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Estimation of COBB-Douglas Production Function in Econometric Model

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Abstract: An important inferential phase in nonlinear production function model problems is the exploration of the estimation of parameters of relationship between the output (dependent variable) and inputs (independent variables). The most frequently used nonlinear production function models in the econometric analysis are the Cobb-Douglas and Constant Elasticity of Substitution (CES) and Trnascendental logarithmic (Translog) production functions. The purpose of this chapter is to describe the various estimation methods for estimating the parameters of these three types of nonlinear production function models.

I. INTRODUCTION

Besides the existing single equation methods of estimation, the deterministic and stochastic specifications in the forms of simultaneous equations have been explained for Cobb-Douglas and CES production function models. The inferential problems including two important methods of estimation namely Generalized Least Squares (GLS) and Zellner's Feasible estimation have been presented for estimating the parameters of the flexible nonlinear production function namely Translog Production.

Some special inferential problems such as the estimation of Cobb-Douglas production function model with multiplicative and additive disturbances; estimation of Cobb-Douglas production function by removing the problem of multicollinearity; and nonlinear method of estimation of Cobb-Douglas production function, have been discussed with reference to the most frequently used nonlinear production function model namely the Cobb-Douglas production function.

II. NONLINEAR METHOD OF ESTIMATION OF COBB-DOUGLAS PRODUCTION FUNCTION MODEL

Consider the Cobb-Douglas production function model as

$$Y = A\, X_1^{\alpha}\, X_2^{\beta} \qquad \ldots (2.1)$$

Where A, α and β are parameters

By selecting some initial values of the unknown parameters, A_0, α_0, and β_0, one may form the following expression by using Taylor series expansion in the neighborhood of $(A - A_0)$, $(\alpha-\alpha_0)$ and $(\beta-\beta_0)$:

$$Y_0 = [\, A_0 X_1^{\alpha_0}\, X_2^{\beta_0}\,] + \left(\frac{\partial Y}{\partial A}\right)_{A=A_0} (A - A_0) + \left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=\alpha_0} (\alpha-\alpha_0) + \left(\frac{\partial Y}{\partial \beta}\right)_{\beta=\beta_0} (\beta-\beta_0) \qquad \ldots (2.2)$$

Adding an error term to this expression and minimize the error sum of squares $S^{|} = \sum_{i=1}^{n} [Y_i - Y_{oi}]^2$, one can obtain the parameter estimates as A_1, α_1 and β_1; and again form a linearized expression :

$$Y_1 = [A_1 X_1^{\alpha_1} \; X_2^{\beta_1}] + \left(\frac{\partial Y}{\partial A}\right)_{A=A_1} (A-A_1) + \left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=\alpha_1} (\alpha - \alpha_1)$$

$$+ \left(\frac{\partial Y}{\partial \beta}\right)_{\beta=\beta_1} (\beta-\beta_1) \qquad \ldots (2.3)$$

Adding again an error term to this expression and the second – round parameter estimates A_2, α_2 and β_2; can be obtained by minimizing error sum of squares

$$S^{II} = \sum_{i=1}^{n} [Y_i - Y_{1i}]^2 . \qquad \ldots (2.4)$$

One can continue this procedure iteratively until the parameter estimates appear to converge to a particular set of values.Another approach is to minimize the residual sums of squares $S = \sum_{i=1}^{n} \left[Y_i - \hat{A} X_{1i}^{\hat{\alpha}} \; X_{2i}^{\hat{\beta}}\right]^2$ with respect to $\hat{A}$, $\hat{\alpha}$ and $\hat{\beta}$ to obtain the following system of three equations

$$\frac{\partial S}{\partial \hat{A}} = 0 \qquad \ldots (2.5)$$

$$\frac{\partial S}{\partial \hat{\alpha}} = 0 \qquad \ldots (2.6)$$

$$\frac{\partial S}{\partial \hat{\beta}} = 0 \qquad \ldots (2.7)$$

These are three simultaneous nonlinear equations in the three unknown parameters. Each of these three equations can be expanded about assumed initial values of the parameters as follows :

$$\frac{\partial S}{\partial A} = \left(\frac{\partial S}{\partial A}\right)_{A=A_0} + \left(\frac{\partial^2 S}{\partial A^2}\right)(A - A_0) + \left(\frac{\partial^2 S}{\partial A \partial \alpha}\right)_{\substack{A=A_0 \\ \alpha=\alpha_0}} (\alpha - \alpha_0)$$

$$+ \left(\frac{\partial^2 S}{\partial A \partial \beta}\right)_{\substack{A=A_0 \\ \beta=\beta_0}} (\beta - \beta_0) = 0 \qquad \ldots (2.8)$$

Similar expressions for $\frac{\partial S}{\partial \alpha}$, $\frac{\partial S}{\partial \beta}$ about A_0, α_0 and β_0 (where A_0, α_0 and β_0 be the initial values of parameters) yield three simultaneous linear equations in the three unknown parameters, which can then be solved for A_1, α_1 and β_1. We then proceed iteratively.

This technique, with modifications to approach the minimum. 'S' efficiently, is the procedure suggested by Eisenpress and Greenstadt (1964). This procedure guarantees convergence to a local extremum, but one can not be certain that one found the global minimum. The number of iterations required will depend upon how accurate the initial guesses of the parameter values turn out to be in the iterative process. The Maximum likelihood estimation of parameters in a system of equations follows the same principles.

III. ESTIMATION OF COBB-DOUGLAS PRODUCTION FUNCTION USING LOGARITHMIC LINEAR TRANSFORMATION

Consider the Cobb-Douglas production functional model is of the form

$$Y_i = A\, X_{1i}^{\alpha}\, X_{2i}^{\beta}\, e^{\in i}\ ,\quad i = 1, 2, \ldots, n \qquad \ldots (3.1)$$

Where Y : Output of the firm

X_1 : Labour input

X_2 : Capital input

A : Technological Coefficient

α : Labour input elasticity parameter

β : Capital input elasticity parameter

$\in$: Classical disturbance variable

To estimate the unknown parameters A, α and β, one may take logarithmic transformation on both sides of the model (3.1), which gives

$$\ln Y_i = \ln A + \alpha \ln X_{1i} + \beta \ln X_{2i} + \in_i \qquad \ldots (3.2)$$

or

$$Y_i^{*} = A^{*} + \alpha X_{1i}^{*} + \beta X_{2i}^{*} + \in_i \qquad \ldots (3.3)$$

$$i = 1, 2, \ldots, n$$

Where, $A^{*} = \ln A;\ X_{1i}^{*} = \ln X_{1i}^{*};\ X_{2i}^{*} = \ln X_{2i},$

Model (3.3) can be considered as a three variable linear model.

The least squares estimates of α, β and A^{*} can be obtained as follows :

$$\begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = \begin{bmatrix} \sum x_1^{*2} & \sum x_1^{*} x_2^{*} \\ \sum x_1^{*} x_2^{*} & \sum x_2^{*2} \end{bmatrix}^{-1} \begin{bmatrix} \sum x_1^{*} y^{*} \\ \sum x_2^{*} y^{*} \end{bmatrix} \qquad \ldots (3.4)$$

and

$$\hat{A}^{*} = \overline{Y}^{*} - \hat{\alpha}\, \overline{X}_1^{*} - \hat{\beta}\, \overline{X}_2^{*} \qquad \ldots (3.5)$$

Where,

$$\sum x_1^{*2} = \left[\sum X_1^{*2} - \frac{\left(\sum X_1^{*}\right)^2}{n} \right]$$

$$\sum x_2^{*2} = \left[\sum X_2^{*2} - \frac{\left(\sum X_2^{*}\right)^2}{n} \right]$$

$$\sum x_1^{*} x_2^{*} = \left[\sum X_1^{*} X_2^{*} - \frac{\left(\sum X_1^{*}\right)\left(\sum X_2^{*}\right)}{n} \right]$$

$$\sum x_1^{*} y^{*} = \left[\sum X_1^{*} Y^{*} - \frac{\left(\sum X_1^{*}\right)\left(\sum Y^{*}\right)}{n} \right]$$

$$\sum x_2^* y^* = \left[\sum X_2^* Y^* - \frac{\left(\sum X_2^*\right)\left(\sum Y^*\right)}{n} \right]$$

and $$\hat{A} = \text{Anti ln} \left(\hat{A}^*\right).$$

The least squares estimated Cobb-Douglas production functional model is given by $\hat{Y}_i = \hat{A}\, X_{1i}^{\hat{\alpha}}\, X_{2i}^{\hat{\beta}}$, i = 1, 2, …, n … (3.6)

IV. ESTIMATION OF COBB-DOUGLAS PRODUCTION FUNCTION THROUGH COST FUNCTION

Consider the Cobb-Douglas production function with labour (X_1) and capital (X_2) inputs as

$$Y = A\, X_1^{\alpha}\, X_2^{\beta} \qquad \ldots (4.1)$$

Let the prices of output, labour and capital be P_Y , P_{X_1} and P_{X_2} respectively. Write the marginal productivity conditions as

$$\frac{\partial Y}{\partial X_1} = \alpha \left(\frac{Y}{X_1} \right) = P_{X_1} \qquad \ldots (4.2)$$

$$\frac{\partial Y}{\partial X_2} = \beta \left(\frac{Y}{X_2} \right) = P_{X_2} \qquad \ldots (4.3)$$

By considering wage rate (w) and rate of interest (r) as proxies for P_{X_1} and P_{X_2} ; and substituting the marginal productivity conditions (4.2) and (4.3) into the production function model (4.1), one can have the cost function corresponds to the Cobb-Douglas production function as

$$C = r \left(\frac{\alpha + \beta}{\beta} \right) \left[\left(\frac{\beta}{\alpha} \cdot \frac{w}{r} \right)^{\alpha} \left(\frac{Y}{A} \right) \right]^{\frac{1}{\alpha + \beta}} \qquad \ldots (4.4)$$

By introducing multiplicative error term into the cost function and using logarithmic linear transformation. The cost function can be estimated. Using the estimates of the parameters of the cost function, the Cobb-Douglas production function can be estimated. In practice, it is more convenient to estimate the cost function rather than to estimate the Cobb-Douglas production function.

V. ESTIMATION OF COBB-DOUGLAS PRODUCTION FUNCTION MODEL WITH MULTIPLICATIVE AND ADDITIVE ERRORS

Consider a general Cobb-Douglas production function model in which there are both additive and multiplicative errors as

$$y = \beta_0\, x_1^{\beta_1}\, x_2^{\beta_2} \ldots x_k^{\beta_k}\, e^{\epsilon} + u \qquad \ldots (5.1)$$

Where y is output; x_i's are inputs; $\in$ and u are errors which are distributed independently and normally with

$E(\in) = E(u) = 0$; $E(\in^2) = \sigma_{\in}^2$, $E(u^2) = \sigma_u^2$;

$E(\in_i \in_j) = E(u_i u_j) = 0, \ \forall\ i \neq j$ and $E(\in_i u_j) = 0\ \forall$ i, j.

Also x_i's are nonstochastic and β_j's are parameters.

Let Z be the random variable such that

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$$Z = \beta_0 x_1^{\beta_1} x_2^{\beta_2} \ldots x_k^{\beta_k} e^{\epsilon} = Q e^{\epsilon} \qquad \ldots (5.2)$$

If $\in \sim N(0, \sigma_{\in}^2)$ then the probability density function (p.d.f) of Z is given by

$$f(z) = \frac{1}{\sqrt{2\pi}\,\sigma_{\in}\, z} \exp\left\{-\frac{\left(\log \frac{z}{Q}\right)^2}{2\sigma_{\in}^2}\right\} \qquad \ldots (5.3)$$

the p.d.f. of u is given by

$$g(u) = \frac{1}{\sqrt{2\pi}\,\sigma_u} \exp\left\{-\frac{u^2}{2\sigma_u^2}\right\} \qquad \ldots (5.4)$$

Now, the p.d.f. of y is the convolution of (5.3) and (5.4) which is given by

$$h(y) = \frac{1}{2\pi\,\sigma_{\in}\,\sigma_u} \int_0^{\infty} \frac{1}{\xi} \exp\left\{-\frac{1}{2}\left[\frac{\left(\log \frac{\xi}{Q}\right)^2}{\sigma_{\in}^2} + \frac{(y-\xi)^2}{\sigma_u^2}\right]\right\} d\xi \qquad \ldots (5.5)$$

Here ξ is simply a dummy variable of integration. The range of integration is $(0, \infty)$ as the lognormally distributed variable is defined only for positive values. For a sample of n observations on y and x_i's, the likelihood function may be written as

$$L = \prod_{i=1}^{n} h(y_i) \qquad \ldots (5.6)$$

Instead of maximizing (5.6), it is convenient to maximize log L as

$$\text{Log } L = \sum_{i=1}^{n} \log\left\{\frac{1}{2\pi\,\sigma_{\in}\,\sigma_u} \int_0^{\infty} \frac{1}{\xi} \exp\left\{-\frac{1}{2}\left[\frac{\left(\log \frac{\xi_i}{Q_i}\right)^2}{\sigma_{\in}^2} + \frac{(y_i-\xi)^2}{\sigma_u^2}\right]\right\}\right\} d\xi \qquad \ldots (5.7)$$

With respect to $\beta_0, \beta_1, \beta_2, \ldots, \beta_k$; $\sigma_{\in}$ and σ_u.

This gives the maximum likelihood estimates of the parameters. However, it is not possible to find analytic expressions for the solutions of maximization of (5.7). The maximization of (5.7) requires on algorithm for numerical maximization of a function of many variables. Goldfeld and Quandt (1970) used Powell's (1964) conjugate Gradient method to obtain solutions. Kelejian (1972) studied further the Cobb-Douglas model with multiplicative and additive errors suggested by Goldfeld and Quandt (1970). He assumed that the errors may follow any probability distribution not necessarily the normal distribution. Let $E(e^{\epsilon}) = \mu$ such that e^{ϵ} may be expressed as

$$e^{\epsilon} = \mu + \psi \qquad \ldots (5.8)$$

where $E(\psi) = 0$.

By substituting (5.8) into (5.9), one can have

$$y = \delta\, x_1^{\beta_1}\, x_2^{\beta_2} \dots x_k^{\beta_k} + \nu \qquad \dots (5.9)$$

Where $\delta = \beta_0 \mu$ and $\nu = \beta_0\, x_1^{\beta_1}\, x_2^{\beta_2} \dots x_k^{\beta_k}\, \psi + u$ .Since ψ and u are independent of x_i's, it follows that $E[\nu/x_i] = 0$, $\forall$ i = 1, 2, …, k.

VI. ESTIMATION OF COBB-DOUGLAS PRODUCTION FUNCTION BY REMOVING MULTICOLLINEARITY

Consider the Cobb-Douglas production function as

$$y_t = A_t\, x_{1t}^{\alpha}\, x_{2t}^{\beta}\, e^{\in t} \quad , \ t = 1, 2, \dots, n \qquad \dots (6.1)$$

Where y_t = output for the time period t ;

X_{1t} = Labour input for the time period t ;

X_{2t} = Capital input for the time period t ;

α, β are the elasticity parameters of labour and capital respectively;

A_t measures the level of technology or technological coefficient;

and $\in_t$ error term.

Taking logarithms on both sides of (6.1), gives

$$\ln y_t = \ln A_t + \alpha \ln x_{1t} + \beta \ln x_{2t} + \in_t \qquad \dots (6.2)$$

By assuming a constant rate of technological change

$A_t = A_0\, e^{\theta t}$, (6.2) may be written as

$$\ln y_t = \ln A_0 + \theta\, t + \alpha \ln x_{1t} + \beta \ln x_{2t} + \in_t \qquad \dots (6.3)$$

Here, θ is a time trend parameter.

Most of the empirical studies indicate very high correlation between $\ln x_{1t}$ and $\ln x_{2t}$, in fact all the estimates are suspect due to the multicollinearity problem.To overcome the multicollinearity problem. One may usually assume the constant returns to scale, i.e., $\alpha + \beta = 1$ and reformulate the Cobb-Douglas production function as

$$\ln y_t = \ln A_0 + \theta\, t + (1-\beta) \ln x_{1t} + \beta \ln x_{2t} + \in_t$$

or $$\ln\left(\frac{y_t}{x_{1t}}\right) = \ln A_0 + \theta\, t + \beta \ln\left(\frac{x_{2t}}{x_{1t}}\right) + \in_t \qquad \dots (6.4)$$

Where $\left(\frac{x_{2t}}{x_{1t}}\right)$ is the capital – labor ratio.

Some empirical studies [for instance, Owyong and Bhanoji Rao (1998)] exhibited that still the multicollinearity present due to the correlation between $\ln\left(\frac{x_{2t}}{x_{1t}}\right)$ and the time trend t.

One remedy to this problem is to omit the time trend variable t from the model (6.4). But, this introduces the problem of omitted variable bias into the estimates. To overcome this problem, consider the first difference of the equation (6.4), and rewrite the models as

$$\left[\ln\left(\frac{y_t}{x_{1t}}\right) - \ln\left(\frac{y_{t-1}}{x_{1t}-1}\right)\right] = \theta + \beta\left[\ln\left(\frac{x_{2t}}{x_{1t}}\right) - \ln\left(\frac{x_{2t-1}}{x_{1t}-1}\right)\right] + u_t$$

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… (6.5)

where $u_t = \epsilon_t - \epsilon_{t-1}$.

$$\Rightarrow \quad \Delta \ln\left(\frac{y_t}{x_{1t}}\right) = \theta + \beta\Delta \ln\left(\frac{x_{2t}}{x_{1t}}\right) + u_t \qquad \ldots (6.6)$$

Thus, the first difference of the natural logarithme output – labour ratio may be regressed on the first difference of the natural logarithmic capital labour ratio without multicollinearity problems. Further, the rate of growth of Total Factor Productivity (TFP) say θ, may be obtained simply from the intercept estimate.

VII. A DETERMINISTIC SIMULTANEOUS EQUATIONS MODEL FOR A COBB-DOUGLAS PRODUCTION FUNCTION

Consider the Cobb-Douglas production function model as

$$Y = A\, X_1^{\alpha}\, X_2^{\beta} \qquad \ldots (7.1)$$

Define the profit function as

$$R = PY - wX_1 - r\,X_2 \qquad \ldots (7..2)$$

Where R : Profit, P : Price of Output
w : Wage rate (chosen as proxy for the price of Labour)
r : rate of interest (chosen as proxy for the price of capital)

Using the marginal productivity conditions, the maximization of profit function (7.2) subject to the production function (7.1) gives the following relationships

$$w = P\frac{\partial Y}{\partial X_1} \quad \Rightarrow \frac{w}{P} = \alpha\frac{Y}{X_1} \text{ or } \frac{w}{P\alpha} = \frac{w}{X_1} \qquad \ldots (7.3)$$

$$r = P\frac{\partial Y}{\partial X_2} \quad \Rightarrow \frac{r}{P} = \beta\frac{Y}{X_2} \text{ or } \frac{w}{P\beta} = \frac{w}{X_2} \qquad \ldots (7.4)$$

From (7.1), (7.3) and (7.4), one can have

$$\left.\begin{aligned} \ln Y &= \ln A + \alpha \ln X_1 + \beta \ln X_2 \\ \ln\left(\frac{w}{P\alpha}\right) &= \ln Y - \ln X_1 \\ \ln\left(\frac{r}{P\beta}\right) &= \ln Y - \ln X_2 \end{aligned}\right\} \qquad \ldots (7.5)$$

The equations (7.5) may be rewritten as

$$\left.\begin{aligned} \ln Y - \alpha \ln X_1 - \beta \ln X_2 &= \ln A \\ \ln Y - \ln X_1 &= \ln\left(\frac{w}{P\alpha}\right) \\ \ln Y - \ln X_2 &= \ln\left(\frac{r}{P\beta}\right) \end{aligned}\right\} \qquad \ldots (7.6)$$

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$$\Rightarrow \begin{bmatrix} 1 & -\alpha & -\beta \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \ln Y \\ \ln X_1 \\ \ln X_2 \end{bmatrix} = \begin{bmatrix} \ln A \\ \ln\left(\frac{w}{P\alpha}\right) \\ \ln \frac{r}{P\beta} \end{bmatrix}$$

or

$$\begin{bmatrix} \ln Y \\ \ln X_1 \\ \ln X_2 \end{bmatrix} = \frac{1}{\alpha+\beta-1} \begin{bmatrix} -1 & \alpha & \beta \\ -1 & 1-\beta & \beta \\ -1 & \alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} \ln A \\ \ln\left(\frac{w}{P\alpha}\right) \\ \ln\left(\frac{r}{P\beta}\right) \end{bmatrix}$$

$$\ln Y = \left[\frac{-\ln A}{\alpha+\beta-1}\right] + \left[\frac{\alpha}{\alpha+\beta-1}\right] \ln \left[\frac{w}{P\alpha}\right] + \left[\frac{\beta}{\alpha+\beta-1}\right] \ln \left[\frac{r}{P\beta}\right]$$

$$\ln X_1 = \left[\frac{-\ln A}{\alpha+\beta-1}\right] + \left[\frac{1-\beta}{\alpha+\beta-1}\right] \ln \left[\frac{w}{P\alpha}\right] + \left[\frac{\beta}{\alpha+\beta-1}\right] \ln \left[\frac{r}{P\beta}\right]$$

$$\ln X_2 = \left[\frac{-\ln A}{\alpha+\beta-1}\right] + \left[\frac{\alpha}{\alpha+\beta-1}\right] \ln \left[\frac{w}{P\alpha}\right] + \left[\frac{1-\alpha}{\alpha+\beta-1}\right] \ln \left[\frac{r}{P\beta}\right] \quad \ldots (7.7)$$

$$\Rightarrow \left.\begin{aligned} \ln Y &= a_0 + a_1 \ln\left(\frac{w}{P\alpha}\right) + a_2 \ln\left(\frac{r}{P\beta}\right) \\ \ln X_1 &= b_0 + b_1 \ln\left(\frac{w}{P\alpha}\right) + b_2 \ln\left(\frac{r}{P\beta}\right) \\ \ln X_2 &= c_0 + c_1 \ln\left(\frac{w}{P\alpha}\right) + c_2 \ln\left(\frac{r}{P\beta}\right) \\ \text{Where} \quad a_0 &= \frac{-\ln A}{\alpha+\beta-1} = b_0 = c_0 \end{aligned}\right\} \quad \ldots (7.8)$$

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$$a_1 = \frac{\alpha}{\alpha+\beta-1} = c_1$$

$$a_2 = \frac{\beta}{\alpha+\beta-1} = b_2$$

$$b_1 = \frac{1-\beta}{\alpha+\beta-1}$$

$$c_2 = \frac{1-\alpha}{\alpha+\beta-1}$$

Since different firms have different outputs and inputs even when confronted with the same set of prices, the deterministic model will be too restrictive for its estimation.
Marschak and Andrews (1944) have introduced stochastic disturbances into model and estimated the parameters by using the least squares estimation. However these estimates are biased and inconsistent. Many Statisticians and Econometricians have tried to obtain at least consistent estimates of the parameters of this simultaneous equations model.

VIII. CORRECTED LEAST SQUARES ESTIMATES FOR THE PARAMETERS OF COBB-DOUGLAS PRODUCTION FUNCTION

By introducing stochastic disturbances into the deterministic simultaneous equations model, (7.5) may be written as

$$\ln Y = \ln A + \alpha \ln X_1 + \beta \ln X_2 + \epsilon_0 \qquad \ldots (8..1)$$

$$\ln X_1 = \ln Y - \ln\left(\frac{w}{P\alpha}\right) + \epsilon_1 \qquad \ldots (8.2)$$

$$\ln X_2 = \ln Y - \ln\left(\frac{r}{P\beta}\right) + \epsilon_2 \qquad \ldots (8.3)$$

Hoch (1958) specified the model (8.1) as model by assuming the uncorrelation between the economic disturbances ϵ_1 and ϵ_2:

$$Y^* = \gamma_0 + \gamma_1 X_1^* + \gamma_2 X_2^* + \epsilon_0 \qquad \ldots (8.4)$$

Where $\gamma_0 = \ln A$; $\gamma_1 = \alpha$, $\gamma_2 = \beta$

$Y^* = \ln Y$; $X_1^* = \ln X_1$ and $X_2^* = \ln X_2$

By applying Least Squares estimation, one can estimate the transformed model and hence obtain the estimates for α and β as $\hat{\gamma}_1$ and $\hat{\gamma}_2$ respectively. Hoch (1958) proposed the generalized formula for the corrected least squares estimates of the parameters of the Cobb-Douglas production function model as

$$\gamma_i^* = \hat{\gamma}_i \left[1 + \frac{M_{00}\left(M_{11} + M_{22} - 2M_{12}\right)}{M_{11}M_{22} - M_{12}^2}\right] - \left[\frac{Q_{00}\left(M_{ij} - M_{12}\right)}{M_{11}M_{22} - M_{12}^2}\right] \qquad \ldots (8.5)$$

i = 1, 2,
j = 1, 2 and $i \neq j$.

Where, $M_{ii} = V_{00} + V_{ii} - 2\,V_{o_i}$, i = 1, 2

$$M_{12} = V_{00} + V_{12} - V_{o_1} - V_{o_2}$$

$$M_{00} = V_{00} + \hat{\delta}_1 V_{o_1} - \hat{\delta}_2 V_{o_2}$$

$$Q_{00} = \frac{M_{00}\left(M_{11}M_{22} - M_{12}^2\right)}{\left(M_{11}M_{22} - M_{12}^2\right) - M_{00}\left(M_{11} + M_{22} - 2M_{12}\right)} \qquad \ldots (8.6)$$

Here, V_{00} is the sample variance of Y^*

V_{ii} is the sample variance of X_i^* , i = 1, 2

V_{12} is the sample covariance of X_1^*, and X_2^* ,

V_{o_i} is the sample covariance of Y^* and X_i^* , i = 1, 2

By substituting Q_{00} in (8.6) into (8.5) one can get the formula for Hoch's (1958) corrected least squares estimates of α and β as

$$\gamma_i^* = \frac{\hat{\gamma}_i\left[M_{11}M_{22} - M_{12}^2\right] - M_{00}\left[M_{ij} - M_{12}\right]}{\left[M_{11}M_{22} - M_{12}^2\right] - M_{00}\left[M_{11} + M_{22} - 2M_{12}\right]} \ldots (8.7)$$

$$i, j = 1, 2, ; i \neq j$$

IX. CONCLUSIONS

In this paper we discuss about different estimation methods for estimating the parameters of three frequently used nonlinear production functions namely Cobb-Douglas, CES and Translog production function models. The deterministic and stochastic specifications of system of simultaneous equations for Cobb-Douglas and CES production function models have been explained besides the existing single equation methods of estimation.

REFERENCES

[1] Aigner, D.J. (1974), "Asymptotic Minimum-MSE Prediction in the Cobb-Douglas Model with a Multiplicative Disturbance Term", Econometrica, 42, 737-748.
[2] Bard, Y. (1974), "Nonlinear Parameter Estimation", Academic Press, New York.
[3] Cavanagh, C.L. (1982), "Hypothesis Testing in Nonlinear Models" Paper Presented at the Econometric Society European Meeting, Durblin.
[4] Davidson, R. and MacKinnon, J.G. (1981), "Several Tests for Model Specification in the Presence of Alternative Hypothesis", Econometrica, 49, 781-793.
[5] Eisenpress, H. and Greenstadt, J. (1966), "The Estimation of Nonlinear Econometric Systems, Econometrica, 34, 851-861.
[6] Goldberger, A.S. (1968), "The Interpretation and Estimation of Cobb-Douglas Functions", Econometrica, 35, 464-473.
[7] Jennrich, R.I. (1969), "Asymptotic Properties of Nonlinear Least Squares Estimators", "The Annals of Statistics 2, 633-643.
[8] Lu, Y.C. and Fletcher, L.B. (1968), "A Generalization of the CES Production Function", Review of Economics and Statistics, 50, 449-452.
[9] Nerlove, M. (1965), "Estimation and Identification of Cobb-Douglas Production Functions", North-Holland, Amsterdam.
[10] Oliver, F.R. (1966), "Aspects of Maximum Likelihood Estimation of the Logistic Growth Function", Journal of the American Statistical Association, 61, 697-705.
[11] Owyong, D.T. and Bhanogi Rao, v.v. (1998), "Total Factor Productivity Growth in the Singapore Economy : Some Econometric Systems", The Singapore Economic Review, 43, 74-81.
[12] Pedda Reddeppa Reddy, M. (1992), "Some Aspects of Nonlinear Econometric Models", Unpublished M.Phil., Thesis, S.V. University, Tirupati, India.

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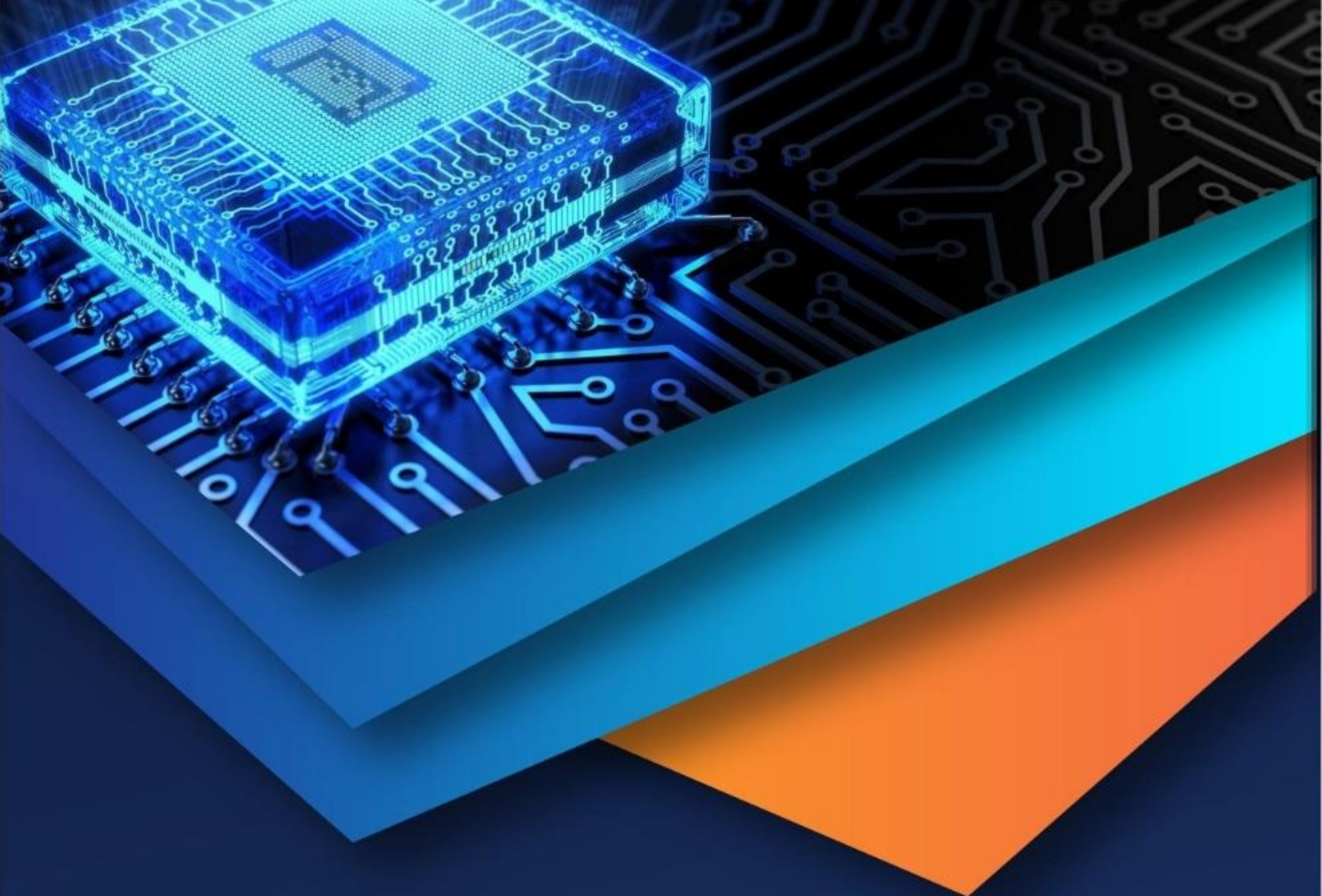
[13] Pearsan, M.H. and Deation, A.S. (1978), "Testing Non-Nested Nonlinear Regression Models", Econometrica, 46, 677-694.
[14] Stephen, M.G. and Quardt, R.e. (1970), "The Estimation of Cobb-Douglas Type Functions with Multiplicative and Additive Errors", International Economic Review, 11, 251-257.
[15] Zellner, A. (1962), "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias", Journal of American Statistical Association, 57, 348-368.



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