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Some characterization on Operations of Anti Fuzzy Graphs

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Abstract: In this paper, we introduce the concept of operations on anti-fuzzy graph such as anti-union, anti-join, anti cartesian product and anti-composition. We applied such operations on some types of anti-fuzzy graphs and obtained the results on them. Anti-complement of anti-fuzzy graph applied on operations on anti-fuzzy graphs and obtained some relations on them.

Keywords: Anti fuzzy graph, Graph operations, vertex degrees.

Mathematical Classification: 05C72, 05C76, 05C07.

I. INTRODUCTION

The concept of fuzzy graph was first introduced by Kaufmann [1] from the fuzzy relation introduced by Zedah [2]. Although Rosenfield [3] introduced another elaborated definition, including fuzzy vertex and fuzzy edge, and also introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Yeh and Bang [4] have also introduced various connected concepts in fuzzy graphs. J.N. Mordeson and P.S. Nair [5] introduced the concept of operations on fuzzy graphs but this concept was extended by M.S. Sunitha and A.Vijayakumar [6]. R. Muthuraj and A. Sasireka [7] introduced the concept of anti fuzzy graph. R. Seethalakshmi and R.B. Gnanajothi[8] discussed the concept some operations such as union and join on anti fuzzy graphs. In this paper, we introduce the concept of operations on anti fuzzy graphs such as anti union, anti join, anti cartesian product and anti composition. The relationship between the anti complement of anti fuzzy graphs on operations on anti fuzzy graphs are discussed. We derived some theorems and results on them.

II. PRELIMINARIES

In this section, basic concepts of an znti fuzzy graph are discussed. Notations and more formal definitions which are followed as in [7] [8].

A. *Definition 2.1 [7]*

A fuzzy graph $G = (\sigma, \mu)$ is said to be an anti fuzzy graph with a pair of functions $\sigma : V \to [0,1]$ and $\mu : V\times V \to [0,1]$, where for all u, $v \in V$, we have $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_A(\sigma, \mu)$.

1) *Note :* μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. i.e., undirected anti fuzzy graphs are only considered.

B. *Definition 2.2 [7]*

An anti fuzzy graph $H_A = (\tau,\rho)$ is called an anti fuzzy subgraph of $G_A(\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u,v) \leq \mu(u,v)$ for all $u,v \in V$.

C. *Definition 2.3 [7]*

An anti fuzzy subgraph $H_A=(\tau,\rho)$ is called an spanning anti fuzzy subgraph of $G_A(\sigma, \mu)$ if $\tau(u) = \sigma(u)$ for all $u \in V$.

D. *Definition 2.4 [8]*

The order p and size q of an anti fuzzy graph $G_A = (V, \sigma, \mu)$ are defined to be $p = \sum_{x\in V}\sigma(x)$ and $q = \sum_{xy\in V}\mu(xy)$.

1) Notation: Without loss of generality let us simply use the letter G_A to denote an anti fuzzy graph.

E. Definition 2.5 [7]

Two vertices u and v in G_A are called adjacent if (½)[σ(u) ∨ σ(v)] ≥ μ(u,v).

F. Definition 2.6 [7]

The underlying anti crisp graph of an anti fuzzy graph $G_A = (\sigma, \mu)$ is denoted by $G_A^* = (\sigma^*,\mu^*)$, where $\sigma^*=\{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V / \mu(u, v) > 0\}$.

G. Definition 2.7 [7]

An anti fuzzy graph $G_A = (\sigma, \mu)$ is a strong anti fuzzy graph of $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v)\in\mu^*$ and is a complete anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u, v) \in \sigma^*$. Two nodes u and v are said to be neighbors if μ (u, v) > 0.

H. Definition 2.8 [7]

u is a node in an anti fuzzy graph G_A then N(u) = {v: (u, v) is a strong arc} is called the open neighborhood of u and N[u] = N(u) ∪{u} is called closed neighborhood of u.

I. Definition 2.9 [7]

The strong neighborhood of an edge e_i in anti fuzzy graph G_A is $N_s(e_i) = \{e_j \in E(G_A) / e_j$ is a strong arc in G_A and adjacent to $e_i\}$.

J. Definition 2.10 [7]

The anti complement of anti fuzzy graph $G_A(\sigma,\mu)$ is an anti fuzzy graph $\overline{G_A} = (\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} = \sigma$ and $\bar{\mu}(u,v) = \mu(u,v) - (\sigma(u) \vee \sigma(v))$ for all u,v in V. and the anti complement of $\overline{G_A}$ is denoted by $\overline{\overline{G_A}} = (\bar{\bar{\sigma}}, \bar{\bar{\mu}})$ where $\bar{\bar{\sigma}} = \sigma$ and $\bar{\bar{\mu}}(u,v) = \mu(u,v) + (\sigma(u) \vee \sigma(v))$.

K. Definition 2.11 [7]

A path P in an anti fuzzy graph is a sequence of distinct vertices $u_0,u_1,u_2,\ldots,u_n$ such that $\mu(u_{i-1},u_i)>0$, $1 \le i \le n$. here h ≥ 0 is called the length of the path P. the consecutive pairs (u_{i-1}, u_i) are called the edges of the path.

L. Definition 2.12 [7]

A cycle in G_A is said to be an anti fuzzy cycle if it contains more than one weakest arc.

III. OPERATIONS ON ANTI FUZZY GRAPHS

In this section, the binary operations on crisp anti fuzzy graphs such as anti union, anti join, anti cartesian product and anti composition are discussed. Let $G_{A_1}(\sigma_1, \mu_1)$ and $G_{A_2}(\sigma_2, \mu_2)$ be two anti fuzzy graphs with the underlying crisp anti fuzzy graphs $G^*_{A_1} = (V_1, E_1)$ $G^*_{A_2} = (V_2, E_2)$ respectively. Some of their properties are studied.

A. Definition 3.1

Let $G_{A_1}(\sigma_1, \mu_1)$ and $G_{A_2}(\sigma_2, \mu_2)$ be two anti fuzzy graphs with $G^*_{A_1}$ and $G^*_{A_2}$. Let $G^*_A = G^*_{A_1} \cup G^*_{A_2} = (V_1\cup V_2, E_1\cup E_2)$ be an anti union of anti fuzzy graph of $G^*_{A_1}$ and $G^*_{A_2}$ and the union of two anti fuzzy graphs G_1 and G_2 is an anti fuzzy graph $G_A=G_{A_1}\cup G_{A_2}$:$(\sigma_1\cup\sigma_2, \mu_1\cup\mu_2)$ is defined by

$$\text{If } V_1\cap V_2=\emptyset \text{ then } (\sigma_1\cup\sigma_2)(u) = \begin{cases}\sigma_1(u) if\ u \in V_1 - V_2\\ \sigma_2(u) if\ u \in V_2 - V_1\end{cases}$$

$$(\mu_1\cup\mu_2)(uv) = \begin{cases}\mu_1(u,v) if\ uv \in E_1 - E_2\\ \mu_2(u,v) if\ uv \in E_2 - E_1\end{cases}$$

and If $V_1\cap V_2\neq\emptyset$ then $(\sigma_1\cup\sigma_2)(u) = \max\{\sigma_1(u), \sigma_2(u)\}$ if $u\in V_1\cap V_2$.

$$(\mu_1\cup\mu_2)(u,v) = \max\{\mu_1(u,v), \mu_2(u,v)\} \text{ if } (u,v)\in E_1\cap E_2.$$

B. Example 3.2

The following anti fuzzy graphs illustrate anti union of $G_{A_1}(\sigma_1, \mu_1)$ and $G_{A_2}(\sigma_2, \mu_2)$. Here Fig.(3) represents an anti union of anti fuzzy graphs of Fig.1 and Fig.2

If $V_1\cap V_2=\emptyset$

G_{A_1}

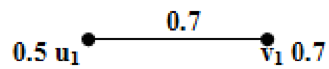

Fig.1 Anti fuzzy graph

G_{A_2}

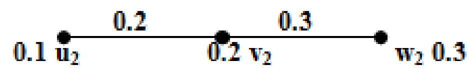

Fig.2 Anti fuzzy graph

$G_{A_1} \cup G_{A_2}$

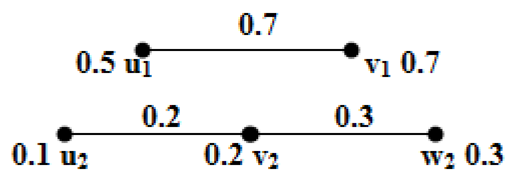

Fig.3 anti union of anti fuzzy graphs G_{A_1} and G_{A_2}

The following Fig.6 represents anti union of anti fuzzy graphs of Fig.4 and Fig.5

If $V_1 \cap V_2 \neq \varnothing$

G_{A_1}

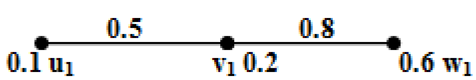

Fig.4 Anti fuzzy graph

G_{A_2}

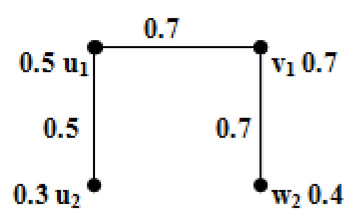

Fig.5 Anti fuzzy graph

$G_{A_1} \cup G_{A_2}$

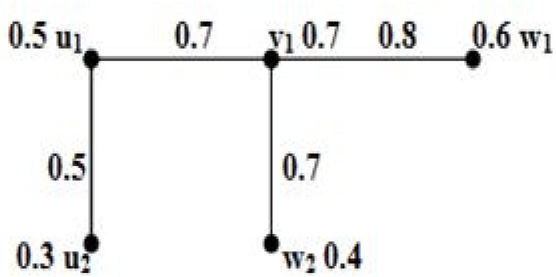

Fig.6 anti union of anti fuzzy graphs G_{A_1} and G_{A_2}

C. Definition 3.3

Consider an anti join $G_A^* = G_{A_1}^* + G_{A_2}^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$ of anti fuzzy graphs where E′ is the set of all edges joining the vertices of V_1 and V_2 where we assume that $V_1 \cap V_2 = \phi$. Then the anti join of anti fuzzy graphs G_{A_1} and G_{A_2} is an anti fuzzy graph $G_A = G_{A_1} + G_{A_2}$: $(\sigma_1+\sigma_2, \mu_1+\mu_2)$ is defined by

$(\sigma_1+\sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u)$ if $u \in V_1 \cup V_2$

$(\mu_1+\mu_2)(u, v) = (\mu_1 \cup \mu_2)(u, v)$ if $(u, v) \in E_1 \cup E_2$.

And $(\mu_1+\mu_2)(u, v) = \max\{(\sigma_1(u), \sigma_2(v)\}$ if $(u,v) \in E'$.

D. Example 3.4

The following anti fuzzy graphs illustrate an anti join of $G_{A_1}(\sigma_1, \mu_1)$ and $G_{A_2}(\sigma_2, \mu_2)$. Here Fig.9 represents the anti join of anti fuzzy graphs of Fig.7 and Fig.8

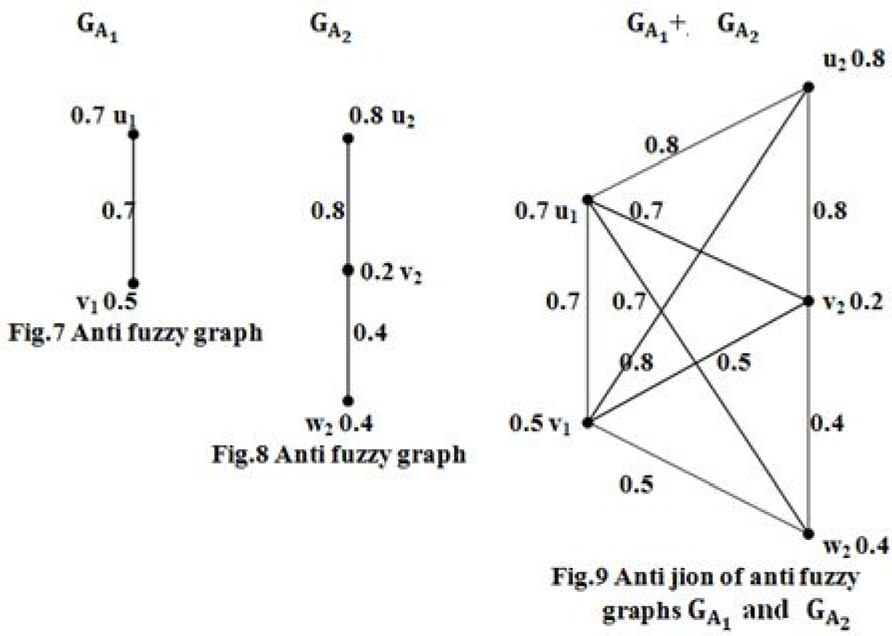

Fig.7 Anti fuzzy graph

Fig.8 Anti fuzzy graph

Fig.9 Anti jion of anti fuzzy graphs G_{A_1} and G_{A_2}

E. Theorem 3.5

Let $G_{A_1}(\sigma_1, \mu_1)$ and $G_{A_2}(\sigma_2, \mu_2)$ be two anti fuzzy graphs then

$$\overline{G_{A_1} + G_{A_2}} \approx \overline{G_1} \cup \overline{G_2}$$
$$\overline{G_{A_1} \cup G_{A_2}} \approx \overline{G_1} + \overline{G_2}$$

1) *Proof:* It is enough to prove that the identity map is isomorphism. Let $I : V_1 \cup V_2 \rightarrow V_1 \cup V_2$ be the identity map.

i. Here it is required to prove that $\overline{\sigma_1 + \sigma_2}(u) = \overline{\sigma_1}(u) + \overline{\sigma_2}(u)$

and $\overline{\mu_1 + \mu_2}(u, v) = \overline{\mu_1} \cup \overline{\mu_2}(u, v)$

By the definition of anti complement of anti fuzzy graph, $\overline{\sigma_1 + \sigma_2}(u) = \overline{\sigma_1}(u) + \overline{\sigma_2}(u)$

$$\overline{\mu_1 + \mu_2}(u, v) = (\mu_1+\mu_2)(u, v) - (\sigma_1+\sigma_2)(u) \vee (\sigma_1+\sigma_2)(v)$$

Consider, $\overline{\sigma_1 + \sigma_2}(u) = (\sigma_1+\sigma_2)(u)$

$$= \begin{cases} \sigma_1(u)\ if\ u \in V_1 \\ \sigma_2(u) if\ u \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\sigma_1}(u)\ if\ u \in V_1 \\ \overline{\sigma_2}\ (u) if\ u \in V_2 \end{cases}$$

$\overline{\mu_1 + \mu_2}(u, v) = (\mu_1+\mu_2)(u, v) - (\sigma_1+\sigma_2)(u) \vee (\sigma_1+\sigma_2)(v)$

$$= \begin{cases} (\mu_1 \cup \mu_2)(u, v) - (\sigma_1 + \sigma_2)(u) \vee (\sigma_1 + \sigma_2)(v)\ \ if\ (u, v) \in E_1 \cup E_2 \\ \sigma_1(u) \vee \sigma_2(v) - \ (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)) if\ (u, v) \in E' \end{cases}$$

$$= \begin{cases} \mu_1(u,v) - \sigma_1(u) \vee \sigma_1(v) & if\ (u,v) \in E_1 \\ \mu_2(u,v) - \sigma_2(u) \vee \sigma_2(v) & if\ (u,v) \in E_2 \\ \sigma_1(u) \vee \sigma_2(v) - (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)) & if\ (u,v) \in E'\ where\ u \in V_1, v \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_1}\,(u,v) & if\ (u,v) \in E_1 \\ \overline{\mu_2}\,(u,v) & if\ (u,v) \in E_2 \\ 0 & if\ (u,v) \in E' \end{cases}$$

$= (\overline{\mu_1} \cup \overline{\mu_2})$ (u, v)

Here it is required to prove that $\overline{\sigma_1 \cup \sigma_2}$ (u) = $(\overline{\sigma_1} + \overline{\sigma_2})(u)$

And $\overline{\mu_1 \cup \mu_2}\,(u,v) = \overline{\mu_1} + \overline{\mu_2}$ (u, v)

By the definition of anti complement of anti fuzzy graph, $\overline{\sigma_1 \cup \sigma_2}$ (u) = $(\sigma_1 \cup \sigma_2)(u)$

$$= \begin{cases} \sigma_1(u)\ if\ u \in V_1 \\ \sigma_2(u) if\ u \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\sigma_1}(u)\ if\ u \in V_1 \\ \overline{\sigma_2}\,(u) if\ u \in V_2 \end{cases}$$

$= (\overline{\sigma_1} \cup \overline{\sigma_2})(u)$

$= (\overline{\sigma_1} + \overline{\sigma_2})(u)$

$\overline{\mu_1 \cup \mu_2}\,(u,v)$ = $(\mu_1 \cup \mu_2)$ (u, v) – $(\sigma_1 \cup \sigma_2)$ (u) $\vee$ $(\sigma_1 \cup \sigma_2)$ (v)

$$= \begin{cases} \mu_1(u,v) - \sigma_1(u) \vee \sigma_1(v) & if\ (u,v) \in E_1 \\ \mu_2(u,v) - \sigma_2(u) \vee \sigma_2(v) & if\ (u,v) \in E_2 \\ \sigma_1(u) \vee \sigma_2(v) & if\ u \in V_1,\ v \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_1}\,(u,v) & if\ (u,v) \in E_1 \\ \overline{\mu_2}\,(u,v) & if\ (u,v) \in E_2 \\ \sigma_1(u) \vee \sigma_2(v) & if\ u \in V_1,\ v \in V_2 \end{cases}$$

$$= \begin{cases} (\overline{\mu_1} \cup \overline{\mu_2})(u,v) & if\ (u,\ v) \in E_1\ or\ E_2 \\ \sigma_1(u) \vee \sigma_2(v) & if\ u \in V_1,\ v \in V_2 \end{cases}$$

$= (\overline{\mu_1} + \overline{\mu_2})$ (u, v)

F. Definition 3.6

Let $G_A^* = G_{A_1}^* \times G_{A_2}^*$ = (V, E′) be the anti cartesian product of anti fuzzy graphs where V= $V_1 \times V_2$ and E′ = {$(u_1,u_2),(u_1,v_2)$ / $u_1 \in V_1$, $(u_2,v_2) \in E_2$}$\cup${$(u_1,w_2),(v_1,w_2)$ / $w_2 \in V_2$, $(u_1,v_1) \in E_1$ }. Then the anti cartesian product of two anti fuzzy graphs, $G_A = G_{A_1} \times G_{A_2}$: $(\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ is an anti fuzzy graph and is defined by

$(\sigma_1 \times \sigma_2)(u_1, u_2)$ = max {$\sigma_1(u_1), \sigma_2(u_2)$} for all $(u_1, u_2) \in V$

$(\mu_1 \times \mu_2)((u_1, u_2), (u_1, v_2))$ = max{$\sigma_1(u_1), \mu_2(u_2, v_2)$} for all $u_1 \in V_1$ and $(u_2, v_2) \in E_2$

$(\mu_1 \times \mu_2)((u_1, w_2), (v_1, w_2))$ = max {$\sigma_2(w_2), \mu_1(u_1, v1)$} for all $w_2 \in V_2$ and $(u_1, v_1) \in E_1$,

Then the fuzzy graph $G_A = (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ is said to be the anti cartesian product of $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$.

G. Example 3.7

The following Fig.10 represents an anti cartesian product of anti fuzzy graphs of Fig.7 and Fig.8.

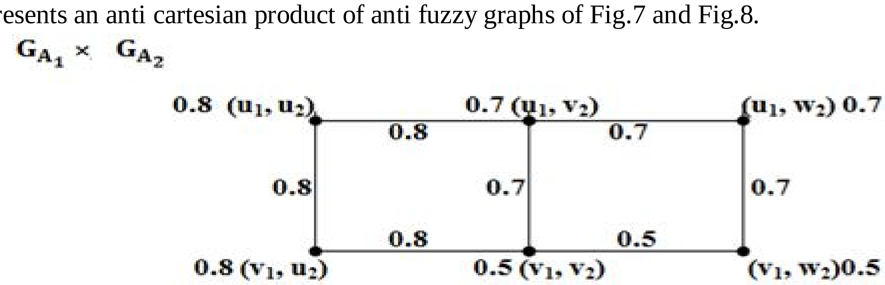

Fig.10 Anti cartesian product of anti fuzzy graphs G_{A_1} and G_{A_2}

H. Theorem 3.8

Let G_A be an anti cartesian product of anti fuzzy graphs G_{A_1} and G_{A_2} where $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$ then $G_A = (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ is an anti fuzzy graph.

Let us consider the two anti fuzzy graphs $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$ and G_A be an anti cartesian product of anti fuzzy graphs G_{A_1} and G_{A_2}. Then $G_A = G_{A_1} \times G_{A_2}$.

$(\mu_1 \times \mu_2)((u_1, u_2), (u_1, v_2)) = \max\{\sigma_1(u_1), \mu_2(u_2, v_2)\}$ for all $u_1 \in V_1$ and $(u_2, v_2) \in E_2$

$\leq \max\{\sigma_1(u_1), \max\{\sigma_2(u_2), \sigma_2(v_2)\}\}$

$= \max\{\sigma_1(u_1), \sigma_2(u_2)\}, \max\{\sigma_1(u_1), \sigma_2(v_2)\}$

$= \max\{(\sigma_1 \times \sigma_2)(u_1, u_2), (\sigma_1 \times \sigma_2)(u_1, v_2)\}$

$\therefore (\mu_1 \times \mu_2)((u_1, u_2), (u_1, v_2)) \leq \max\{(\sigma_1 \times \sigma_2)(u_1, u_2), (\sigma_1 \times \sigma_2)(u_1, v_2)\}$

Consider, $(\mu_1 \times \mu_2)((u_1, w_2), (v_1, w_2)) = \max\{\sigma_2(w_2), \mu_1(u_1, v_1)\}$ for all $w_2 \in V_2$ and $(u_1, v_1) \in E_1$,

$\leq \max\{\sigma_2(w_2), \max\{\sigma_1(u_1), \sigma_1(v_1)\}\}$

$= \max\{\sigma_1(u_1), \sigma_2(w_2)\}, \max\{\sigma_1(v_1), \sigma_2(w_2)\}$

$= \max\{(\sigma_1 \times \sigma_2)(u_1, w_2), (\sigma_1 \times \sigma_2)(v_1, w_2)\}$

$(\mu_1 \times \mu_2)((u_1, w_2), (v_1, w_2)) \leq \max\{(\sigma_1 \times \sigma_2)(u_1, w_2), (\sigma_1 \times \sigma_2)(v_1, w_2)\}$

$(\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ is an anti fuzzy graph.

I. Definition 3.9

Let $G_A^* = G_{A_1}^* \, o \, G_{A_2}^* = (V, E)$ be an anti composition of anti fuzzy graphs where $V = V_1 \times V_2$ and $E = \{(u_1,u_2),(u_1,v_2) / u_1 \in V_1, (u_2,v_2) \in E_2\} \cup \{(u_1,w_2),(v_1,w_2) / w_2 \in V_2, (u_1,v_1) \in E_1\} \cup \{(u_1,u_2), (v_1,v_2) / (u_1,v_1) \in E_1, u_2 \neq v_2\}$. Then the anti composition of anti fuzzy graphs, $G_A = G_{A_1} \, o \, G_{A_2}$: $(\sigma_1 o \sigma_2, \mu_1 o \mu_2)$ is an anti fuzzy graph and is defined by

$(\sigma_1 o \sigma_2)(u_1, u_2) = \max\{\sigma_1(u_1), \sigma_2(u_2)\}$ for all $(u_1, u_2) \in V$

$(\mu_1 o \mu_2)((u_1, u_2), (u_1, v_2)) = \max\{\sigma_1(u_1), \mu_2(u_2, v_2)\}$ for all $u_1 \in V_1$ and $(u_2, v_2) \in E_2$

$(\mu_1 o \mu_2)((u_1, w2), (v_1, w_2)) = \max\{\sigma_2(w_2), \mu_1(u_1, v_1)\}$ for all $w_2 \in V_2$ and $(u_1, v_1) \in E_1$,

$(\mu_1 o \mu_2)((u_1, u_2), (v_1, v_2)) = \max\{\sigma_2(u_2), \sigma(v_2), \mu_1(u_1,v_1)\}$ for all $(u_1,u_2)(v_1,v_2) \in E - E''$.

Where $E'' = \{(u_1,u_2),(v_1,v_2) / u_1 \in V_1, (u_2,v_2) \in E_2\} \cup \{(u_1,w_2),(v_1,w_2) / w_2 \in V_2, (u_1,v_1) \in E_1\}$

J. Theorem 3.10

Let $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$ be two anti fuzzy graphs then $G_A = G_{A_1} \, o \, G_{A_2}$: $(\sigma_1 \, o \, \sigma_2, \mu_1 \, o \, \mu_2)$ is a strong anti fuzzy graph and $\overline{G_{A_1} \, o \, G_{A_2}} = \overline{G_{A_1}} \, o \, \overline{G_{A_2}}$.

1) Proof: Let us consider $G_A(\sigma, \mu) = G_{A_1} \, o \, G_{A_2}$ be an anti fuzzy graph where $\sigma = \sigma_1 \, o \, \sigma_2$ and $\mu = \mu_1 \, o \, \mu_2$.

Now, $\mu((u_1, u_2), (u_1, v_2)) = \sigma_1(u_1) \vee \mu_2(u_2, v_2)$

$= \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2)$

$= (\sigma_1(u_1) \vee \sigma_2(u_2)) \vee (\sigma_1(u_1) \vee \sigma_2(v_2))$

{Since G_{A_2} is strong anti fuzzy graph}

$= (\sigma_1 \, o \, \sigma_2)(u_1, u_2) \vee (\sigma_1 \, o \, \sigma_2)(u_1, v_2)$

$= \sigma(u_1, u_2) \vee \sigma(u_1, v_2) \quad \longrightarrow (1)$

$\mu((u_1, w_2), (v_1, w_2)) = \sigma_2(w_2) \vee \mu_1(u_1, v_1)$

$= \sigma_2(w_2) \vee \sigma_1(u_1) \vee \sigma_1(v_1)$

$= (\sigma_2(w_2) \vee \sigma_1(u_1)) \vee (\sigma_2(w_2) \vee \sigma_1(v_1))$

{Since G_{A_1} is strong anti fuzzy graph}

$= (\sigma_1(u_1) \vee \sigma_2(w_2)) \vee (\sigma_1(v_1) \vee \sigma_2(w_2))$

$= (\sigma_1 \, o \, \sigma_2)(u_1, w_2) \vee (\sigma_1 \, o \, \sigma_2)(v_1, w_2)$

$= \sigma(u_1, w_2) \vee \sigma(v_1, w_2) \quad \longrightarrow (2)$

$\mu((u_1, u_2), (v_1, v_2)) = \sigma_2(u_2) \vee \sigma_2(v_2) \vee \mu_1(u_1, v_1)$

$= \sigma_2(u_2) \vee \sigma_2(v_2) \vee \sigma_1(u_1) \vee \sigma_1(v_1)$ {since G_{A_1} is strong anti fuzzy graph}

$= [\sigma_1(u_1) \vee \sigma_2(u_2)] \vee [\sigma_1(v_1) \vee \sigma_2(v_2)]$

$= (\sigma_1 \, o \, \sigma_2)(u_1, u_2) \vee (\sigma_1 \, o \, \sigma_2)(v_1, v_2)$

$= \sigma(u_1, u_2) \vee \sigma(u_1, v_2)$ ⟶ (3)

From (1), (2) and (3), we get that G_A is a strong anti fuzzy graph.

Now to prove that $\overline{G_{A_1} o\, G_{A_2}} = \overline{G_{A_1}}\, o\, \overline{G_{A_2}}$

Let $\overline{G_A}(\sigma, \overline{\mu}) = \overline{G_{A_1} o\, G_{A_2}}$ $(\sigma_1 o\, \sigma_2, \overline{\mu_1 \; o\, \mu_2})$ and

$\overline{G_{A_1}}: (\sigma_1, \overline{\mu_1})$ o $\overline{G_{A_2}}: (\sigma_2, \overline{\mu_2}) = \overline{G_{A_1}}\, o\, \overline{G_{A_2}}$ $(\sigma_1 o\, \sigma_2, \overline{\mu_1}\, o\, \overline{\mu_2})$

From the definition of anti composition of anti fuzzy graphs G_A, the presence of edges in G_A, depending upon this we classify the following cases.

Case 1 us consider $e = ((u_1,u_2),(u_1,v_2))$ for all $(u_2,v_2) \in E_2$. Then $e \in E$ and G_A is a strong anti fuzzy graph. Therefore, $\overline{\mu}(e) = 0$. Also, $\overline{\mu_1}\, o\, \overline{\mu_2} = 0$ since $(u_2, v_2) = \overline{E_2}$.

If $u_2 \neq v_2$ then $\bar{\mu}(e) = \sigma(u_1, u_2) \vee \sigma(u_1, v_2)$

$= \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_1(u_1) \vee \sigma_2(v_2)$

$= \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2)$

$= \sigma_1(u_1) \vee \mu_2(u_2, v_2)$ ⟶ (4)

But $(u_2, v_2) \in \overline{E_2}$ we have $\overline{\mu_1}\, o\, \overline{\mu_2}\,(e) = \sigma_1(u_1) \vee \overline{\mu_2}\,(u_2, v_2)$

$= \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2)$

{Since G_{A_2} is strong anti fuzzy graph}

$= \bar{\mu}(e)$ {by (4)}

Case 2 Let us consider $e = ((u_1, w_2), (v_1, w_2))$ for all $(u_1, v_1) \in E_1$. Then $e \in E$ and G_A is a strong anti fuzzy graph. Therefore, $\bar{\mu}(e) = 0$ as in case 1. Since $(u_1, v_1) \notin E_1$. We have $\overline{\mu_1}\, o\, \overline{\mu_2} = 0$.

Case 3:

Let us consider $e = ((u_1, w_2),(v_1,w_2))$ for all $(u_1,v_1) \notin E_1$. Then $e \notin E$ and G_A is a strong anti fuzzy graph. Hence $\mu(e) = 0$ and $\bar{\mu}(e) = \sigma(u_1, w_2) \vee \sigma(v_1, w_2)$

$= (\sigma_1(u_1) \vee \sigma_2(w_2)) \vee (\sigma_1(v_1) \vee \sigma_2(w_2))$

$= \sigma_1(u_1) \vee \sigma_2(w_2) \vee (\sigma_1(v_1)$

{Since $(u_1, v_1) \in \overline{E_1}$ }

$\bar{\mu}(e) = \sigma_1(u_1) \vee (\sigma_1(v_1) \vee \sigma_2(w_2)$ ⟶ (5)

But $\overline{\mu_1}\, o\, \overline{\mu_2}\,(e) = \mu_1(u_1, v_1) \vee \sigma_2(w_2)$

$= \sigma_1(u_1) \vee (\sigma_1(v_1) \vee \sigma_2(w_2)$ {since G_{A_1} is strong anti fuzzy graph}

$= \bar{\mu}(e)$ {by (5)}

$\therefore \overline{\mu_1}\, o\, \overline{\mu_2}\,(e) = \bar{\mu}(e)$

Case 4:

Let us consider $e = ((u_1, u_2),(v_1,v_2))$ for all $(u_1,v_1) \in E_1$, $u_2 \neq v_2$.

If $e \in E$ then $\bar{\mu}(e) = 0$ as in case (1), therefore $\overline{\mu_1}\, o\, \overline{\mu_2}\,(e) = 0$ since $(u_1,v_1) \in \overline{E_1}$

If $e \neq E$ then $\mu(e) = 0$, $\bar{\mu}(e) = \sigma(u_1,u_2) \vee \sigma(v_1,v_2)$

$\bar{\mu}(e) = \sigma_1(u_1) \vee \sigma_1(v_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2)$ ⟶ (6)

Since $(u_1, v_1) \in \overline{E_1}$, we have

$\overline{\mu_1}\, o\, \overline{\mu_2}\,(e) = \overline{\mu_1}\,(u_1, v_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2)$

$= \sigma_1(u_1) \vee \sigma_1(v_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2)$

$= \bar{\mu}(e)$ {by (6)}

If $e = ((u_1, u_2), (v_1, v_2))$ and $(u_1, v_1) \notin E_1$, $(u_2, v_2) \notin E_2$ then $e \notin E$ hence $\mu(e)=0$.

$\bar{\mu}(e) = \sigma(u_1, u_2) \vee \sigma(v_1, v_2)$

$= \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_1(v_1) \vee \sigma_2(v_2)$

But $(u_1, v_1) \notin E_1$ then $(u_1, v_1) \in \overline{E_1}$ and $u_2=v_2=w_2$ then we have case 3. If $(u_1, v_1) \in \overline{E_1}$ and $u_2 \neq v_2$ then we have case 4(i). Therefore, from case (1) to (4), we get $\overline{G_{A_1} o\, G_{A_2}} = \overline{G_{A_1}}\, o\, \overline{G_{A_2}}$

K. Remark

Suppose $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$ are not strong anti fuzzy graphs $\overline{G_{A_1} o\, G_{A_2}} \neq \overline{G_{A_1}}\, o\, \overline{G_{A_2}}$.

L. Example 3.11

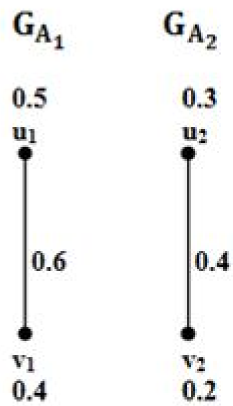

Fig.11 Anti fuzzy graphs

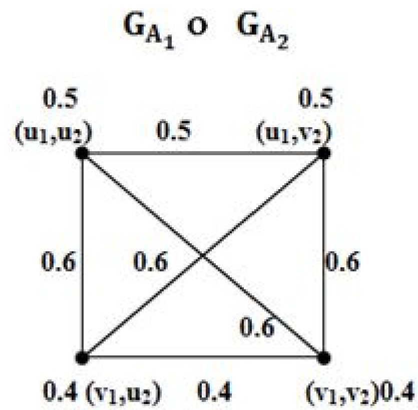

Fig.12 Anti composition of anti fuzzy graphs of Fig.11

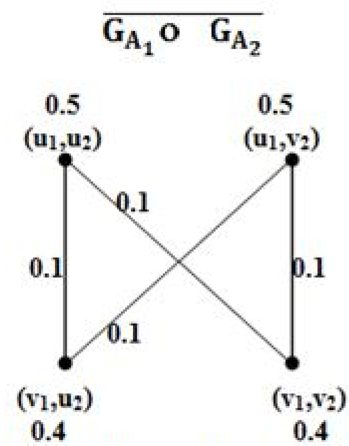

Fig.13 Anti complement of anti fuzzy graph of Fig.12

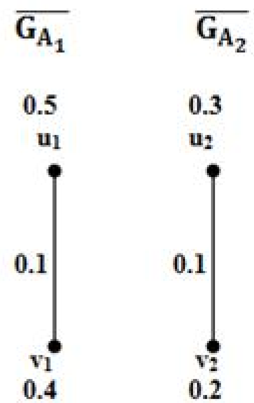

Fig.14 Anti complement of Anti fuzzy graphs of Fig.11

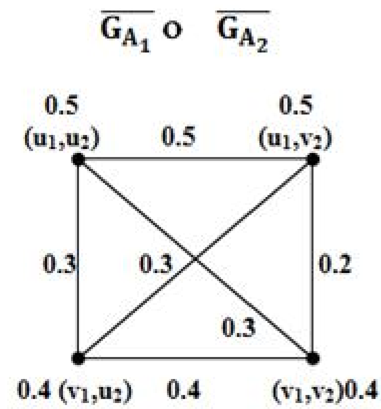

Fig.15 Anti composition of Anti fuzzy graphs of Fig.14

From Fig.13 and Fig.15, we observe that $\overline{G_{A_1} o\, G_{A_2}} \neq \overline{G_{A_1}}\, o\, \overline{G_{A_2}}$ because the anti fuzzy graphs G_{A_1} and G_{A_2} are not strong anti fuzzy graphs.

V. CONCLUSION

The concept of operations applied on anti fuzzy graphs such as anti union, anti join, anti cartesian product and anti composition. Some theorems and results are obtained on them. Anti complement of anti fuzzy graph applied on operations on anti fuzzy graphs. The relationships between the resulting graphs are discussed and derived some theorems on them.

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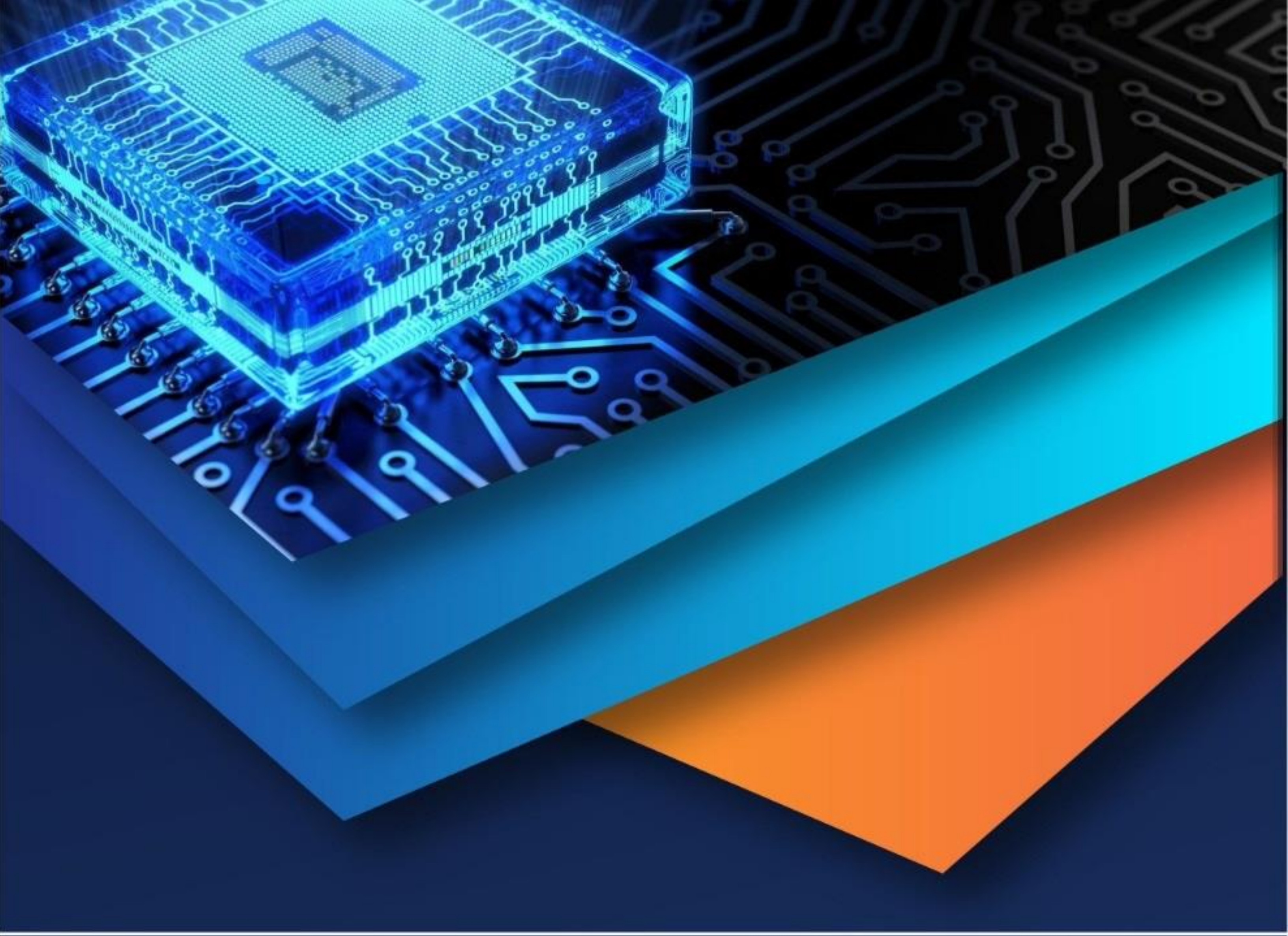

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