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# ON (k,d) – HERONIAN MEAN GRAPHS

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***Abstract: Mean labeling was first introduced by S. Somasundram and R. Ponraj. Heronian mean labeling was introduced by S. S. Sandhya, E. Ebin Raja Merly and S. D. Deepa. We have extended this notion to a labeling called k-Heronian mean labeling. In this paper, we introduce (k,d)-Heronian mean labeling of some graphs. Here k and d are denoted as any positive integer greater than or equal to 1.***
***Keywords: (k,d)-Heronian mean labeling, (k,d)-Heronian mean graph, Path, $L_n \odot K_1$, $T_n \odot K_1$, $Q_n \odot K_1$, $TL_n \odot K_1$, $H_n$ , Peterson graph.***

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## I. INTRODUCTION

We begin with simple, finite, connected and undirected graph G (V, E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian [2]. Terms are not defined here are used in the sense of Harary [3]. S. Somasundram and R. Ponraj were introduced mean labeling of graphs in [6] [7]. The concept of Heronian mean labeling was introduced by S.S. Sandhya et.al [4] [5].

We introduced the concept of k-Heronian mean labeling in [1]. In this paper, we introduce and investigate (k, d)-Heronian mean labeling of some graphs. For brevity, we use (k,d)-HML for (k,d)-Heronian mean labeling and k and d are any positive integer greater than or equal to 1.

### A. *Definiton 1.1*

A graph G=(V,E) with p vertices and q edges is said to be a (k,d)-Heronian Mean graph if it is possible to label the vertices$x \in V$with distinct labels $\mathbf{f(x)}$ from k, k+d, k+2d,…,k+qd in such a way that when each edge **e**=**uv** is labeled with, $f^*(e) = \left\lfloor \frac{f(u)+f(v)+\sqrt{f(u)f(v)}}{3} \right\rfloor$ or $\left\lceil \frac{f(u)+f(v)+\sqrt{f(u)f(v)}}{3} \right\rceil$, then the resulting edge labels are distinct. In this case **f** is called a (k,d)-Heronian Mean labeling of G.

### B. *Definition 1.2*

If G has order n, the corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i th vertex of G with an edge to every vertex in the i th copy of H

### C. *Definition 1.3*

A triangular snake ($T_n$) is obtained from a path by identifying each edge of the path with an edge of the cycle $C_3$.

### D. *Definition 1.4*

The H – graph of a path $P_n$ denoted by $H_n$ is the graph obtained from two copies of $P_n$ with vertices $v_1, v_2, \dots, v_n$ and $u_1, u_2, \dots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$; if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$; if n is even.

### E. *Definition 1.5*

A Triangular Ladder $T(L_n)$is a graph obtained from $L_n$by adding the edges $u_iv_{i+1}, 1 \le i \le n-1$, where $1 \le i \le n$ are the vertices of $L_n$ such that $u_1u_2u_3 \dots u_n$and$v_1v_2v_3 \dots v_n$ are two paths of length n in the graph $L_n$.

### F. *Definition 1.6*

The ladder graph $L_n$ , is obtained from the cartesian product of two path graphs.

## II. MAIN RESULTS

*A. Theorem 2.1*

Any path $P_n$ is a (k, d)-Heronian mean graph, for all $n \geq 2$.

Proof:

Let $V(P_n) = \{u_i; 1 \leq i \leq n\}$ and $E(P_n) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\}$ be the vertices and edges of $P_n$ respectively.

Define $f: V(P_n) \rightarrow \{k, k+d, k+2d, \ldots, k+(n-1)d\}$ by

$f(u_i) = k + d(i-1);\ 1 \leq i \leq n$.

Now the induced edge labels are

$f^*(e_i) = k + d(i-1);\ 1 \leq i \leq n-1$.

Here p = n and q = n-1.

Clearly, f is (k, d)-Heronian mean labeling of $P_n$.

Hence $P_n$ is a (k, d)-Heronian mean graph, for all $n \geq 2$.

*1) Example 2.2:* (50,2)-Heronian mean labeling of $P_5$ is given in the figure 2.1:

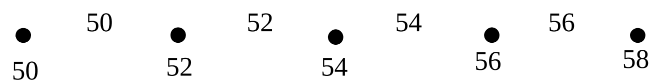


Figure 2.1: (50,2)-HML of $P_5$

*B. Theorem 2.3*

The graph $L_n \odot K_1$ is a (k,d)- Heronian mean labeling, for all $n \geq 2$.

Proof;

Let $V(L_n \odot K_1) = \{u_i, u_i', v_i, v_i'; 1 \leq i \leq n\}$ and

$$E(L_n \odot K_1) = \{e_i = (u_i, u_i'), \quad e_i' = (u_i', v_i'), \quad e_i'' = (v_i, v_i'); 1 \leq i \leq n\} \cup$$

$\{e_i''' = (u_i', u_{i+1}'), e_i^{iv} = (v_i', v_{i+1}'); 1 \leq i \leq n-1\}$ be the vertices and edges of $L_n \odot K_1$ respectively.

Define $f: V(L_n \odot K_1) \rightarrow \{k, k+d, \ldots, k+(5n-2)d\}$ by

$$f(u_1) = k$$
$$f(u_i) = k + d(5i-6);\ 2 \leq i \leq n$$
$$f(u_1') = k + d$$
$$f(u_i') = k + 5d(i-1);\ 2 \leq i \leq n$$
$$f(v_1') = k + 2d$$
$$f(v_i') = k + d(5i-4);\ 2 \leq i \leq n$$
$$f(v_i) = k + d(5i-2);\ 1 \leq i \leq n$$

Now the induced edge labels are

$$f^*(e_i) = k + 5d(i-1);\ 1 \leq i \leq n$$
$$f^*(e_i') = k + d(5i-4);\ 1 \leq i \leq n$$
$$f^*(e_i'') = k + d(5i-3);\ 1 \leq i \leq n$$
$$f^*(e_i''') = k + d(5i-2);\ 1 \leq i \leq n-1$$
$$f^*(e_i^{iv}) = k + d(5i-1);\ 1 \leq i \leq n-1$$

Here p = 4n and q = 5n-2.

Clearly f is a (k,d)- Hernonian mean labeling.

Hence $L_n \odot K_1$ is a (k,d)-Heronian mean graph for all $n \geq 2$.

*1) Example 2.4*

(75,2) – Heronian mean labeling of $\square_7 \odot \square_1$ is given in figure 2.2:

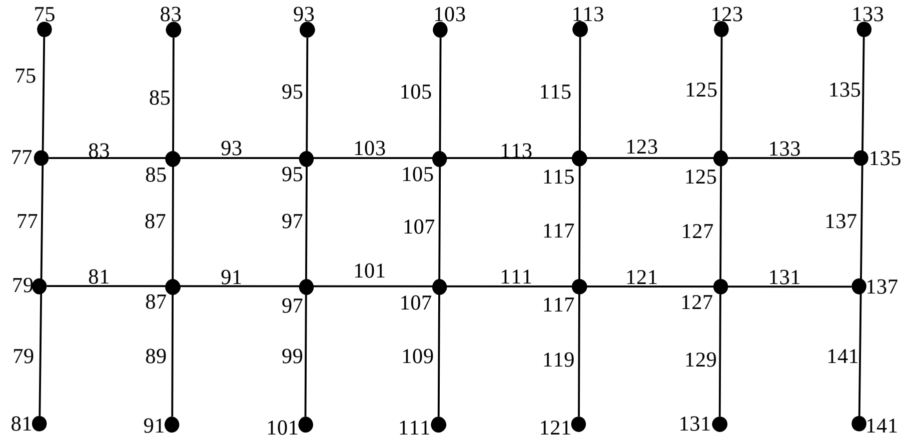


Figure 2.2: (75, 2)-HML of $L_7 \odot K_1$

*C. Theorem 2.5*

The graph $T_n \odot K_1$ is a (k,d)- Heronian mean labeling, for all $n \geq 2$.

Proof;

Let $V(T_n \odot K_1) = \{u_i, v_i; 1 \leq i \leq n\} \cup \{w_i, x_i; 1 \leq i \leq n-1\}$ and

$$E(T_n \odot K_1) = \{e_i = (u_i, v_i), \quad 1 \leq i \leq n\} \cup$$

$$\{e_i' = (v_i, w_i), \quad e_i'' = (w_i, x_i), e_i''' = (v_i, v_{i+1}), e_i^{iv} = (w_i, x_i, ); 1 \leq i \leq n-1\}$$

be the vertices and edges of $T_n \odot K_1$ respectively.

Define $f: V(T_n \odot K_1) \to \{k, k+d, \dots, k+(5n-4)d\}$ by

$$f(u_i) = k + 5d(i-1); \; 1 \leq i \leq n$$

$$f(v_i) = k + d(5i-4); \; 1 \leq i \leq n$$

$$f(w_i) = k + d(5i-3); \; 1 \leq i \leq n-1$$

$$f(x_i) = k + d(5i-2); \; 1 \leq i \leq n-1$$

Now the induced edge labels are

$$f^*(e_i) = k + 5d(i-1); \; 1 \leq i \leq n$$

$$f^*(e_i') = k + d(5i-4); \; 1 \leq i \leq n-1$$

$$f^*(e_i'') = k + d(5i-3); \; 1 \leq i \leq n-1$$

$$f^*(e_i''') = k + d(5i-2); \; 1 \leq i \leq n-1$$

$$f^*(e_i^{iv}) = k + d(5i-1); \; 1 \leq i \leq n-1$$

Here p = 4n-2 and q = 5n-4.

Clearly f is a (k,d)- Hernonian mean labeling.

Hence $T_n \odot K_1$ is a (k,d)-Heronian mean graph for all $n \geq 2$.

1) *Example 2.6:* (150,3)-Heronian mean labeling of $T_6 \odot K_1$ is given in figure 2.3:

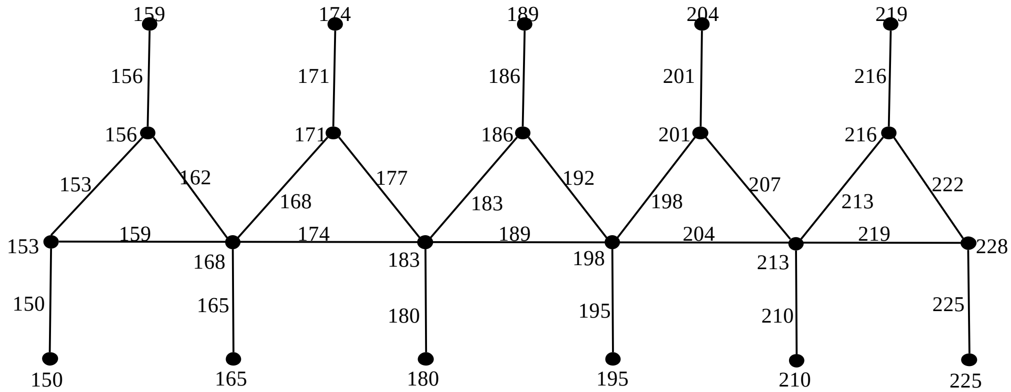


Figure 2.3: (150,3)-HML of $T_6 \odot K_1$

*D. Theorem 2.7*

The graph $Q_n \odot K_1$ is a (k,d)- Heronian mean labeling, for all $n \geq 2$.

Proof;

Let $V(Q_n \odot K_1) = \{u_i, v_i; 1 \leq i \leq n\} \cup \{w_i, x_i, w_i', x_i'; 1 \leq i \leq n-1\}$ and $E(Q_n \odot K_1) = \{e_i = (u_i, v_i),\ 1 \leq i \leq n\} \cup \{e_i' = (v_i, w_i),\ e_i'' = (w_i, x_i), e_i''' = (w_i, w_i'), e_i^{iv} = (w_i', x_i'), e_i^{v} = (v_i, v_{i+1}), e_i^{vi} = (w_i', x_i); 1 \leq i \leq n-1\}$ be the vertices and edges of $Q_n \odot K_1$ respectively.

Define $f: V(Q_n \odot K_1) \rightarrow \{k, k+d, \dots, k+(7n-6)d\}$ by

$$f(u_i) = k + 7d(i-1);\ \ 1 \leq i \leq n$$
$$f(v_i) = k + d(7i-6);\ 1 \leq i \leq n$$
$$f(w_i) = k + d(7i-5);\ 1 \leq i \leq n-1$$
$$f(x_i) = k + d(7i-4);\ \ 1 \leq i \leq n-1$$
$$f(w_i') = k + d(7i-3);\ 1 \leq i \leq n-1$$
$$f(x_i') = k + d(7i-2);\ 1 \leq i \leq n-1$$

Now the induced edge labels are

$$f^*(e_i) = k + 7d(i-1);\ 1 \leq i \leq n$$
$$f^*(e_i') = k + d(7i-6);\ 1 \leq i \leq n-1$$
$$f^*(e_i'') = k + d(7i-5);\ 1 \leq i \leq n-1$$
$$f^*(e_i''') = k + d(7i-4);\ 1 \leq i \leq n-1$$
$$f^*(e_i^{iv}) = k + d(7i-3);\ 1 \leq i \leq n-1$$
$$f^*(e_i^{v}) = k + d(7i-2);\ 1 \leq i \leq n-1$$
$$f^*(e_i^{vi}) = k + d(7i-1);\ 1 \leq i \leq n-1$$

Here p = 6n-4 and q = 7n-6.

Clearly f is a (k,d)- Hernonian mean labeling.

Hence $Q_n \odot K_1$ is a (k,d)-Heronian mean graph for all $n \geq 2$.

1) *Example 2.8:* (200,6)-Heronian mean labeling of $Q_4 \odot K_1$ is given in figure 2.4:

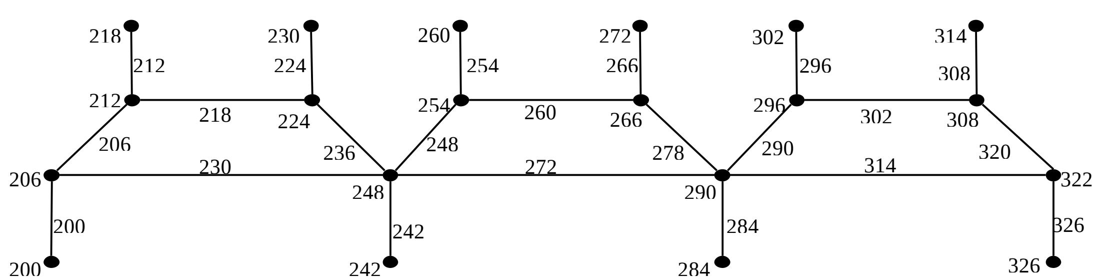


Figure 2.4: (200, 6)-HML of $Q_4 \odot K_1$

*E. Theorem 2.9*

The Total graph $TL_n \odot K_1$ is a (k,d)- Heronian mean labeling, for all $n \geq 2$.

Proof:

Let $V(TL_n \odot K_1) = \{u_i, v_i, w_i, x_i; 1 \leq i \leq n\}$ and

$$E(TL_n \odot K_1) = \{e_i = (u_i, v_i), e_i' = (v_i, w_i), \quad e_i'' = (w_i, x_i);\ 1 \leq i \leq n\} \cup \{e_i''' = (v_i, v_{i+1}), e_i^{iv} = (v_i, w_{i+1}), e_i^{v} = (w_i, w_{i+1}); 1 \leq i \leq n-1\}$$

be the vertices and edges of $TL_n \odot K_1$ respectively.

Define $f: V(TL_n \odot K_1) \rightarrow \{k, k+d, \dots, k+(6n-3)d\}$ by

$$f(u_i) = k + 6d(i-1);\ \ 1 \leq i \leq n$$
$$f(v_i) = k + d(6i-5);\ 1 \leq i \leq n$$
$$f(w_i) = k + d(6i-4);\ 1 \leq i \leq n$$
$$f(x_i) = k + d(6i-3);\ \ 1 \leq i \leq n$$

Now the induced edge labels are

$$f^*(e_i) = k + 6d(i-1);\ 1 \leq i \leq n$$
$$f^*(e_i') = k + d(6i-5);\ 1 \leq i \leq n$$
$$f^*(e_i'') = k + d(6i-4);\ 1 \leq i \leq n$$
$$f^*(e_i''') = k + d(6i-3);\ 1 \leq i \leq n-1$$
$$f^*(e_i^{iv}) = k + d(6i-2);\ 1 \leq i \leq n-1$$
$$f^*(e_i^{v}) = k + d(6i-1);\ 1 \leq i \leq n-1$$

Here p = 4n and q = 6n-3.

Clearly f is a (k,d)- Hernonian mean labeling.

Hence $TL_n \odot K_1$ is a (k,d)-Heronian mean graph for all $n \geq 2$.

*1) Example 2.10*

(185,7)-Heronian mean labeling of $TL_7 \odot K_1$ is given in figure 2.5:

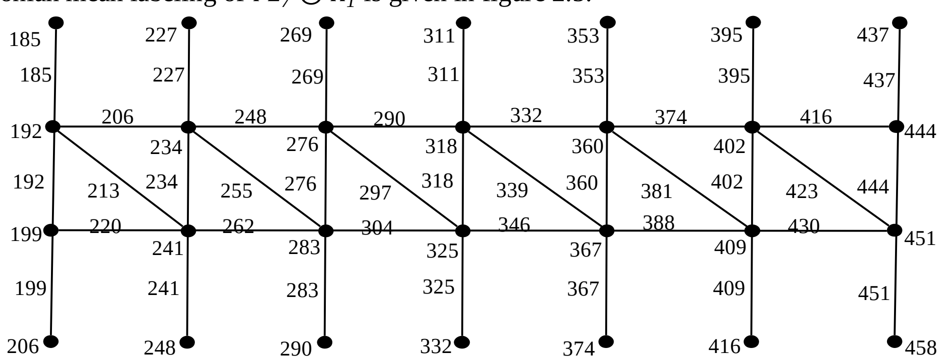


Figure 2.5: (185,7)-HML of $TL_7 \odot K_1$

*F. Theorem 2.11*

The graph $H_n$ is a (k,d)-Heronian mean graph for all $n \geq 2$.

Proof:

Let $V(H_n) = \{u_i, v_i; 1 \leq i \leq n\}$ and

$$E(H_n) = \{e_i = (u_i, u_{i+1}), e_i' = (v_i, v_{i+1});\ 1 \leq i \leq n-1\} \cup \{e\}$$

be the vertices and edges of $H_n$ respectively.

Define $f: V(H_n) \rightarrow \{k, k+d, \dots, k+(2n-1)d\}$ by

$$f(u_i) = k + d(i-1);\ \ 1 \leq i \leq n$$
$$f(v_i) = k + d(n+i-1);\ 1 \leq i \leq n$$

Now the induced edge labels are

$$f^*(e_i) = k + d(i-1);\ 1 \leq i \leq n-1$$
$$f^*(e_i') = k + d(n+i-1);\ 1 \leq i \leq n-1$$
$$f^*(e) = k + d(n-1)$$

Here p = 2n and q = 2n-1.

Clearly f is a (k,d)- Hernonian mean labeling.
Hence $H_n$ is a (k,d)-Heronian mean graph for all $n \geq 2$.

*1) Example 2.12:* (125,8)-Heronian mean labeling of $H_6$ is given in figure 2.6:

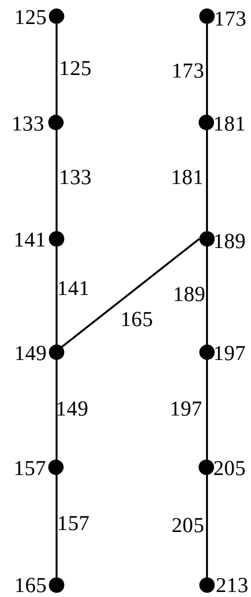


Figure 2.6: (125, 8)-HML of $H_6$

*G. Theorem 2.13*

The Peterson graph is a (k,d)-Heronian mean graph.

Proof:

Let G be Peterson Graph.

Let $V(G) = \{u_i, v_i; 1 \leq i \leq 5\}$ and $E(G) = \{e_1 = (u_1, u_3), e_2 = (u_3, u_5), e_3 = (u_5, u_2), e_4 = (u_2, u_4)\} \cup \{e_5 = (u_4, u_1)\} \cup \{e_i' = (v_i, v_{i+1}); 1 \leq i \leq 4\} \cup \{e_5' = (v_1, v_5)\} \cup \{e_i'' = (v_i, u_i)\ ; 1 \leq i \leq 5\}$ be the vertices and edges of G.

Define $f: V(G) \to \{k, k + d, \ldots, k + 15d\}$

$$f(u_i) = k + d(i - 1);\ \ 1 \leq i \leq 4$$
$$f(u_5) = k + 5d$$
$$f(v_i) = k + d(i + 9);\ 1 \leq i \leq 5$$

Now the induced edge labels are

$$f^*(e_i) = k + d(i - 1);\ 1 \leq i \leq 5$$
$$f^*(e_i') = k + d(i + 9); 1 \leq i \leq 2$$
$$f^*(e_i') = k + d(i + 10); 3 \leq i \leq 4$$
$$f^*(e_5') = k + 12d$$
$$f^*(e_i'') = k + d(i + 4);\ 1 \leq i \leq 5$$

Here p = 2n and q = 2n-1.
Clearly f is a (k,d)- Hernonian mean labeling.
Hence $H_n$ is a (k,d)-Heronian mean graph for all $n \geq 2$.

*1) Example 2.14*

(10,9)-Heronian mean labeling of Peterson Graph.

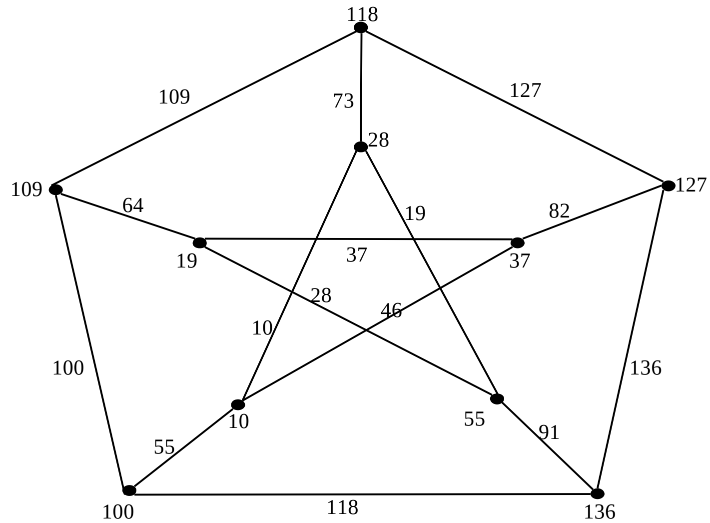


Figure 2.7: (k,d)-HML of Peterson Graph

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