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On $\hat{\mu}\beta$ Lower Separation and $\hat{\mu}\beta$ Regularity Axioms in Fuzzy Topological Spaces

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Abstract: The aim of this paper is to introduce a new class of sets on $\hat{\mu}\beta$ lower separation and $\hat{\mu}\beta$ regularity axioms in fuzzy topological spaces and study their basic properties in fuzzy topological spaces. We introduce and investigate some separation axioms by using $\hat{\mu}\beta$ closed set.

I. INTRODUCTION

The fundamental concept of a fuzzy set was introduced by Zadeh in 1965, [16]. Subsequently, in 1968, Chang [1] introduced fuzzy topological spaces (in short, fts). In Chang's fuzzy topological spaces, each fuzzy set is either open or not. Later on, Chang's idea was developed by Goguen [3], who replaced the closed interval $I = [0, 1]$ by a more general lattice L. In 1985, Kubiak [4], and ˇSostak [11], in separated works, made topology itself fuzzy besides their dependence on fuzzy sets. In 1991, from a logical point of view, Ying [15] studied Hohles topology and called it fuzzifying topology. This fuzzification opened a rich field for research. As it is well known, the neighborhood structure is not suitable to I-topology, and Pu and Liu [10] broke through the classical theory of neighborhood system and established the strong and powerful method of quasicoincident neighborhood system in I-topology. Zhang and Xu [17] established the neighborhood structure in fuzzifying topological spaces. Considering the completeness and usefulness of theory of I-fuzzy topologies, Fang [2] established I-fuzzy quasicoincident neighborhood system in I-fuzzy topological spaces and gave a useful tool to study I-fuzzy topologies. In ordinary topology, $\hat{\mu}\beta$ open sets were introduced and studied by J.Subashini and K.Indirani [13]. Separation is an essential part of fuzzy topology, on which a lot of work has been done. In the framework of fuzzifying topologies, Shen[12], Yue and Fang [18], Li and Shi [6], and Khedr et al. [5] introduced some separation axioms and their separation axioms are discussed on crisp points not on fuzzy points. In 2004, Mahmoud et al. [8] introduced fuzzy semi continuity and fuzzy semi separation axioms and examined the validity of some characterization of these concepts. Further, they also defined fuzzy generalized semi open set and introduced fuzzy separation axioms by using the semi open sets concept. In the same paper, the authors also discussed fuzzy semi connected and fuzzy semi compact spaces and some of their properties. In this paper, we introduce the notions of some lower separation axioms such as the $\hat{\mu}\beta T_0$, $\hat{\mu}\beta T_1$ and $\hat{\mu}\beta T_2$ axioms with instigating some of their properties and the relations between them in the general framework of fuzzy topological spaces. And also introduce the notions of some lower regularity axioms such as the $\hat{\mu}\beta R_0$ and $\hat{\mu}\beta R_1$ with instigating some of their properties and the relations between them in the general framework of fuzzy topological spaces.

II. PRELIMINARIES

Throughout this paper, X represents a nonempty fuzzy set and fuzzy subset A of X, denoted by $A \leq X$, then it is characterized by a membership function in the sense of Zadeh [16]. The basic fuzzy sets are the empty set, the whole set, and the class of all fuzzy sets of X which will be denoted by 0_X, 1_X, and I^X, respectively. A subfamily τ of I^X is called a fuzzy topology described by Chang [1]. Moreover, the pair $(X, \tau) = (I^X, \tau)$ will be meant as a fuzzy topological space, on which no separation axioms are assumed unless explicitly stated. The fuzzy closure, the fuzzy interior, and the fuzzy complement of any set A in (X, τ) are denoted by cl(A), int(A), and $1 - A$, respectively. A fuzzy set which is a fuzzy point [14] with support $x \in X$ and value t $(0 < t \leq 1)$ is denoted by x_t, and Pt(X) will denote the family of all point fuzzy sets $x_t \in I^X$. For any two fuzzy sets A and B in (X, τ), $A \leq B$ if and only if $A(x) \leq B(x)$ for each $x \in X$.

A. Definition 2.1

([17]). In a fuzzy topological space (X, τ), a fuzzy set A is called a quasi coincident with a fuzzy set B, denoted by AqB, if $A(x) + B(x) > 1$ for some $x \in X$. A fuzzy point $x_t \leq A$ is called quasicoincident with the fuzzy set A, denoted by x_tqA, if $t + A(x) > 1$. Relation "does not quasicoincide with" or "not quasicoincident with" is denoted by $\neg q$. A fuzzy set A in (X, τ) is called quasi-neighborhood of x_t if there is a fuzzy open set U such that x_tqU *and* $U \leq A$.

B. Definition 2.2

(see [13]). A subset A of a topological space (X,τ) is called $\hat{\mu}\beta$ closed set if $\hat{\mu}$cl(A)$\subseteq U$, whenever A$\subseteq U$ and U is β open in X.

C. Definition 2.3

Let A be a fuzzy set in fuzzy topological space (X, τ). $int_{\hat{\mu}\beta}$ $(A) = \vee\{B \in F\hat{\mu}\beta O(X, \tau) : B \leq A\}$ is called the $\hat{\mu}\beta$-interior of A, and $cl_{\hat{\mu}\beta}$ (A) = $\wedge\{B \in F\hat{\mu}\beta C(X, \tau) : A \leq B\}$ is called the $\hat{\mu}\beta$-closure of A.

III. $\hat{\mu}\beta$ – SEPARATION AXIOMS

In this section, the notions of some lower separation axioms such as the $\hat{\mu}\beta T_0$, $\hat{\mu}\beta T_1$, $\hat{\mu}\beta T_{1/2}$, and $\hat{\mu}\beta T_2$ axioms are introduced. Furthermore some of their properties and the relations between them in the general framework of fuzzy topological spaces is studied.

A. Definition 3.1

A fuzzy topological space (X, τ) is called

Fuzzy $\hat{\mu}\beta T_0$-space if for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, there exist $U \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t qU$ *and* $y_r \leq 1 - y_r$ or $y_r qU$ *and* $x_t \leq 1 - x_t$

Fuzzy $\hat{\mu}\beta T_1$-space if for every pair of fuzzy points x_t , y_r in X $(x \neq y)$, there exist $U,V \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t qU$ *and* $y_r \leq 1 - y_r$ and $y_r qV$ *and* $x_t \leq 1 - x_t$

Fuzzy $\hat{\mu}\beta T_2$-space if for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, there exist $U,V \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t qU$ *and* $y_r \leq 1 - y_r$, $y_r qV$ *and* $x_t \leq 1 - x_t$, and $U \neg qV$.

B. Theorem 3.2 .

Let (X, τ) be a fuzzy topological space. If (X, τ) is fuzzy $\hat{\mu}\beta T_i$-space, then it is fuzzy $\hat{\mu}\beta T_{i-1}$-space, where i = 1, 2.

1) *Proof.* Obvious.

C. Theorem3.3

A fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta T_0$-space if and only if for every pair of fuzzy points x_t, y_r in X (x ≠ y), $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$.

Suppose that (X, τ) is $\hat{\mu}\beta T_0$-space. Then for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, there exists $U \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t qU \leq 1 - y_r$ or $y_r qU \leq 1 - x_t$. If $x_t qU \leq 1 - y_r$, then $x_t \nleq 1 - U$ and $U \leq 1 - y_r$, that is, $x_t \nleq 1 - U$ and $y_r \leq 1 - U$. Since $1 - U$ is fuzzy $\hat{\mu}\beta$-closed and $cl_{\hat{\mu}\beta}$ (y_r) is the smallest fuzzy $\hat{\mu}\beta$-closed set containing y_r, then $cl_{\hat{\mu}\beta}$ $(y_r) \leq 1 - U$. Since $x_t \nleq 1 - U$ and $x_t \leq cl_{\hat{\mu}\beta}$ (x_t), then $cl_{\hat{\mu}\beta}$ $(x_t) \neq cl_{\hat{\mu}\beta}$ (y_r).Conversely, suppose that x_t, y_r be a pair of fuzzy points in X with ($x \neq y$) and $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}$ (y_r). Let $z_\lambda \in Pt(X)$ such that $z_\lambda \leq cl_{\hat{\mu}\beta}$ (x_t) and $z_\lambda \nleq cl_{\hat{\mu}\beta}$ (y_r). We claim that $x_t \nleq cl_{\hat{\mu}\beta}$ (y_r). For, if $x_t \leq cl_{\hat{\mu}\beta}$ (y_r), then $cl_{\hat{\mu}\beta}$ $(x_t) \nleq cl_{\hat{\mu}\beta}$ (y_r). This contradicts the fact that $z_\lambda \nleq cl_{\hat{\mu}\beta}$ (y_r). Hence $x_t \nleq cl_{\hat{\mu}\beta}$ (y_r), that is, $x_t q(1 - cl_{\hat{\mu}\beta}$ $(y_r))$. And since $U := 1 - cl_{\hat{\mu}\beta}(y_r) \in F\hat{\mu}\beta O(X, \tau)$ and $1 - y_r \leq 1 - cl_{\hat{\mu}\beta} = U$, then $x_t qU \leq 1 - y_r$ .That is, (X, τ) is fuzzy $\hat{\mu}\beta T_0$-space.

D. Theorem 3.4

A fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta T_1$-space if and only if every singleton fuzzy points x_t in X is fuzzy $\hat{\mu}\beta$- closed in X.

1) *Proof.* Suppose that (X, τ) is $\hat{\mu}\beta T_1$-space. Let $y_r \leq 1 - x_t$, $(x \neq y)$. Then there exist $U,V^y \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t qU \leq 1 - y_r$ and $y_r qV^y \leq 1 - x_t$. Let $y_r qV^y \leq 1 - x_t$, we have $V^y \leq 1 - x_t$. Let $A = \vee\{V^y : y_r q(1 - x_t)\}$. Therefore $A = 1 - x_t$. Hence $1 - x_t$, is fuzzy $\hat{\mu}\beta$-open set, that is, x_t is fuzzy $\hat{\mu}\beta$ closed set. Conversely, let x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$. Then x_t and y_r are fuzzy $\hat{\mu}\beta$-closed sets. Consequently, $1 - x_t$ and $1 - y_r$ are fuzzy $\hat{\mu}\beta$-open sets. Let U = 1 - x_t and V = 1 -y_r. Hence there exist U, V$\in \hat{\mu}\beta O(X,\tau)$ such that $x_t q$U and U $\leq (1 - y_r)$ and $y_r q$V and V $\leq (1 - x_t)$. Therefore, (X, τ) is fuzzy $\hat{\mu}\beta T_1$-space.

D. Definition 3.5

A fuzzy subset A of fuzzy topological space (X, τ) is called fuzzy $\hat{\mu}\beta$-symmetric if for every pair of fuzzy points x_t, y_r in X, $x_t \leq cl_{\hat{\mu}\beta}$ (y_r) implies $y_r \leq cl_{\hat{\mu}\beta}$ (x_t) (i.e., $x_t \leq cl_{\hat{\mu}\beta}$ (y_r) implies $cl_{\hat{\mu}\beta}$ $(y_r) = cl_{\hat{\mu}\beta}(x_t)$).

E. Corollary 3.6.

If fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta T_1$- space, then it is $\hat{\mu}\beta$-symmetric.

F. Corollary 3.7

A fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta$-symmetric and fuzzy $\hat{\mu}\beta T_0$-space if and only if it is fuzzy $\hat{\mu}\beta T_1$-space.

1) *Proof.* If (X, τ) is fuzzy $\hat{\mu}\beta T_1$-space, then by Theorem 3.2 and Corollary 3.6, it is fuzzy $\hat{\mu}\beta$-symmetric and fuzzy $\hat{\mu}\beta T_0$-space. Conversely, suppose that x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$. Then by fuzzy $\hat{\mu}\beta T_0$-space, we may assume that there exists an open set U such that $x_t qU$ *and* $U \leq 1 - y_r$ for some $U \in F\hat{\mu}\beta O(X, \tau)$, hence $x_t \nleq cl_{\hat{\mu}\beta}(y_r)$, which implies, by $\hat{\mu}\beta$-symmetric, $y_r \nleq cl_{\hat{\mu}\beta}(x_t)$. Let V = $1-cl_{\hat{\mu}\beta}(x_t)$. Therefore $y_r qV$ and V $\leq 1-x_t$. Hence, (X, τ) is fuzzy $\hat{\mu}\beta T_1$-space.

G. *Theorem 3.8*

For fuzzy $\hat{\mu}\beta$-symmetric topological space (X, τ), the following properties are equivalent:

1) (X, τ) is fuzzy $\hat{\mu}\beta T_0$-space;
2) (X, τ) is fuzzy $\hat{\mu}\beta T_1$-space
3) Proof. Obvious.

H. *Theorem 3.9*

For fuzzy $\hat{\mu}\beta$-symmetric topological space (X, τ), the following properties are equivalent:

(X, τ) is fuzzy $\hat{\mu}\beta T_2$-space;

for every pair of fuzzy points x_t , y_r in X (x ≠ y), there exists U ∈ $F\hat{\mu}\beta O(X, \tau)$ such that $x_t qU \leq 1 - y_r$ and $y_r \nleq cl_{\hat{\mu}\beta}(U)$;

for every fuzzy point x_t in X, $\wedge \{cl_{\hat{\mu}\beta}(U) : U \in F\hat{\mu}\beta O(X, \tau), x_t \leq U\} = x_t$.

$qV \leq 1 - x_t$ and $U \neg qV$, hence, $x_t \leq U$. Since $U \leq 1-V$ and $1-V$ is fuzzy $\hat{\mu}\beta$-closed, then $cl_{\hat{\mu}\beta}(U) \leq cl_{\hat{\mu}\beta}(1-V) = 1 - v$. And since $y_r \nleq 1 - V$, then $y_r \nleq cl_{\hat{\mu}\beta}(U)$.

(2)→(3): It is clear that $x_t \leq \wedge\{cl_{\hat{\mu}\beta}(U) : U \in F\hat{\mu}\beta O(X, \tau), x_t \leq U\}$. Now if $x_t \neq y_r$, then there exists $U \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t qU \leq 1 - y_r$ and $y_r \nleq cl_{\hat{\mu}\beta}(U)$. This implies that $y_r \nleq \wedge \{cl_{\hat{\mu}\beta}(U) : U \in F\hat{\mu}\beta O(X, \tau), x_t \leq U\}$. Hence $\wedge\{cl_{\hat{\mu}\beta}(U) : U \in F\hat{\mu}\beta O(X, \tau), x_t \leq U\} = x_t$.

(3)→(1): Let x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$. Since $\wedge\{cl_{\hat{\mu}\beta}(U) : U \in F\hat{\mu}\beta O(X, \tau), x_t \leq U\} = x_t$, then there is fuzzy $U \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t \leq U$ and $y_r \nleq cl_{\hat{\mu}\beta}(U)$.Hence $x_t qU \leq cl_{\hat{\mu}\beta}(U) \leq 1-y_r$. Put $V = 1-cl_{\hat{\mu}\beta}(U)$, then $y_r qV \leq 1 - V \leq 1 - x_t$, and it is clear that $V \neg U$. Hence (X, τ) is fuzzy $\hat{\mu}\beta T_2$-space.

IV. $\hat{\mu}\beta$-REGULARITY AXIOMS

In this section, we introduce the notions of some lower regularity axioms such as the $\hat{\mu}\beta R_0$ and $\hat{\mu}\beta R_1$ with instigating some of their properties and the relations between them in the general framework of fuzzy topological spaces.

A. *Definition 4.1*

A fuzzy topological space (X, τ) is called fuzzy $\hat{\mu}\beta R_0$-space if for every $U \in F\hat{\mu}\beta O(X, \tau)$ and for every fuzzy point $x_t \leq U$, $cl_{\hat{\mu}\beta}(x_t) \leq U$.

B. *Theorem 4.2*

A fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space if and only if for every pair of fuzzy points x_t , y_r in X with (x ≠ y) and $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$, $cl_{\hat{\mu}\beta}(x_t) \neg q\, cl_{\hat{\mu}\beta}(y_r)$.

Suppose that a fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space. Let x_t , y_r be a pair of fuzzy points in X with $(x \neq y)$ and $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$. Then there exists fuzzy point z_μ in X such that $z_\mu \leq cl_{\hat{\mu}\beta}(x_t)$ and $z_\mu \nleq cl_{\hat{\mu}\beta}(y_r)$. If $x_t \leq cl_{\hat{\mu}\beta}(y_r)$, then $cl_{\hat{\mu}\beta}(x_t) \leq cl_{\hat{\mu}\beta}(y_r)$.Hence, $z_\mu \leq cl_{\hat{\mu}\beta}(y_r)$, but this is a contradiction. Then $x_t \nleq cl_{\hat{\mu}\beta}(y_r)$, that is, $x_t \leq 1-cl_{\hat{\mu}\beta}(y_r)$.Since $1 -cl_{\hat{\mu}\beta}(y_r)$ is fuzzy $\hat{\mu}\beta$-open and (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space, then $cl_{\hat{\mu}\beta}(x_t) \leq 1 - cl_{\hat{\mu}\beta}(y_r)$. Hence $cl_{\hat{\mu}\beta}(x_t) \neg q\, cl_{\hat{\mu}\beta}(y_r)$.Conversely, Let $V \in F\hat{\mu}\beta O(X, \tau)$ and $x_t \leq V$. We will prove that $cl_{\hat{\mu}\beta}(x_t) \leq V$. Let $y_r \nleq V$. Then $y_r \leq 1 – V$ and $x \neq y$. This implies that $cl_{\hat{\mu}\beta}(y_r) \leq cl_{\hat{\mu}\beta}(1 - V) = 1 - V$. Since $x_t \leq V$, then $x_t \nleq cl_{\hat{\mu}\beta}$, that is, $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$. Then by assumption, $cl_{\hat{\mu}\beta}(x_t) \neg q\, cl_{\hat{\mu}\beta}(y_r)$. That is, $cl_{\hat{\mu}\beta}(x_t) \leq 1 - cl_{\hat{\mu}\beta}(y_r) \leq V$. Hence (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space.

C. *Definition 4.3.*

Let A be a fuzzy subset of fuzzy topological space (X, τ). The fuzzy $\hat{\mu}\beta$-kernel of A, denoted by $Fker_{\hat{\mu}\beta}(A)$ is defined to be the set $Fker_{\hat{\mu}\beta}(A) = \wedge\{U \in F\hat{\mu}\beta O(X, \tau) : A \leq U\}$. In particular, the fuzzy $\hat{\mu}\beta$-kernel of fuzzy point $x_t \in Pt(X)$ is defined to be the set $Fker_{\hat{\mu}\beta}(x_t) = \wedge\{U \in F\hat{\mu}\beta O(X, \tau) : x_t \leq U\}$.

D. Lemma 4.6

Let (X, τ) be a fuzzy topological space and x_t, $y_r \in Pt(X)$. Then $Fker_{\hat{\mu}\beta}(x_t) \neq Fker_{\hat{\mu}\beta}(y_r)$ if and only if $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$.

1) *Proof.* Suppose that $Fker_{\hat{\mu}\beta}(x_t) \neq Fker_{\hat{\mu}\beta}(y_r)$. Then there exists fuzzy point z_μ in X such that $z_\mu \leq Fker_{\hat{\mu}\beta}(x_t)$ and $z_\mu \nleq Fker_{\hat{\mu}\beta}(y_r)$. In the part $z_\mu \leq Fker_{\hat{\mu}\beta}(x_t)$, by Lemma 4.5, $x_t q\ cl_{\hat{\mu}\beta}(z_\mu)$. This implies that $x_t \leq cl_{\hat{\mu}\beta}(z_\mu)$, that is, $cl_{\hat{\mu}\beta}(x_t) \leq cl_{\hat{\mu}\beta}(z_\mu)$. And similarly, in the part $z_\mu \nleq Fker_{\hat{\mu}\beta}(y_r)$ we get $y_r \neg q\ cl_{\hat{\mu}\beta}(z_\mu)$. This implies $y_r \nleq cl_{\hat{\mu}\beta}(z_\mu)$. Since $cl_{\hat{\mu}\beta}(x_t) \leq cl_{\hat{\mu}\beta}(z_\mu)$, then $y_r \nleq cl_{\hat{\mu}\beta}(x_t)$. Hence $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$.Conversely, suppose that $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$. Then there exists fuzzy point z_μ in X such that $z_\mu \leq cl_{\hat{\mu}\beta}(x_t)$ and $z_\mu \nleq cl_{\hat{\mu}\beta}(y_r)$. If $x_t \leq cl_{\hat{\mu}\beta}(y_r)$, then $cl_{\hat{\mu}\beta}(x_t) \leq cl_{\hat{\mu}\beta}(y_r)$. Hence $z_\mu \leq cl_{\hat{\mu}\beta}(y_r)$ but this is a contradiction. Then $x_t \nleq cl_{\hat{\mu}\beta}(y_r)$, that is, $x_t \leq 1 - cl_{\hat{\mu}\beta}(y_r)$. Hence $1 - cl_{\hat{\mu}\beta}(y_r) \in F\hat{\mu}\beta O(X, \tau)$ containing x_t and not y_r. Then $y_r \nleq Fker_{\hat{\mu}\beta}(x_t)$ and $y_r \leq Fker_{\hat{\mu}\beta}(y_r)$. Hence $Fker_{\hat{\mu}\beta}(x_t) \neq Fker_{\hat{\mu}\beta}(y_r)$.

E. Theorem 4.7

A fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space if and only if for every pair of fuzzy points x_t, y_r in X with $(x \neq y)$ and $Fker_{\hat{\mu}\beta}(x_t) \neq Fker_{\hat{\mu}\beta}(y_r)$, $Fker_{\hat{\mu}\beta}(x_t) \neg q Fker_{\hat{\mu}\beta}(y_r)$.

Proof. Suppose that a fuzzy topological space (X, τ) is $\hat{\mu}\beta R_0$- space. Let x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$ and $Fker_{\hat{\mu}\beta}(x_t) \neq Fker_{\hat{\mu}\beta}(y_r)$. By Lemma 4.6, $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$. Suppose that $Fker_{\hat{\mu}\beta}(x_t) q Fker_{\hat{\mu}\beta}(y_r)$ for some $z \in X$. Take $\mu = Fker_{\hat{\mu}\beta}(x_t)(z) \vee Fker_{\hat{\mu}\beta}(y_r)(z) \in (0, 1]$. Then $z_\mu \leq Fker_{\hat{\mu}\beta}(x_t)$, $z_\mu \leq Fker_{\hat{\mu}\beta}(y_r)$. In the part $z_\mu \leq Fker_{\hat{\mu}\beta}(x_t)$, by Lemma 4.5 we get that $x_t \leq cl_{\hat{\mu}\beta}(z_\mu)$, which implies $cl_{\hat{\mu}\beta}(x_t) \leq cl_{\hat{\mu}\beta}(z_\mu)$. Then by Theorem 4.2, $cl_{\hat{\mu}\beta}(x_t) = cl_{\hat{\mu}\beta}(z_\mu)$. Similarly, in the part $z_\mu \leq Fker_{\hat{\mu}\beta}(y_r)$, we get that $cl_{\hat{\mu}\beta}(y_r) = cl_{\hat{\mu}\beta}(z_\mu) = cl_{\hat{\mu}\beta}(x_t)$. This is a contradiction. Therefore, $Fker_{\hat{\mu}\beta}(x_t) \neg q Fker_{\hat{\mu}\beta}(y_r)$. Conversely, we will use Theorem 4.2 to prove that (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space. Let x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$ and $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$. Then by Lemma 4.6, $Fker_{\hat{\mu}\beta}(x_t) \neq Fker_{\hat{\mu}\beta}(y_r)$. Hence by assumption, we get that $Fker_{\hat{\mu}\beta}(x_t) \neg q Fker_{\hat{\mu}\beta}(y_r)$. Suppose that $cl_{\hat{\mu}\beta}(x_t) q\ cl_{\hat{\mu}\beta}(y_r)$ for some $z \in X$. Take $\mu = cl_{\hat{\mu}\beta}(x_t)(z) \vee cl_{\hat{\mu}\beta}(y_r)(z) \in (0, 1]$. Then $z_\mu \leq cl_{\hat{\mu}\beta}(x_t)$ and $z_\mu \leq cl_{\hat{\mu}\beta}(y_r)$. Hence by Lemma 4.5, $x_t \leq Fker_{\hat{\mu}\beta}(z_\mu)$ and $y_r \leq Fker_{\hat{\mu}\beta}(z_\mu)$. Then by Lemma 4.4, $Fker_{\hat{\mu}\beta}(x_t) \leq Fker_{\hat{\mu}\beta}(z_\mu)$, $Fker_{\hat{\mu}\beta}(y_r) \leq Fker_{\hat{\mu}\beta}(z_\mu)$, that is, $Fker_{\hat{\mu}\beta}(x_t) q Fker_{\hat{\mu}\beta}(z_\mu)$, $Fker_{\hat{\mu}\beta}(y_r) q Fker_{\hat{\mu}\beta}(z_\mu)$. Hence by assumption, $Fker_{\hat{\mu}\beta}(x_t) = Fker_{\hat{\mu}\beta}(z_\mu)$, $Fker_{\hat{\mu}\beta}(y_r) = Fker_{\hat{\mu}\beta}(z_\mu)$. Hence $Fker_{\hat{\mu}\beta}(x_t) = Fker_{\hat{\mu}\beta}(y_r)$, that is, $Fker_{\hat{\mu}\beta}(x_t) q Fker_{\hat{\mu}\beta}(y_r)$. But this is a contradiction. Hence $cl_{\hat{\mu}\beta}(x_t) \neg q\ cl_{\hat{\mu}\beta}(y_r)$. Therefore, by Theorem 4.2, (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space.

F. Theorem 4.8

For fuzzy topological space (X, τ), the following properties are equivalent:

1) (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space
2) for every fuzzy set $A \neq 0_X$ and $U \in F\hat{\mu}\beta O(X, \tau)$ such that AqU, there exists $V \in F\hat{\mu}\beta C(X, \tau)$ such that AqV and $V \leq U$
3) for every $U \in F\hat{\mu}\beta O(X, \tau)$, $U = \vee\{V \in F\hat{\mu}\beta O(X, \tau) : V \leq U$
4) for every $U \in F\hat{\mu}\beta O(X, \tau)$, $U = \wedge\{V \in F\hat{\mu}\beta O(X, \tau) : U \leq V$
5) for every fuzzy point $x_t \in Pt(X)$, $cl_{\hat{\mu}\beta}(x_t) \leq F\ Fker_{\hat{\mu}\beta}(x_t)$
6) *Proof.* (1)→(2): Let $A \neq 0_X$ be fuzzy set in X and $U \in F\hat{\mu}\beta O(X, \tau)$ such that AqU for some $z \in X$. Take $\mu = A(z) \vee U(z)$. Then $z_\mu \leq A$ and $z_\mu \leq U$. Since $U \in F\hat{\mu}\beta O(X, \tau)$ and (X, τ) is fuzzy $\hat{\mu}\beta R_0$ space, then $cl_{\hat{\mu}\beta}(z_\mu) \leq U$. Take $V = cl_{\hat{\mu}\beta}(z_\mu)$. Then $V \in F\hat{\mu}\beta C(X, \tau)$ and $V \leq U$. Since $z_\mu \leq A$, then $cl_{\hat{\mu}\beta}(z_\mu) qA$, that is, AqV.

(2)→(3): It is clear that $\vee\{V \in F\hat{\mu}\beta O(X, \tau) : V \leq U\} \leq U$. Let $x_t \leq U$. Since $U \in F\hat{\mu}\beta O(X, \tau)$ and $x_t \neq 0_X$, then there exists $V \in F\hat{\mu}\beta C(X, \tau)$ such that $x_t \leq V$ and $V \leq U$. Then $x_t \leq \vee\{V \in F\hat{\mu}\beta O(X, \tau) : V \leq U\}$, that is, $U \leq \vee\{V \in F\hat{\mu}\beta O(X, \tau) : V \leq U\}$.

(3)→(4): Obvious.

(4)→(5): Let $x_t \in Pt(X)$ and $y_r \nleq Fker_{\hat{\mu}\beta}(x_t)$. Then there exits $V \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t \leq V$ and $y_r \nleq V$. Hence $y_r \leq 1 - V$, which implies that $cl_{\hat{\mu}\beta}(y_r) \leq cl_{\hat{\mu}\beta}(1-V) = 1-V$. That is, $cl_{\hat{\mu}\beta}(y_r) \leq 1 - \wedge\{U \in F\hat{\mu}\beta O(X, \tau) : V \leq U\}$. Hence there exists $U \in F\hat{\mu}\beta O(X, \tau)$ such that $x_t \nleq U$ and $cl_{\hat{\mu}\beta}(y_r) \leq U$. Hence $cl_{\hat{\mu}\beta}(x_t) \leq 1 - U$. Therefore, $y_r \nleq cl_{\hat{\mu}\beta}(x_t)$. That is, $cl_{\hat{\mu}\beta}(x_t) \leq Fker_{\hat{\mu}\beta}(x_t)$.

(5)→(1): Let $U \in F\hat{\mu}\beta O(X, \tau)$ and $x_t \leq U$. Then $cl_{\hat{\mu}\beta}(x_t) \leq Fker_{\hat{\mu}\beta}(x_t) \leq U$. Hence (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space.

G. Corollary 4.9

A fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta R_0$- space if and only if $cl_{\hat{\mu}\beta}(x_t) = Fker_{\hat{\mu}\beta}(x_t)$ for all $x_t \in Pt(X)$.

Suppose that (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space. Then by Theorem 4.8, $cl_{\hat{\mu}\beta}(x_t) \leq Fker_{\hat{\mu}\beta}(x_t)$ for all $x_t \in Pt(X)$. Let $y_r \leq Fker_{\hat{\mu}\beta}(x_t)$. Then by Lemma 4.5, $x_t \in cl_{\hat{\mu}\beta}(y_r) \leq Fker_{\hat{\mu}\beta}(y_r)$. Hence $x_t \leq Fker_{\hat{\mu}\beta}(y_r)$, which implies, by the same lemma, that $y_r \in cl_{\hat{\mu}\beta}(x_t)$. Therefore, $cl_{\hat{\mu}\beta}(x_t) = Fker_{\hat{\mu}\beta}(x_t)$ for all $x_t \in Pt(X)$. Conversely, it is obvious by Theorem 4.8.

H. Theorem 4.10

For fuzzy topological space (X, τ), the following properties are equivalent:

(X, τ) is fuzzy $\hat{\mu}\beta R_0$-space;

$x_t \leq cl_{\hat{\mu}\beta}(y_r)$ if and only if $y_r \leq cl_{\hat{\mu}\beta}(x_t)$ for all x_t , $y_r \in Pt(X)$.

1) Proof. (1)→(2): Let $x_t \leq cl_{\hat{\mu}\beta}(y_r)$. Since (X, τ) is fuzzy $\hat{\mu}\beta R_0$- space, then, by Corollary 4.9, $cl_{\hat{\mu}\beta}(y_r) = Fker_{\hat{\mu}\beta}(y_r)$. Hence by Lemma 4.5, $y_r \leq cl_{\hat{\mu}\beta}(x_t)$. Similarly, the converse part.

(2)→(1): Let $U \in F\hat{\mu}\beta O(X, \tau)$ and $x_t \leq U$. If $y_r \nleq U$, then $x_t \nleq cl_{\hat{\mu}\beta}(y_r)$. Then $y_r \nleq cl_{\hat{\mu}\beta}(x_t)$. Hence $cl_{\hat{\mu}\beta}(x_t) \leq U$. That is, (X, τ) is fuzzy $\hat{\mu}\beta R_0$-space.

I. Definition 4.12.

A fuzzy topological space (X, τ) is called fuzzy $\hat{\mu}\beta R_1$-space if for every pair of fuzzy points x_t , y_r in X with $cl_{\hat{\mu}\beta}(x_t) \neq cl_{\hat{\mu}\beta}(y_r)$, there exists $U,V \in F\hat{\mu}\beta O(X, \tau)$ such that $cl_{\hat{\mu}\beta}(x_t) \leq U$, $cl_{\hat{\mu}\beta}(y_r) \leq V$ and $U \neg q V$.

J. Theorem 4.13

A fuzzy topological space (X, τ) is fuzzy $\hat{\mu}\beta R_1$- space if and only if for every pair of fuzzy points x_t , y_r in X with $Fker_{\hat{\mu}\beta}(x_t) \neq Fker_{\hat{\mu}\beta}(y_r)$, there exists $U,V \in F\hat{\mu}\beta O(X, \tau)$ such that $cl_{\hat{\mu}\beta}(x_t) \leq U$, $cl_{\hat{\mu}\beta}(y_r) \leq V$ and $U \neg q V$.

1) Proof. Obvious, by Lemma 4.6.

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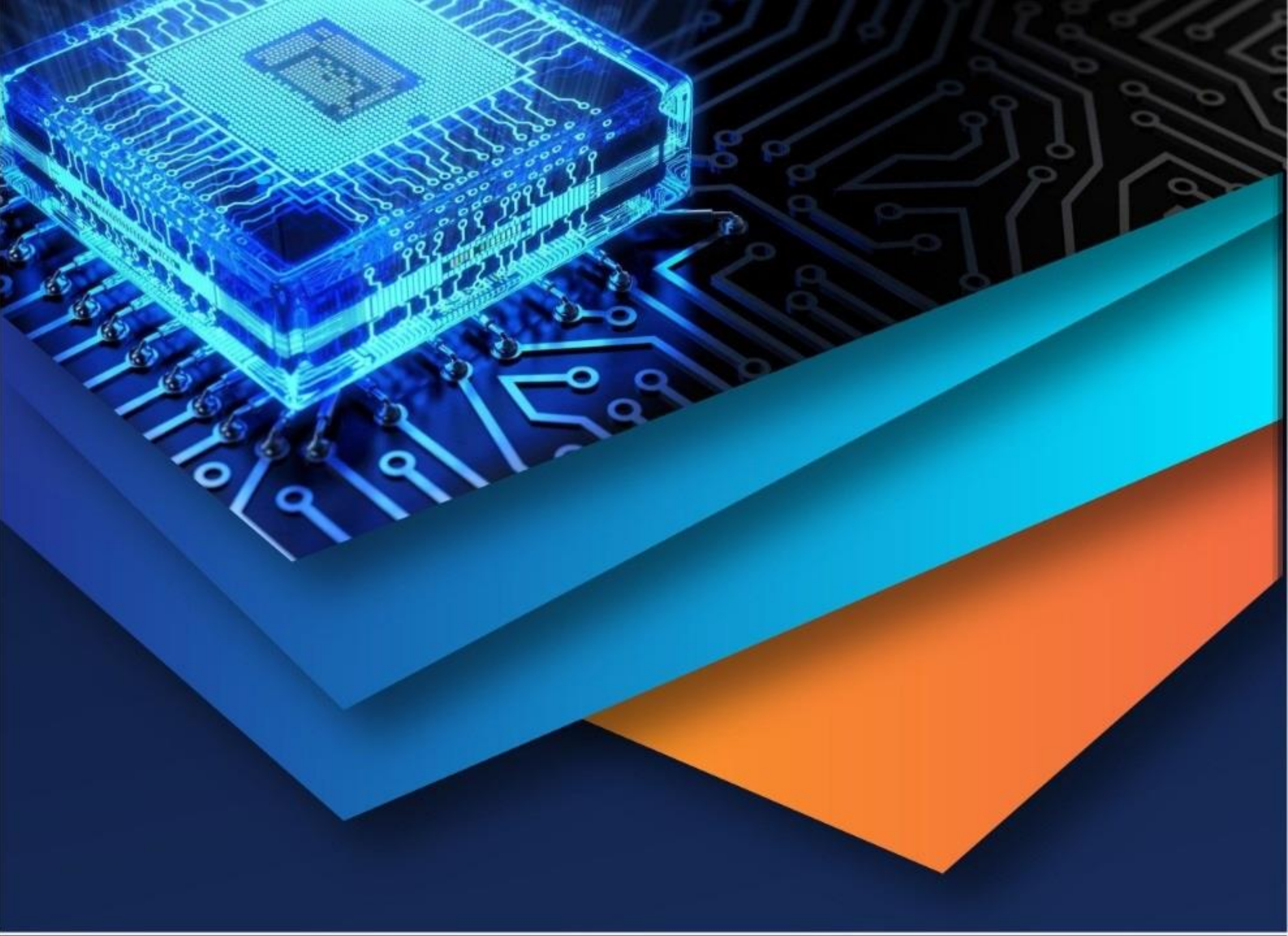

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