ &\leq t\\ (f^{-1}(\eta_\alpha))(X-Y) &\leq t\\ X-Y &\in L(f^{-1}(\eta_\alpha);\ t).\end{aligned}$$

Let $f(X) \in L(\eta_\alpha ;\ t)$ and $f(Y)\in\Re_2$ then $f(X)f(Y)\in L(\eta_\alpha ;\ t)$, as $L(\eta_\alpha ;\ t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$.

$$\begin{aligned}(f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(Y)\\ &= \eta_\alpha(f(Y))\\ &\leq t\\ (f^{-1}(\eta_\alpha))(XY) &\leq t\\ XY &\in L(f^{-1}(\eta_\alpha);\ t).\end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha);\ t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal of a HX ring $\Re_1$.

F. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX ideal on $\Re_2$. If $f : \Re_1\rightarrow \Re_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha ;\ t)$ be a lower level HX ideal of an anti-fuzzy HX ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha);\ t)$ is a lower level HX ideal of an anti-fuzzy HX ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: It is clear.

G. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX right ideal on $\Re_1$. If $f : \Re_1\rightarrow \Re_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX right ideal $L(\lambda_\mu ;\ t)$ of an anti-fuzzy HX right ideal λ_μ of a HX ring $\Re_1$ is a lower level HX left ideal $L(f(\lambda_\mu);\ t)$ of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.
Let λ_μ be an anti-fuzzy HX right ideal of a HX ring $\Re_1$. Clearly, $f(\lambda_\mu)$ is an anti-fuzzy HX left ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.
Let $L(\lambda_\mu ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring $\Re_1$.
Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_\mu ; t)$ and hence $Y-X \in L(\lambda_\mu ; t)$.
Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(Y-X) \leq t$.
For this $t \in [0,1]$, let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring $\Re_1$.
Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.
We have to prove that $L(f(\lambda_\mu) ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.
Let $X, Y \in L(\lambda_\mu ; t)$ and hence $Y-X \in L(\lambda_\mu ; t)$.
For $f(X), f(Y) \in L(f(\lambda^\mu) ; t)$,

$$\begin{aligned} (f(\lambda_\mu))(f(X)-f(Y)) &= (f(\lambda_\mu))(f(Y-X)), \\ &= \lambda_\mu(Y-X) \\ &\leq t \\ (f(\lambda_\mu))(f(X)-f(Y)) &\leq t. \\ (f(X)-f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal λ_μ of a HX ring $\Re_1$.
For, $f(X) \in L(f(\lambda_\mu) ; t)$ and $f(Y) \in \Re_2$,

$$\begin{aligned} (f(\lambda_\mu))(f(Y)\, f(X)) &\leq (f(\lambda_\mu))(f(X)) \\ &\leq t \\ (f(\lambda_\mu))(f(Y)(f(X)) &\leq t. \\ (f(Y)\, f(X)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

Hence, $L(f(\lambda_\mu) ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

H. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX left ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX left ideal $L(\lambda_\mu ; t)$ of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$ is a lower level HX right ideal $L(f(\lambda_\mu) ; t)$ of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.
Let λ_μ be an anti-fuzzy HX left ideal of a HX ring $\Re_1$. Clearly, $f(\lambda_\mu)$ is an anti-fuzzy HX right ideal of a HX ring $\Re_2$. Let X and Y in $\Re_1$, implies f (X) and f (Y) in $\Re_2$.
Let $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.
Choose $t \in [0,1]$ in such a way that $X, Y \in L(\lambda_\mu ; t)$ and hence $Y-X \in L(\lambda_\mu ; t)$.
Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(Y) \leq t$ and $\lambda_\mu(Y-X) \leq t$.
For this $t \in [0,1]$, let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $XY \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.
Then, $\lambda_\mu(X) \leq t$ and $\lambda_\mu(XY) \leq t$.
We have to prove that $L(f(\lambda_\mu) ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.
Let $X, Y \in L(\lambda_\mu ; t)$ and hence $Y-X \in L(\lambda_\mu ; t)$.
For $f(X), f(Y) \in L(f(\lambda^\mu) ; t)$,

$$\begin{aligned} (f(\lambda_\mu))(f(X) - f(Y)) &= (f(\lambda_\mu))(f(Y-X)), \\ &= \lambda_\mu(Y-X) \\ &\leq t \\ (f(\lambda_\mu))(f(X) - f(Y)) &\leq t. \\ (f(X) - f(Y)) &\in L(f(\lambda_\mu); t). \end{aligned}$$

let $X \in L(\lambda_\mu ; t)$ and $Y \in \Re_1$ then $YX \in L(\lambda_\mu ; t)$, as $L(\lambda_\mu ; t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal λ_μ of a HX ring $\Re_1$.

For, $f(X) \in L(f(\lambda_\mu) ; t)$ and $f(Y) \in \Re_2$,

$$(f(\lambda_\mu))(f(X)\, f(Y)) \leq (f(\lambda_\mu))f(X) \leq t$$

$$(f(\lambda_\mu))(f(X)(f(Y)) \leq t.$$

$$(f(X)\, f(Y)) \in L(f(\lambda_\mu); t).$$

Hence, $L(f(\lambda_\mu) ; t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

I. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let λ_μ be an anti-fuzzy HX ideal on $\Re_1$. If $f : \Re_1 \rightarrow \Re_2$ is an anti homomorphism and onto, then the anti-image of a lower level HX ideal $L(\lambda_\mu ; t)$ of an anti-fuzzy HX ideal λ_μ of a HX ring $\Re_1$ is a lower level HX ideal $L(f(\lambda_\mu); t)$ of an anti-fuzzy HX ideal $f(\lambda_\mu)$ of a HX ring $\Re_2$.

1) Proof: It is clear.

J. Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX right ideal on $\Re_2$. If $f: \Re_1 \rightarrow \Re_2$ is an anti homomorphism on HX rings. Let $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.

Let η_α be an anti-fuzzy HX right ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX left ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.Let $X ,Y \in \Re_1$ then $f(X) ,f(Y) \in \Re_2$.

Choose $t \in [0 , 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence ,$f(Y)-f(X) \in L(\eta_\alpha ; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(Y)-f(X)) \leq t$.

For this $t \in [0 , 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$. Now, Let $X , Y \in L(f^{-1}(\eta_\alpha) ; t)$.

$$(f^{-1}(\eta_\alpha)) (X-Y) = \eta_\alpha(f(X-Y)) = \eta_\alpha(f(Y)-f(X)) \leq t$$

$$(f^{-1}(\eta_\alpha))(X-Y) \leq t$$

$$X-Y \in L(f^{-1}(\eta_\alpha); t).$$

Let $f(X) \in L(\eta_\alpha ; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha ; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX right ideal of an anti-fuzzy HX right ideal η_α of the HX ring $\Re_2$.

$$(f^{-1}(\eta_\alpha)) (YX) \leq f^{-1}(\eta_\alpha)(X) = \eta_\alpha(f(X)) \leq t$$

$$(f^{-1}(\eta_\alpha)) (YX) \leq t$$

$$YX \in L(f^{-1}(\eta_\alpha); t).$$

Hence, $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX left ideal of an anti-fuzzy HX left ideal of a HX ring $\Re_1$.

K. Theorem

Let R_1 and R_2 be any two rings , $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX left ideal on $\Re_2$. If $f : \Re_1 \rightarrow \Re_2$ is a homomorphism on HX rings. Let $L(\eta_\alpha ; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: Let R_1 and R_2 be any two rings and $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism.

Let η_α be an anti-fuzzy HX left ideal of a HX ring $\Re_2$. Clearly, $f^{-1}(\eta_\alpha)$ is an anti-fuzzy HX right ideal of a HX ring $\Re_1$. Let X and Y in $\Re_1$, implies f(X) and f(Y) in $\Re_2$.

Let $L(\eta_\alpha ; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$. Let $X, Y \in \Re_1$ then $f(X), f(Y) \in \Re_2$.

Choose $t \in [0, 1]$ in such a way that $f(X), f(Y) \in L(\eta_\alpha; t)$ and hence ,$f(Y)-f(X) \in L(\eta_\alpha ; t)$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(Y)) \leq t$ and $\eta_\alpha(f(Y)-f(X)) \leq t$.

For this $t \in [0, 1]$, $f(X) \in L(\eta_\alpha; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$.

Then, $\eta_\alpha(f(X)) \leq t$, $\eta_\alpha(f(X)f(Y)) \leq t$.

We have to prove that $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$. Now, Let $X, Y \in L(f^{-1}(\eta_\alpha) ; t)$.

$$\begin{aligned}(f^{-1}(\eta_\alpha))(X-Y) &= \eta_\alpha(f(X-Y)) \\ &= \eta_\alpha(f(Y)-f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(X-Y) &\leq t \\ X-Y &\in L(f^{-1}(\eta_\alpha); t).\end{aligned}$$

Let $f(X) \in L(\eta_\alpha ; t)$ and $f(Y) \in \Re_2$ then $f(X)f(Y) \in L(\eta_\alpha ; t)$, as $L(\eta_\alpha ; t)$ be a lower level HX left ideal of an anti-fuzzy HX left ideal η_α of the HX ring $\Re_2$.

$$\begin{aligned}(f^{-1}(\eta_\alpha))(XY) &\leq (f^{-1}(\eta_\alpha))(X) \\ &= \eta_\alpha(f(X)) \\ &\leq t \\ (f^{-1}(\eta_\alpha))(XY) &\leq t \\ XY &\in L(f^{-1}(\eta_\alpha); t).\end{aligned}$$

Hence, $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX right ideal of an anti-fuzzy HX right ideal of a HX ring $\Re_1$.

L. Theorem

Let R_1 and R_2 be any two rings, $\Re_1$ and $\Re_2$ be HX rings on R_1 and R_2 respectively. Let η_α be an anti-fuzzy HX ideal on $\Re_2$. If $f : \Re_1 \rightarrow \Re_2$ is an anti homomorphism on HX rings. Let $L(\eta_\alpha ; t)$ be a lower level HX ideal of an anti-fuzzy HX ideal η_α of a HX ring $\Re_2$ then $L(f^{-1}(\eta_\alpha); t)$ is a lower level HX ideal of an anti-fuzzy HX ideal $f^{-1}(\eta_\alpha)$ of a HX ring $\Re_1$.

1) Proof: It is clear.

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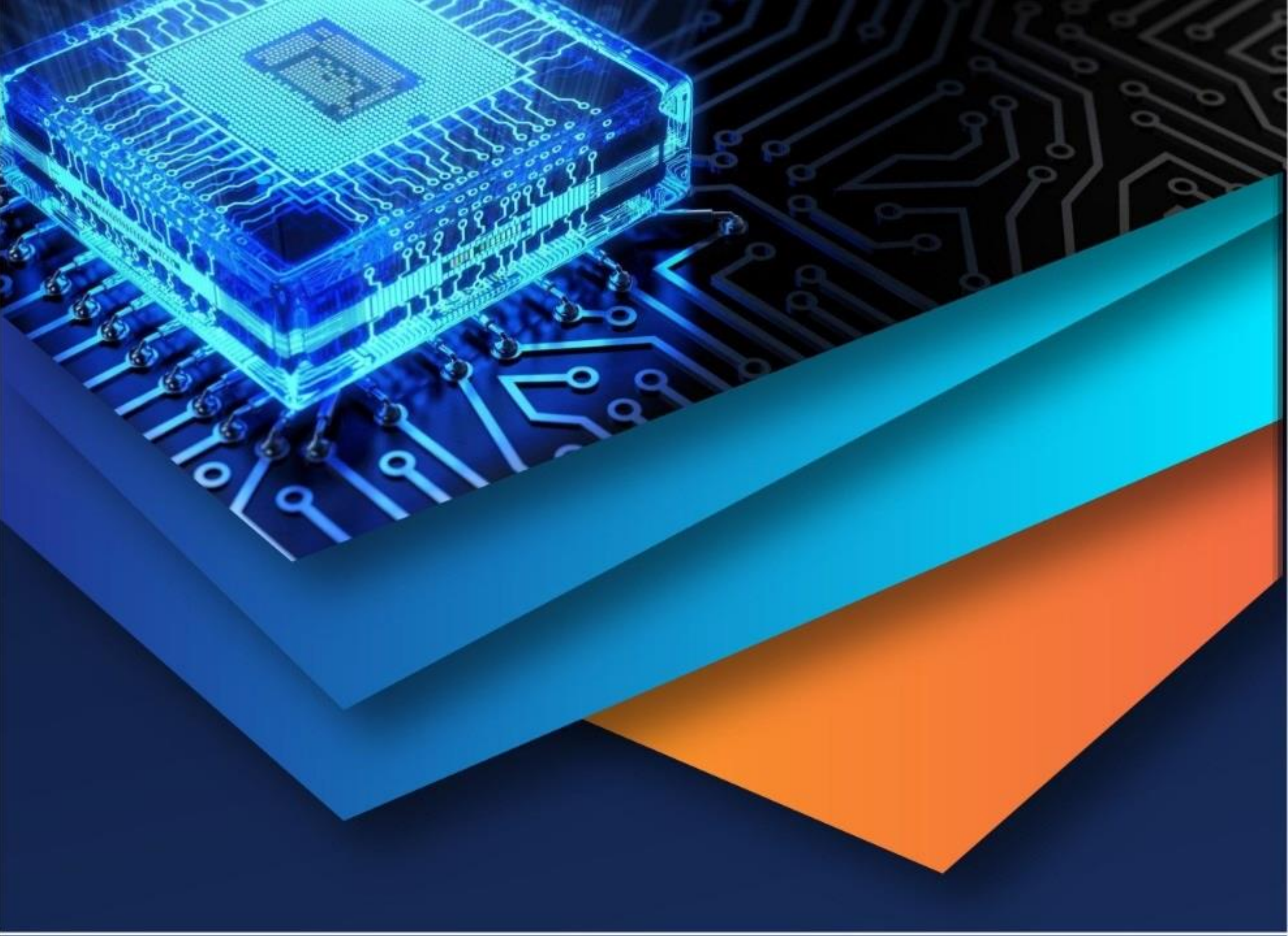

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