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International Journal For Research in  
Applied Science and Engineering Technology



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# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

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**Volume: 5      Issue: XII      Month of publication: December 2017**

**DOI:**

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# Moving Average Models in Forecasting Methods

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**Abstract:** Forecasting inevitable in any kind of planning and administration. It is an important part of decision making and many of our decisions are based on predictions of future unknown events. Statistical planning must be based on forecasts. Forecasting has always been necessary and to put forecasting on a scientific basis. This means to forecast by reference to past history and statistics rather than by pure intention and guess-work. Forecasting is becoming increasingly important both for the regulation of developed economics as well as for the planning of the economic development of underdeveloped countries. In formulating policy decisions, it is essential to be able to forecast the value of the economic magnitudes. Such forecasts will enable the policy maker to judge whether it is necessary to take any measures in order to influence the relevant economic variables.

## I. INTRODUCTION

The three words 'prediction, projection and forecast' have seems to be synonymous, but there were some distinctions. A 'prediction' is an estimate based only on past data on study variable. It may be considered as pure mechanical extrapolation. A 'projection' is a prediction where the extrapolated values are subject to certain assumptions. In the demography, where the projection of number of births, deaths, marriages etc. involve assumptions about changes in the birth rate, death rates, marriage rate etc., A 'forecast' is an estimate which relates the series in which one may interested to external factors. Forecasts are thus made by estimating future values of the external factors by means of prediction, projection or forecast and from these values, finding the estimates of the dependent variable. Hence, forecasting involves using all our knowledge, from all the sources about the situation. The general forecasting methods include guessing, thumb rule or informal models; expert judgment; extrapolation; leading indicators; surveys; time series models; regression models and economic systems. In the past four decades, a considerable amount of research has been developed in the field of forecasting.

### A. Qualitative vs. Quantitative Methods

Qualitative forecasting techniques are subjective based on the opinion and judgment of consumers, experts; appropriate when past data is not available. It is usually applied to intermediate – long range decisions.

Example of qualitative forecasting methods :

- 1) Informed opinion and judgment.
- 2) elphi Method
- 3) Market research
- 4) Historical life – Cycle analogy.
- 5) Quantitative forecasting models are used to estimate future demands as a function of past data; appropriate when past data is available. It is usually applied to short – intermediate range decisions.

### B. Example of quantitative forecasting methods

Last period demand

- 1) Arithmetic Average.
- 2) Simple moving average (N- Period)
- 3) Weighted moving average (N – Period)

- 4) Simple exponential smoothing
- 5) Multiplicative seasonal indexes

*C. " Naive Approach*

Naive forecasts are the most cost – effective and effective objective forecasting model, and provide a benchmark against which more sophisticated models can be compared. For stable time series data, this approach says that the forecast for any period equals the previous period’s actual value. A naïve forecasting model simply assumes the revenue available at time ‘t’ is the same amount available at time ‘t-1’.

This is also known as the “random walk approach

$$F_t = A_{t-1} \tag{1.1}$$

where Ft is the forecast at time t, and

At-1 is the actual value at time (t-1)

A variation of this involves averaging the two prior periods to generate the estimate. Yet another variation involves adjusting for any seasonality that may be present. Naïve forecasting is often used when the data series is unpredictable. It is also used in expert forecasting as the starting point for estimates that are then adjusted mentally.

*D. Time Series Methods*

Time series approach is the “bread and butter” of forecasting. They have been used extensively in the private sector and have been subject to substantial evaluation. In using time series techniques, Frank (1993) identifies several essential concepts that need consideration prior to the selection of technique.

**II. MOVING AVERAGE**

A moving average is a type of finite impulse response filter used to analyze a set of data points by creating a series of averages of different subsets of the full data set. Moving average is commonly used with time series data to smooth out short - term fluctuations and high light longer – term trends or cycles.As implied by the name, the future value to be forecast is based on the average of N previous periods. It is a moving average because the oldest data points are dropped off as new ones are added.

$$F_t = \frac{\sum_{i=1}^N A_{t-i}}{N} \tag{2.1}$$

where Ft is the forecast at time t,

At-i is the actual value at time (t-i),

and N is the number of time periods averaged.

**III. SINGLE MOVING AVERAGE**

The most commonly used forecasting moving average model is Single Moving Average Model.The moving average forecast is based on the assumption of a constant model.

$$X_t = b + \epsilon_t \tag{3.1}$$

Estimate the Single Parameter of the model at time T as average of the last m observations, where m is the moving average interval.

$$\hat{b}_T = \frac{\sum_{t=T-m+1}^T x_t}{m} \tag{3.2}$$

Since the model assumes a constant underlying mean, the forecast for any number of periods in the future is the same as the estimate of the parameter.

$$\hat{x}_{\tau+t} = \hat{b}_{\tau} \text{ for } \tau=1,2,\dots \tag{3.3}$$

For a continuously increasing series with trend a, the values of lag and bias of the estimator of the mean is given in the equations below.

$$\text{Lag} = \frac{m-1}{2}, \text{ Bias} = \frac{-a(m-1)}{2} \dots \tag{3.4}$$

The moving average forecast of  $\tau$  periods into the future is represented by shifting the curves to the right. The lag and bias increase proportionally. The equations below indicate the lag and bias of a forecast  $\tau$  periods into the future when compared to the model parameters.

Again, these formulas are for a time series with a constant linear trend.

$$\text{Lag}(\tau) = \tau + \frac{m-1}{2}, \text{ Bias} = -a \left[ \tau + \frac{(m-1)}{2} \right] \tag{3.5}$$

The error is the difference between the actual data and the forecasted value.

If the time series is truly a constant value the expected value of the error is zero and the variance of the error is comprised of a term that is a function of  $\alpha$  and a second term that is the variance of the noise  $\sigma^2$ .

$$e_{\tau} = x_{T+\tau} - \hat{x}_{T+\tau} \tag{3.6}$$

$$E[e_{\tau}] = 0 \tag{3.7}$$

$$\text{Var}[e_{\tau}] = \sigma_{e_{\tau}}^2 = \frac{\sigma^2}{m} + \sigma^2 \tag{3.8}$$

#### A. Double Moving Average Forecasting Model

The Double Moving Average forecasting model (or 2nd moving averages of the 1st order moving averages) gives unequally weighted averages, and intended to handle data series with trend better than the Single Moving Averages. Double Moving Average can be denoted as MA (MxN) which Means an M- period MA of an N – Period of MA .

The Double Moving Averages for available averaging periods using the following equation:

$$M2_{n,t} = M1_{n,t} + M1_{n,t-1} + \dots + M1_{n,t-n+1} \tag{3.9}$$

where  $M2_{n,t}$  = An n–period double moving average calculated in period t.

$M1_{n,t}$  = An n - Period single moving average calculated in period t.

and n = Number of periods in the moving average.

For developing a forecast, we can use the Double Moving Average as

$$FM2_{n,t,t+h} = A_{n,t} + B_{n,t} + h \tag{3.10}$$

Where

$FM2_{n,t,t+h}$  = The n – period, double moving average

Forecast made in period t for period t+h ,

$A_{n,t}$ = The intercept for an n–period double moving average forecast, calculated :

$A_{n,t} = 2M1_{n,t} - M2_{n,t}$

$B_{n,t}$  = The slope for an n – period double moving average forecast, calculated :

$$B_{n,t} = \frac{2}{n-1} (M1_{n,t} - M2_{n,t}) \tag{3.11}$$

Where h = The horizon, the number of periods used for forecasting into the future.

**B. Cumulative Moving Average**

In a cumulative moving average, the data arrives in an ordered data stream and getting the average of all the data up until the current data point. As each new transaction occurs, the average at the time of the transaction can be calculated for all of the transactions up to that point using the cumulative average, typically an unweighted average of the sequence of i values

$x_1, x_2, \dots, x_i$  up to the current time.

$$CA_i = \frac{x_1 + x_2 + \dots + x_i}{i} \tag{3.12}$$

The brute – force method to calculate this would be to store all of the data and calculate the sum and divide by a number of data points every time a new data point arrived.

However, it is possible to simply update cumulative average as a new value  $X_{i+1}$  becomes available, using the formula:

$$CA_{i+1} = \frac{X_{i+1} + iCA_i}{i+1} \tag{3.13}$$

Where CAo can be taken to be equal to zero.

When all of the data points arrive (i=N), the cumulative average will equal the final average.

The derivation of the cumulative average formula is straight forward.

Using  $x_1 + x_2 + \dots + x_i = iCA_i$ ,

and similarly for i+1, it is seen that

$$x_{i+1} = (x_1 + x_2 + \dots + x_{i+1}) - (x_1 + x_2 + \dots + x_i) = (i+1)CA_{i+1} - iCA_i$$

Solving this equation for CAi+1 results in:

$$CA_{i+1} = \frac{(x_{i+1} + iCA_i)}{i+1} = CA_i + \frac{x_{i+1} - CA_i}{i+1} \tag{3.14}$$

**C. Weighted Moving Average**

A weighted average is any average that has multiplying factors to give different weights to data at different positions in the sample window. Mathematically, the moving average is the convolution of the data points with a fixed weighting function. A weighted Moving Average (WMA) has the specific meaning of weights that decrease in Arithmetical Progression. In an n – day WMA the latest day has weight n, the second latest n-1, etc., down to one.

$$WMAM = \frac{nP_M + (n-1)P_{M-1} + \dots + 2P_{(M-n+2)} + P_{(M-n+1)}}{n + (n-1) + \dots + 2 + 1} \tag{3.15}$$

The denominator is a triangle number equal  $\frac{n(n+1)}{2}$

When calculating the WMA across successive values, the difference between the numerators of WMAM+1 and WMAM is

$$nP_{M+1} - P_M - \dots - P_{M-n+1}$$

If we denote the sum  $P_M + \dots + P_{M-n+1}$  by total M, then

$$\text{Total M+1} = \text{TotalM} + P_{M+1} - P_{M-n+1}$$

Numerator  $M+1 = \text{Numerator}_M + n_{PM+1} - \text{Total}_M$

$$WM_{M+1} = \frac{\text{Numerator}_{M+1}}{n + (n-1) + \dots + 2 + 1} \tag{3.16}$$

**D. Exponential Moving Average**

An Exponential Moving Average (EMA), also known as an Exponentially Weighted Moving Average (EWMA), is a type of infinite impulse response filter that applies weighting factors which decrease exponentially. The weighting for each order data point decreases exponentially, never reaching zero.

The EMA for a series  $y$  may be calculated recursively:

$$S_1 = Y_1$$

$$\text{For } t > 1, S_t = \alpha \times Y_{t-1} + (1 - \alpha) \times S_{t-1} \tag{3.17}$$

where the co-efficient  $\alpha$  represents the degree of weighting decrease,

$Y_t$  is the observation at a time period  $t$ ,

$S_t$  is the value of the EMA at any time period  $t$ .

**E. Modified Moving Average**

A modified Moving Average (MMA), running Moving Average (RMA), or Smoothed Moving Average is defined as :

$$\text{MMA today} = \frac{(N - 1) \times \text{MMA}_{\text{yesterday}} + \text{Price}}{N} \tag{3.18}$$

In short, this is exponential moving average, with  $\alpha = 1/N$

**IV. EXPONENTIAL SMOOTHING**

Exponential smoothing is a technique that can be applied to time series data, either to produce smoothed data for presentation, or to make forecasts. The raw data sequence is often represented by  $\{X_t\}$ , and the output of the exponential smoothing algorithm is commonly written as  $\{S_t\}$ , which may be regarded as a best estimate of what the next value of  $X$  will be. When the sequence of observations begins at time  $t = 0$ , the simplest form of exponential smoothing is given by the formulas:

$$\left. \begin{aligned} S_1 &= x_0 \\ S_t &= \alpha x_{t-1} + (1-\alpha) S_{t-1}, \quad t > 1 \end{aligned} \right\} \tag{4.1}$$

where  $\alpha$  is the smoothing factor, and  $0 < \alpha < 1$ .

**A. Double Exponential Smoothing**

Simple exponential smoothing does not do well when there is a trend in the data. In such situations, a method was devised under the name “Double Exponential Smoothing”. One method, sometimes referred to as “Holt winters double exponential smoothing”, works as follows: The raw data sequence of observations is represented by  $\{X_t\}$ , beginning at time  $t = 0$ .  $\{S_t\}$  is to represent the smoothed value for time  $t$ , and  $\{b_t\}$  is the best estimate of the trend at time  $t$ . Double exponential smoothing is given by the formulas.

$$\left. \begin{aligned} S_0 &= x_0 \\ S_t &= \alpha x_t + (1-\alpha) (S_{t-1} + b_{t-1}) \\ b_t &= \beta (S_t - S_{t-1}) + (1-\beta) b_{t-1} \end{aligned} \right\} \tag{4.2}$$

$$F_{t+m} = S_t + m b_t$$

where  $\alpha$  is the data smoothing factor,  $0 < \alpha < 1$

$\beta$  is the trend smoothing factor,  $0 < \beta < 1$ .

and  $b_0$  is taken as  $(X_{n-1} - X_0) / (n-1)$  for some  $n > 1$ .

Note that  $F_0$  is undefined (there is no estimation for time 0), and according to the definition  $F_1 = S_0 + b_0$ , which is well defined, thus further values can be evaluated. A second method, referred to as either Brown's Linear Exponential Smoothing (LES) or Brown's double exponential smoothing work as follows:

$$\begin{aligned}
 S_{01} &= x_0 \\
 S_{011} &= x_0 \\
 S_{t1} &= \alpha x_t + (1-\alpha) S_{t-1} \\
 S_{t11} &= \alpha S_{t1} + (1-\alpha) S_{t11} \\
 F_{t+m} &= a_t + mb_t
 \end{aligned} \tag{4.3}$$

Where  $a_t$ , the estimated level at time  $t$ , and  $b_t$ , the estimated trend at time  $t$  are :

$$\begin{aligned}
 a_t &= 2S_{t1} - S_t^{11} \\
 b_t &= \frac{\alpha}{1-\alpha} (S_t^1 - S_t^{11})
 \end{aligned}$$

### B. Triple Exponential Smoothing

Triple exponential smoothing takes into account seasonal changes as well as trends. It was first suggested by Holt's Student Peter wintners, in 1960. Triple exponential smoothing is given by the formulas:

$$\left. \begin{aligned}
 S_0 &= x_0 \\
 S_t &= \alpha \frac{x_t}{C_t - L} + (1-\alpha) F_t \\
 b_t &= \beta (S_t - S_{t-1}) + (1-\beta) b_{t-1} \\
 C_t &= \gamma \frac{x_t}{S_t} + (1-\gamma) C_{t-L}
 \end{aligned} \right\} \tag{4.4}$$

The output of the algorithm is written as  $F_{t+m}$ , an estimate of the value of  $X$  at time  $t+m$ ,  $m > 0$  based on the raw data up to time  $t$ .

$$F_{t+m} = (S_t + mb_t) C_{(t+m)(\text{mod } L)} \tag{4.5}$$

where  $\alpha$  is the data smoothing factor,  $0 < \alpha < 1$

$\beta$  is the trend smoothing factor,  $0 < \beta < 1$

and  $\gamma$  is the seasonal change smoothing factor  $0 < \gamma < 1$ .

## V. CONCLUSIONS

Forecasting techniques that are based on regression analysis are substantially different in their underlying concepts and theory from the techniques of time series analysis, smoothing and decomposition. Regression techniques are generally referred to as causal or explanatory approaches to forecasting. They attempt to predict the future by discovering and measuring the effect of important independent variables on the dependent variable to be forecast. Because of their costs, these methods are generally used in long-run planning and in situations where the value of increased accuracy warrants the additional expense. In this paper we discuss about various basic forecasting models such as Naive, Moving averages, Simple smoothing, Double moving averages and Double smoothing, triple smoothing and adaptive smoothing forecasting models.

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