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Two Layered Model of Air Mucus Interface through Constricted Human Airways under the Influence of Time Varying Pressure Gradient

M. Chitra[1], S. Shabana[2]
[1]Associate Professor, [2]Research Scholar
[1, 2] Department of Mathematics, Thiruvalluvar University, Vellore – 632 115, TamilNadu.

Abstract: This paper deals with a two layered mathematical model for air flow through constricted trachea in the human respiratory tract with time varying pressure gradient. The transport of mucus as a viscoelastic fluid. The governing equations are solved by analytically. The effect of air flow due to inspiration and expiration is considered by prescribing shear stress at the mucus-air interface. Effects of slip parameter, viscosity on axial velocity profile, flow rate and wall shear stress of air region and mucus region for different constricted height and length of the human trachea are discussed graphically.
Keywords: Air mucus interface, Aerosol, Time varying pressure gradient, Viscoelastic fluid, Constriction.

I. INTRODUCTION

The respiratory system has several means protecting itself from dust. As the inhaled air is drawn in through the mouth or nose, it starts to travel a path that has numerous branches. While small particles move with the air around then bends and branches, larger particles strike the sides of the airways at these bends. Here they get stuck in the mucus lining and are removed from the system. Several other features enhance the effectiveness of the structure, keeping dust out of the system, or working to remove it after entry.Mucus covers the ciliated epithelium of the airways, which includes the nose, trachea, sinuses and the proximal bronchioles. The mucus continuously moves upwards towards the upper end of the trachea. The regular airflow reversals are obliviously very important in contributing to the particle deposition on the surface of the mucus layer, which is one of the main functions of the mucus.

V.S.Verma (2010) has studied a planar model for mucus transport and the effects of air flow, porosity and mucus visco-elasticity in human respiratory tract. P. Nirmala Ratchagar and M. Chitra (2014) have discussed effects of air-mucus interface through a human trachea with mild constriction of aerosols. Chong et al., (1986) investigated continuous flow model of mucus transport in the airways by two-phase gas-liquid flow mechanism. M. King et.al. (1974) have also proposed on the transport of mucus and its rheological stimulants in ciliated systems.

The objective of the paper is to study the effects of slip parameter (α) and viscosity (μ) on axial velocity profile, flow rate and wall shear stress of the flow of air and mucus region of the constricted trachea in the human airways, with time varying pressure gradient.

II. MATHEMATICAL FORMULATION

We consider the laminar of unsteady fully developed flow of air and mucus region of the constricted human trachea.

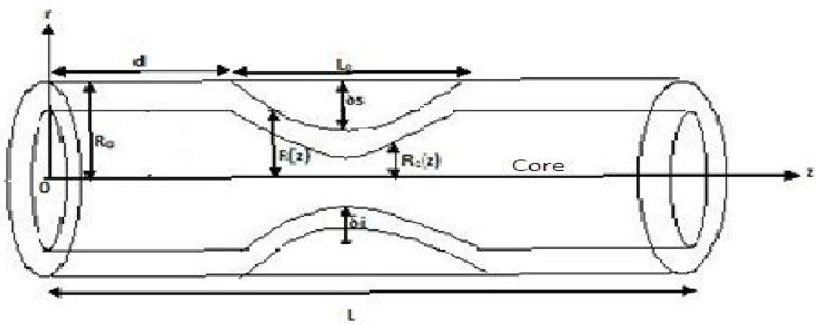

Figure.1 Geometry of constricted trachea with mucus region

Geometry of constricted trachea can be defined as

$$R(z) = \frac{R'(Z)}{R_0} = \begin{cases} 1 - \frac{\delta_h}{2R_0}\left[1 + \cos\frac{2\pi}{L_0}\left(z - d - \frac{L_0}{2}\right)\right]; & \text{if } d \le z \le d + L_0 \\ 1 & ; \text{otherwise} \end{cases} \quad (1)$$

$$R'(z) = \frac{\overline{R'(z)}}{R_0} = \begin{cases} \alpha - \frac{\alpha\delta_h}{2R_0}\left[1 + \cos\frac{2\pi}{L_0}\left(z - d - \frac{L_0}{2}\right)\right]; & \text{if } d \le z \le d + L_0 \\ \alpha & ; \text{otherwise} \end{cases} \quad (2)$$

Where $R(z)$ the radius of the tube with constricted trachea , R_0 is the radius of the trachea. L is the length of the trachea and $L_0$is the length of the constricted trachea. δ_h is the thickness of the deposition on the trachea,$R'(z)$ represents the geometry of the interface. Where α represents the ratio of the central layer radius to the tube radius in the unconstructed region, where $R'(z) = \alpha R(z)$.The physical contribution of the constriction and mucus region of trachea is shown in Fig. 1.

The governing equation of the air and mucus region in the trachea assuming fully developed unsteady flow where the flows are driven by the time varying pressure gradient.

Region: I Air region $[0 \le r \le R'(z)]$

$$\frac{\partial u_a}{\partial t} = -\frac{\partial p}{\partial z} + \frac{\mu_a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_a}{\partial t}\right) \quad (3)$$

Region: II Mucus region $[R'(z) \le r \le R(z)]$

$$\frac{\partial u_m}{\partial t} = -\frac{\partial p}{\partial z} + \frac{\mu_m}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_m}{\partial t}\right) \quad (4)$$

To solve the above system of equations (3) and (4), we use the following boundary conditions.

The symmetry condition

$$\frac{\partial u_a}{\partial r} = 0 \text{ at } r = 0 \quad (5)$$

Interface condition

$$u_a = u_m \text{ at } r = R'(z) \quad (6)$$

Slip on the boundary

$$u_m = -\beta\tau_m \text{ at } r = R(z) \quad (7)$$

A. Method of Solution

We assumed the pressure gradient to be exponential we can take

$$-\frac{\partial p}{\partial z} = e^{-\lambda^2 t}$$

$$u_a = u_a(r, t)e^{-\lambda^2 t}$$

$$u_m = u_m(r, t)e^{-\lambda^2 t}$$

The governing equations are solved by using Bessel function method.

The solution of equation (3) using the boundary condition (5) and (6), in the air region is

$$u_a = \left[-\frac{1}{\lambda^2} + \left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\frac{J_0\left(\frac{\lambda^2}{\mu_m}r\right)}{J_0\left(\frac{\lambda^2}{\mu_m}R(z)\right)}\frac{J_0\left(\frac{\lambda^2}{\mu_a}r\right)}{J_0\left(\frac{\lambda^2}{\mu_a}R'(z)\right)}\right]e^{-\lambda^2 t} \quad \text{where } 0 < r < R'(z) \quad (8)$$

The solution of equation (4) using the boundary condition (5) and (6), in the mucus region is

$$u_m = \left[-\frac{1}{\lambda^2} + \left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\frac{J_0\left(\frac{\lambda^2}{\mu_m} r\right)}{J_0\left(\frac{\lambda^2}{\mu_m} R(z)\right)}\right] e^{-\lambda^2 t} \qquad \text{where } R'(z) < r < R(z) \qquad (9)$$

The velocity of the constricted trachea is

$$u = u_a + u_m$$

$$u = \left[-\frac{1}{\lambda^2} + \left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\frac{J_0\left(\frac{\lambda^2}{\mu_m} r\right)}{J_0\left(\frac{\lambda^2}{\mu_m} R(z)\right)}\frac{J_0\left(\frac{\lambda^2}{\mu_a} r\right)}{J_0\left(\frac{\lambda^2}{\mu_a} R'(z)\right)}\right] e^{-\lambda^2 t}$$

$$+\left[-\frac{1}{\lambda^2} + \left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\frac{J_0\left(\frac{\lambda^2}{\mu_m} r\right)}{J_0\left(\frac{\lambda^2}{\mu_m} R(z)\right)}\right] e^{-\lambda^2 t}$$

B. Flow Rate

Flow rate of air region is $[0 \le r \le R'(z)]$

$$Q_a = \int_0^{R'(z)} 2\pi r u_a \, dr$$

$$Q_a = \int_0^{R'(z)} 2\pi r \left[-\frac{1}{\lambda^2} + \left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\frac{J_0\left(\frac{\lambda^2}{\mu_m} r\right)}{J_0\left(\frac{\lambda^2}{\mu_m} R(z)\right)}\frac{J_0\left(\frac{\lambda^2}{\mu_a} r\right)}{J_0\left(\frac{\lambda^2}{\mu_a} R'(z)\right)}\right] e^{-\lambda^2 t} \, dr$$

$$Q_a = -\pi e^{-\lambda^2 t}\frac{R^2(z)}{\lambda^2} + \pi\left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\frac{e^{-\lambda^2 t}}{R'^2(z)\, J_0{}^2\left(\frac{\lambda^2}{\mu_m} R(z)\right)}\frac{J_1{}^2\left(\frac{\lambda^2}{\mu_m}\right)}{J_0\left(\frac{\lambda^2}{\mu_a} R'(z)\right)} \qquad (11)$$

low rate of mucus region is $[R'(z) \le r \le R(z)]$

$$Q_m = \int_{R'(z)}^{R(z)} 2\pi r u_m \, dr$$

$$Q_m = \int_{R'(z)}^{R(z)} 2\pi r \left[-\frac{1}{\lambda^2} + \left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\frac{J_0\left(\frac{\lambda^2}{\mu_m} r\right)}{J_0\left(\frac{\lambda^2}{\mu_m} R(z)\right)}\right] e^{-\lambda^2 t} \, dr$$

$$Q_m = \frac{2\pi e^{-\lambda^2 t}}{\left(\frac{\lambda^2}{\mu_m}\right)^2}\left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\left(\frac{R(z) J_1\left(\frac{\lambda^2}{\mu_m} R(z)\right) - R'(z) J_1\left(\frac{\lambda^2}{\mu_m} R'(z)\right)}{J_0{}^2\left(\frac{\lambda^2}{\mu_m} R(z)\right)}\right) \qquad (12)$$

The total flow rate (Q) of the constricted trachea is obtained by adding equation (11) and (12)

$$Q = \int_0^{R(z)} 2\pi\, r u \, dr$$

$$Q = \int_0^{R'(z)} 2\pi r u_a \, dr + \int_{R'(z)}^{R(z)} 2\pi r u_m \, dr$$

$$Q = -\pi e^{-\lambda^2 t}\frac{R^2(z)}{\lambda^2} + \pi\left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}} + \frac{1}{\lambda^2}\right)\frac{e^{-\lambda^2 t}}{R'^2(z)\, J_0{}^2\left(\frac{\lambda^2}{\mu_m} R(z)\right)}\frac{J_1{}^2\left(\frac{\lambda^2}{\mu_m}\right)}{J_0\left(\frac{\lambda^2}{\mu_a} R'(z)\right)}$$

$$+\frac{2\pi e^{-\lambda^2 t}}{\left(\frac{\lambda^2}{\mu_m}\right)^2}\left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}}+\frac{1}{\lambda^2}\right)\left(\frac{R(z)J_1\left(\frac{\lambda^2}{\mu_m}R(z)\right)-R'(z)J_1\left(\frac{\lambda^2}{\mu_m}R'(z)\right)}{J_0{}^2\left(\frac{\lambda^2}{\mu_m}R(z)\right)}\right) \qquad (13)$$

C. Wall Shear Stress

The wall shear stress (τ) of the constricted trachea is

$$\tau = \left[-\mu\frac{\partial u}{\partial r}\right]_{r=R(z)}$$

$$= \mu\frac{\lambda^2}{\mu_m}\left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}}+\frac{1}{\lambda^2}\right)e^{-\lambda^2 t}\frac{J_1\left(\frac{\lambda^2}{\mu_m}R(z)\right)}{J_0\left(\frac{\lambda^2}{\mu_m}R(z)\right)}$$

$$+\mu\left(\frac{-\beta\tau_m}{e^{-\lambda^2 t}}+\frac{1}{\lambda^2}\right)e^{-\lambda^2 t}\left[\frac{\frac{\lambda^2}{\mu_a}J_0\left(\frac{\lambda^2}{\mu_m}R(z)\right)J_1\left(\frac{\lambda^2}{\mu_a}R(z)\right)+\frac{\lambda^2}{\mu_m}J_0\left(\frac{\lambda^2}{\mu_a}R(z)\right)J_1\left(\frac{\lambda^2}{\mu_m}R(z)\right)}{J_0\left(\frac{\lambda^2}{\mu_m}R'(z)\right)}\right]$$

III. CONCLUSION

The fig.2 shows that the velocity u decreases as the increase of thickness (δ) of the stenosed with radial position r for fixed slip parameter $\beta = 0.05$ and fixed length L=0.5. Fig.3 shows that the flow rate increases as the increase of slip parameter β for fixed thickness $(\delta = 0.2)$. Fig.4 shows that the wall shear stress (τ) increases as the increase of thickness (δ) for fixed slip parameter $\beta = 0.1$ with axial position of length z.

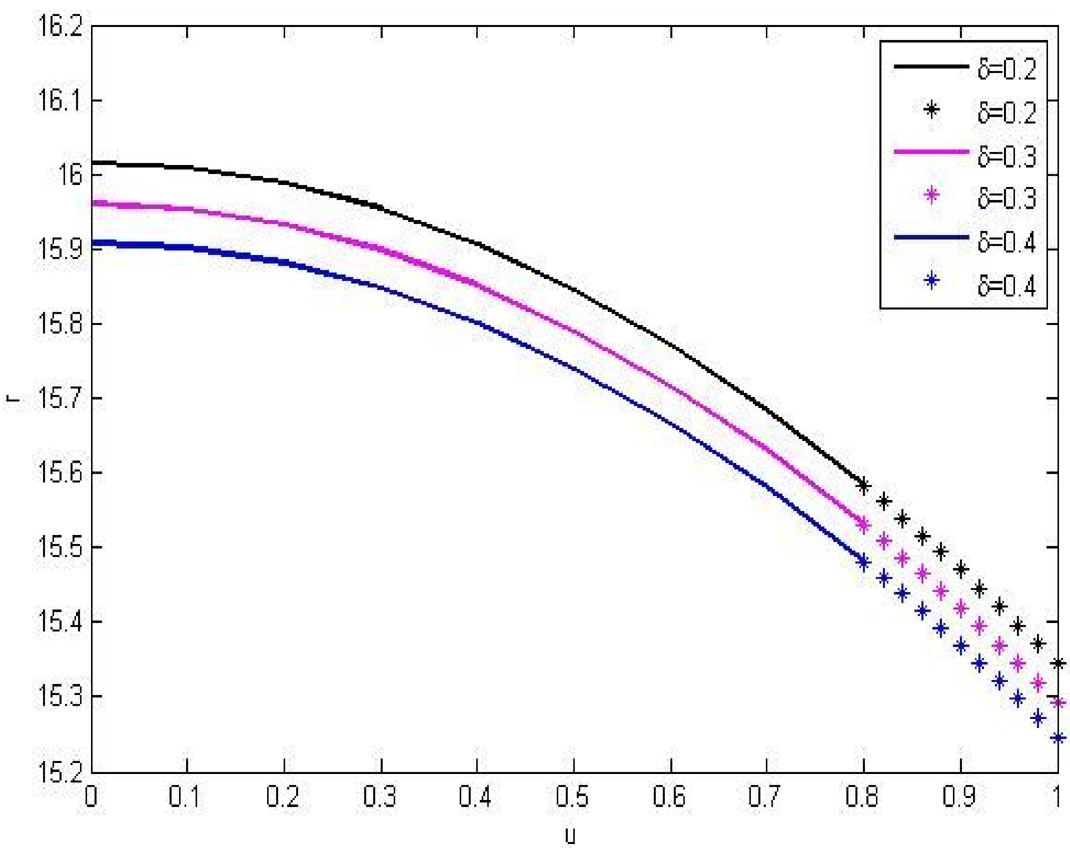

Fig.2: Variation of axial velocity u with radial position r for different thickness of the constricted trachea

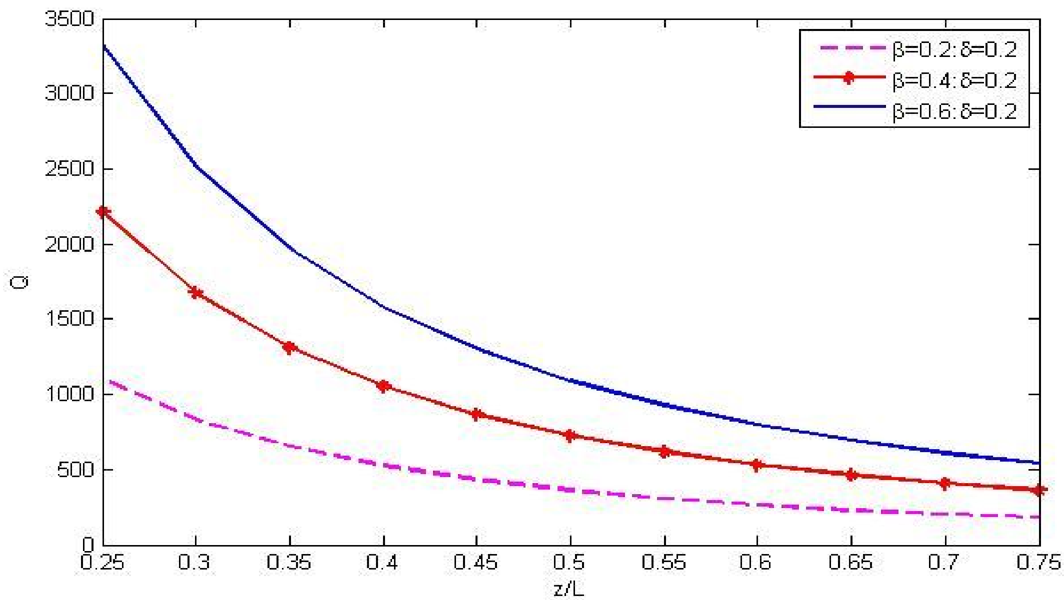

Fig.3: Variation of flow rate Q the axial position z for different slip parameter β of the constricted trachea

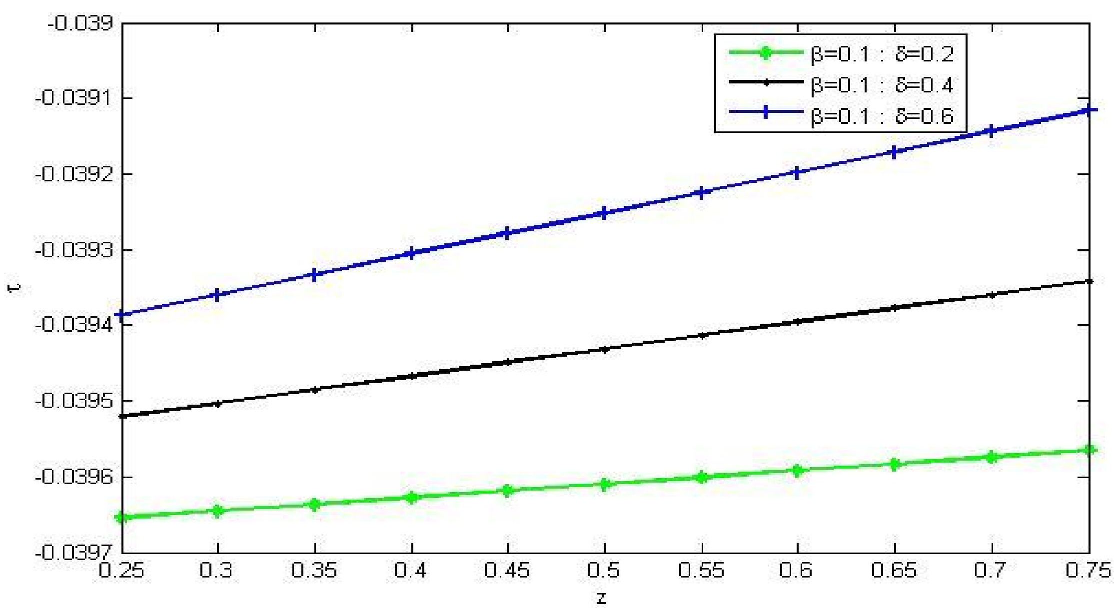

Fig.4: Variation of wall shear stress τ with axial position z for different thickness δ of the constricted trachea.

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