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Intersection - Empty Domination Number of a Graph

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Abstract: Let G=(V, E) be a graph with vertex set V and edge set E. A set S ⊆ V is a dominating set of G if every vertex in V-S is adjacent to a vertex in D.A dominating set S⊆ V is said to be a intersection empty dominating set if $\gamma(G) > 1$, then for every $u \in S$, there exists a vertex $v \in S$ such that $N(u) \cap N(v) \cap (V - S) = \emptyset$.The minimum cardinality of a intersection empty dominating set is called the intersection empty domination number and is denoted by γ_{ied}. In this paper, we define a new parameter called independent empty domination number and we study about this new parameter and we obtain several results on independent empty domination set.

Keywords: Intersection- Empty Domination Number,Tree

I. INTRODUCTION

Graph Theory is a major tool in mathematics research, electrical engineering, computer programming and networking, business administration, sociology ,economics, marketing and communications; the list can go on and on. Graphs are used to represent many real life situations. The dominating set concept in graphs has been used in many applications.

The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it because of its many and varied applications in such fields as linear algebra and optimization, design and analysis of communication networks, and social sciences and military surveillance. Many variants of dominating models are available in the existing literature. For a comprehensive bibliography of papers on the concept of domination, readers are referred to Hedetniemi and Laskar .

Throughout this paper, G = (V,E) stands for a finite, connected, undirected graph with

neither loops nor multiple edges. A subset D of V is called a dominating set in G, if every vertex in V − D is adjacent to a vertex in D. The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by γ(G).

In this paper, we define a new parameter called intersection empty domination number. Hence a study about this new parameter will give more bounds and inequalities connected the domination related parameters.

A. Definition

Let G be a non-trivial connected graph.A dominating set S⊆ V is said to be a intersection empty dominating set if $\gamma(G) > 1$, then for every $u \in S$, there exists a vertex $v \in S$ such that $N(u) \cap N(v) \cap (V - S) = \emptyset$.The minimum cardinality of a intersection empty dominating set is called the intersection empty domination number and is denoted by γ_{ied}.

The maximum cardinality of a intersection empty dominating set is called the upper common point dominating set denoted by Γ_{ied}.

B. Theorem:1

For p ≥ 4, $\gamma_{ied}(P_p) = \left\lceil \frac{p}{3} \right\rceil$

C. Proof:

Form S, the set of all vertices in G such that no two vertices have common vertex with cardinality $\left\lceil \frac{p}{3} \right\rceil$. Clearly,S is the intersection empty dominating set.

$$\gamma_{ied}(C_p) = \begin{cases} \left\lceil \frac{p}{3} \right\rceil + 1, & k \equiv 2(mod\ 3) \\ \left\lceil \frac{p}{3} \right\rceil & otherwise \end{cases}$$

Case(i) k≡2(mod 3)

Let S be the independent minimal dominating set .

By result" $\gamma(C_p) = \left\lceil \frac{p}{3} \right\rceil$

Now,Add one vertex {w} to S with the condition that w satisfy $N(u) \cap N(v) = \{w\}, u, v \in S$ then $S \cup \{w\}$ forms a intersection empty dominating set with cardinality $\left\lceil \frac{k}{3} \right\rceil + 1$.

Case(ii) k≡0(mod 3),k≡1(mod 3)

By result" $\gamma(C_p) = \left\lceil \frac{p}{3} \right\rceil$

Clearly,S is an intersection empty dominating set.

$$\gamma_{ied}(C_p) = \begin{cases} \left\lceil \frac{p}{3} \right\rceil + 1, & k \equiv 2(mod\ 3) \\ \left\lceil \frac{p}{3} \right\rceil & otherwise \end{cases}$$

D. Theorem:3

For $K_{1,n}$,$\gamma_{ied} = 2$

Proof:

Let u be the center vertex of G.

S=$\{u, v_i \ / \ i = 1,2,3, \dots n\}$ forms intersection empty dominating set with cardinality two.

E. Theorem:4

$$\gamma_{ied}(Bistar) = 2$$

1) Proof: Let S be the set of non pendant vertices of T.

Clearly, S forms intersection empty dominating set with cardinality two.

F. Theorem:5

If S is intersection empty dominating set then V-S need not be dominating set.

G. Example:6

In, C_8 , Let {1,2,3,4,5,6,7,8} be the vertices of C_8.

Now, S={1,4,7,8} V-S={2,3,5,6}

{1,4,7,8} is intersection empty dominating set.

{2,3,5,6} is not dominating set.

Obseravation:7

$$1 \le \gamma \le \gamma_{ied} \le p$$

H. Theorem:8

If S is an intersection empty dominating set then either (V-S) or (V-S)∪{u},u∈S such that atleast one neighbourhood $N(u) \in S$ is dominating set for G.

H. Theorem:9

$$\gamma + \gamma_{ied} \le p + 1$$

1) Proof: Let S be the intersection empty dominating set.

Obviously, $p = |S| + |V - S|$

Case(i) If (V-S) is an dominating set for G.

Then $\gamma \le |V - S| \le p - |S|$

$$\gamma + \gamma_{ied} \le p$$

Case(ii) If (V-S) is not dominating set for G.

Then,(V-S)∪ $\{u\}$ where $u \in S$ such that $N(u) \in S$

$$\gamma \le |V - S| + 1$$

$$\gamma \leq p - \gamma_{ied} + 1$$
$$\gamma + \gamma_{ied} \leq p + 1$$

I. Theorem:10

Every connected graph G has atleast one independent empty dominating set.

1) Proof: Let G be connected graph.
Clearly,The vertex set V(G) forms a dominating set for G, Let it be S.
Moreover,V-S=Φ
Henceforth, $N(u) \cap N(v) \cap (V-S) = \emptyset$ for every $u, v \in S$
Therefore, V(G) is also a independent empty dominating set.

J. Theorem:11

For any p, $\gamma_{ied}(W_p) = p$

1) Proof: Let S be minimal independent empty dominating set with p-1 vertices.
For this case, $|V-S| = 1$
Suppose $u \in V-S$ is center vertex of W_p
Since,every outer vertex is adjacent to centre vertex.
Therefore,for every $u \in S, v \in S, N(u) \cap N(v) \cap (V-S)$ has a centre vertex as common element.
Case(ii) Suppose $v \in V-S$ is a vertex belonging to outer vertex in W_p
We observe that the degree of every outer vertex is three.
Then v is adjacent to two vertices, except centre vertex,take them as v_1, v_2.
Since $v_1, v_2 \in S$, Both $N(v_1)$ and $N(v_2)$ contain the vertex v.
Now, $N(v_1) \cap N(v_2) \cap (V-S) = \{v\}$
Thus in all cases, we get common element which is a contradiction to S is a minimal independent empty dominating set with cardinality p-1.
Hence $\gamma_{ied} \geq p$
By observation[7] ,$\gamma_{ied} \leq p$
Hence $\gamma_{ied}(W_p) = p$

K. Theorem:12

$$\gamma_{ied}(K_p) = p$$

Clearly, $\gamma_{ied}(K_p) \leq p$
Take $v_1, v_2, \dots, v_p$ be the vertices of K_p
Consider, S={$v_1, v_2, \dots, v_{p-1}$} is the set of all collection of vertices of K_p
Hence V-S={v_p} and S produces a dominating set for the graph.
With the reason, K_p is complete
Each vertex of K_p has degree p-1.
Moreover for every vertex $u, v \epsilon s, N(u)$ *and* $N(v)$ contains v_p as one of the vertex.
This yields $N(u) \cap N(v) \cap (V-S)$ is non empty set
Hence $\gamma_{ied} \geq p$
Therefore, $\gamma_{ied}(K_p) = p$
Ă has the property that every non pendant vertices are adjacent to atleast one pendant vertex.

L. Theorem:13

For any tree TϵĂ then $\gamma_{ied} = \gamma$

1) Proof: Obviously , $\gamma_{ied} \geq \gamma$

It is enough to prove that $\gamma_{ied} \leq \gamma$

Let N be the set of all non pendant vertices of T with cardinality $p - l$

For some $u, v \epsilon V$ is such that u and v are adjacent
Now, $N(u) \cap N(v) \cap (V - S)$ does not contain common element
Since $N(u)$ and $N(v)$ contain common element then T contains a cycle which is a contradiction to tree.
Hence $N(u) \cap N(v) \cap (V - S) = \emptyset$
Hence N also form a independent empty dominating set for T.
Also,clearly, N is a minimal dominating set for T.

$$\gamma_{ied} \leq |N| = \gamma$$

$$\gamma_{ied} \leq \gamma$$

Therefore,$\gamma_{ied} = \gamma$

M. Theorem:14
For any tree T, $\gamma_{ied} \leq p - l$.Equality holds if and only if either T is isomorphic to bistar or All probable non pendant vertices of T are adjacent to no less than one pendant vertex of T.
1) Proof:
Let L be the set of all pendant vertices in T with $|L| = l$ and S be the set of all non pendant vertices of T.
Then S=V-L frame a dominating set for T.
Hence $\gamma \leq |V - L| = p - l$
By theorem[13], $\gamma_{ied} \leq p - l$
Be of the opinion, $\gamma_{ied} = p - l$
Whenever diam T=3,
T is isomorphic to bistar accompanied by $\gamma_{ied} = p - l$
Presuppose, diam T >3
Let u be the such vertex which is not adjacent to pendant vertex in T.
Case(i) No less than one element of N(u) may adjacent to not less than one pendant vertex say v
Then u could be dominated by $v\epsilon s$
Hence S-{u} frame a independent empty dominating set with $p - l - 1$ vertices which is a contradiction to $\gamma_{ied} = p - l$
Case(ii) Suppose no less than one element of N(u) is adjacent to only non pendant vertices say k
Then u could be dominated by $k\epsilon S$
Now,Abolish u from S frame a independent empty dominating set with the condition that $\gamma_{ied} = p - l - 1$ vertices which gives a contradiction to $\gamma_{ied} = p - l$
Converse is obvious.

N. Theorem:15
For a tree TϵĂ then $\gamma_{ied} \leq \frac{p}{2}$.Equality holds if and only if $p = 2l$
1) Proof: Let L be non pendant vertices of T with $|L| = l$
Since L forms a independent empty dominating set
Then one could construct two independent empty dominating sets V-L and L using any one of the vertices in T.
Since $\gamma_{ied} \leq \min\{ |L|, |V - L|\} \leq \min\{l, p - l\}$
We obtain $\gamma_{ied} \leq \frac{p}{2}$
Now,Assume $p = 2l$

$$\gamma_{ied} \leq \min\{l, p - l\} \leq \min\left\{\frac{p}{2}, p - \frac{p}{2}\right\} = \frac{p}{2}$$

Now, By abolish one vertex , say t, from L,it will collapse the independent empty dominating set property in L.
Hence L-{t} can not be independent empty dominating set for T.
Therefore, $\gamma_{ied} \geq l$

$\gamma_{ied} \geq \frac{p}{2}$ which gives $\gamma_{ied} = \frac{p}{2}$
Conversely, Assume $\gamma_{ied} = \frac{p}{2}$

Since $T \epsilon$ Ă
By Theorem[14], $\gamma_{ied} \leq p - l$
$\frac{p}{2} \leq p - l$, $\frac{p}{2} - p \leq -l$, $p \leq 2l$
Since $T \epsilon$ Ă ,clearly, $l \leq p - l, p \geq 2l$
This yields a result that $p = 2l$.

O. Theorem:16

For a connected graph G, $\gamma_{ied} \leq p$
1) Proof: Since the entire vertex set of G is a dominating set for G.
Also ,$N(u) \cap N(v) \cap (V - S) = \emptyset \; for \; every \; u, v \; \epsilon V$
Clearly,V(G) forms a independent empty dominating set for G

$$\gamma_{ied} \leq |V(G)|$$

$$\gamma_{ied} \leq p$$

P. Theorem:17

Let $K_{m,n}$ (m=n) be complete bipartite graph.Then $\gamma_{ied}(K_{m,n}) = 2m$
Truly, $\gamma_{ied} \leq 2m$
Let S be the minimal independent empty dominating set with 2m-1 vertices
Theorefore, V-S surely contains single element say {t}
Obviously degree of t should be greater than or equal to two
Let $u_1, u_2 \epsilon S$ be the vertex which are adjacent the vertex t.
Easily we could tell that, for $u_1, u_2 \epsilon S$, $N(u_1) \cap N(u_2) \cap (V - S) = \{t\}$ which takes into contradiction to S is the minimal independent dominating set with 2m-1 vertices.

$$\gamma_{ied} \geq 2m$$

Hence $\gamma_{ied} = 2m$ for complete bipartite graph with m=n.

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