Volume: 5 **Issue:** XII **Month of publication:** December 2017

DOI:

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

Auto Regressive (AR) Models in Forecasting Methods

J. Prabhakara Naik[1], T. Gangaram[2], R.V.S.S. Nagabhushana Rao[3], K.Vasu[4], G. Mokesh Rayalu[5]
[1]*Lecturer in Statistics, SSBN Degree & PG College(Autonomous), Anantapur*
[2]*Lecturer , Department of Statistics, PVKN Govt College, Chittoor*
[3]*Assistant professor, Department of Statistics, Vikrama Simhapuri University, Nellore*
[4]*Assistant Professor, Vidya Educational Institutions, Yadamari , Chittoor*
[5]*Assistant Professor, Department of Mathematics, School of Advanced sciences ,VIT, Vellore*

Abstract: Forecasting is the process of making statements about the events whose actual outcomes (typically) have not yet been observed. A common place example might be estimation for some variable of interest at some specified future date.Prediction is similar, but more general term. Both might refer to formal statistical methods employing time–series, cross–sectional or longitudinal data, or alternatively to less formal judgement methods. Forecasting is used in the practice of customer demand planning in everyday business forecasting for manufacturing companies. Forecasting has also been used to predict the development of conflict situations. Forecasting can be described as predicting what the future will look like, whereas planning predicts what the future should look like. There is no single right forecasting method to use. Selection of a method should be based on objectives and conditions (data etc.,). A good place to find a method, is by visiting a selection tree .

I. INTRODUCTION

In Forecasting, in the case of moving averages, single, double , cumulative and more historical and complicated exponential smoothing methods are studied. The moving averages are used to eliminate the seasonality and randomness from data series. In this sense, moving averages are the back bone of decomposition methods. In all the cases the objective is to make use of past data to develop a forecasting system for future periods. The Box–Jenkins methodology only applies to what are called stationary time series. Stationary time series have constant means and variances that do not vary over time. Stationary time series can be correlated, however, and it is this autocorrelation that the Box – Jenkins methods try to capture. Consequently, any time series that trends up or down is not stationary since the mean of the series depend on time. Two techniques are generally used to transform a trending series to stationary form: linear detrending (regressing on a time period) and differencing.

Box and Jenkins recommend differencing a time series, rather than detrending by regressing on a time trend, to remove the trend and achieve stationary. This approach views the trend in a series as erratic and not very predictable. The Box – Jenkins method is a univariate time series modelling and forecasting method. In this chapter, discussing the advanced forecasting techniques , were used in many problematic situations in different sectors are like AR Model, MA Model, ARMA, ARIMA, Non–Linear AR, VAR and BVAR , and additional Box – Jenkins methods.

II. AUTO REGRESSIVE (AR) FORECASTING MODEL WITH ORDER P

The Auto Regressive (AR) forecasting model with order P, is given by

$$Z_t = \phi_1 Z_{t-1} + \ldots\ldots\ldots\ldots + \phi_p Z_{t-p} + a_t \qquad (2.1)$$

When Zt , the deviation from the mean at time t, as being regressed on the P previous deviations Zt-1, ……, Zt-p

In backshift operator notation, the AR (P) model can be written as

$$\left(1-\phi_1 B - \ldots\ldots - \phi_p B^p\right) Z_t^1 = a_t \qquad (2.2)$$

or $\phi\left(B\right) Z_t = a_t$

Like simple AR models ,

$$\gamma_0 = \phi_1 \gamma_1 + \ldots\ldots + \phi_p \gamma_p + \sigma^2 \qquad (2.3)$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

$$\gamma_0 = \frac{\sigma^2}{1-\phi_1\rho_1-\dots\dots\dots-\phi_p\rho_p}$$

Which implies that (2.4)

The auto covariances and autocorrelations follow a pthorder difference equation

$$\gamma_k = \phi_l r_{k-l} + \dots\dots\dots + \phi_p r_{k-p} \quad , \qquad k > o$$

$$\rho_k = \phi_l \rho_{k-l} + \dots\dots\dots + \phi_p \rho_{k-p}, \qquad k > o$$

(2.5)

The first P – equations, (K=1,………, P) in equation (2.5) are called the Yule – Walker Equations, as

K = 1 : $\rho_1 = \phi_1 + \rho_1\phi_2 + \dots\dots\dots + \rho_{p-1}\phi_p$

K = 2 : $\rho_2 = \rho_1\phi_1 + \phi_2 \dots\dots\dots + \rho_{p-2}\phi_p$

⋮

K = P : $\rho_p = \rho_{p-1}\phi_1 + \rho_{P-2}\phi_2 \dots\dots\dots + \phi_p$

In matrix notation, we can write these equations, as

ρ = Pϕ (2.6)

where ρ = (ρl, ρ2, ……….., ρP)l

= (ϕl, ϕ2, ………..,ϕP)l

and P = $\begin{bmatrix} 1 & \rho_1 & \rho_2 & \dots & \rho_{p-1} \\ \rho_1 & 1 & \rho_1 & \dots & \rho_{p-2} \\ . & . & . & & . \\ \rho_{p-1} & \rho_{p-2} & \rho_{p-3} & \dots & 1 \end{bmatrix}$

The autoregressive parameters ϕ can be expressed as a function of the first P auto correlations by solving the equation system (2.6).

ϕ = P-1 ρ (2.7)

The difference equation in (2.5), $\phi(B)\rho_k = 0 \;,\; k = 1, 2, \dots$ determines the behaviour of the autocorrelationfunction. It can be shown that its solution is

$$\rho_k = A_1G_1^k + \dots\dots + A_pG_p^k \;,\quad k = 0, 1, 2, \dots\dots$$

whereGi-l (i=1,2 ………, P) are the distinct roots of

$\phi(B) = (1-G_1B)(1-G_2B)\dots\dots(1-G_pB) = 0$, and the Ai's are constants.

III. MOVING AVERAGE (MA) FORECASTING MODEL WITH ORDER Q

The moving average forecasting model with order q is given by

$$Z_t - \mu = a_t - \theta_1 a_{t-1} - \dots\dots - \theta_q a_{t-q}$$

or

$$Z_t - \mu = \left(1-\theta_1 B - \dots\dots - \theta_q B^q\right) a_t = \theta(B) a_t \qquad (3.1)$$

Its auto covariances and autocorrelations can be derived by setting φo = l

$\varphi_1 = -\theta_1, \dots\dots, \varphi_q = -\theta_q, \varphi_j = 0,\; j > q$, or by considering $E(Z_t-\mu)(Z_{t-k}-\mu)$ *directly*

The auto covariancesare :

$$\gamma_0 = (1+\theta_1^2 + \dots\dots\dots\dots + \theta_q^2)\ \sigma^2$$

$$\gamma_k = (-\theta_k + \theta_1\theta_{K+1} + \dots\dots + \theta_{q-k}\ \theta_q)\sigma^2, K = 1, 2, \dots\dots q, \quad K > q$$

$\gamma_{lc} = 0$

The Auto correlation function (ACF) of the MA (q) process cuts off after lag q.In the infinite Auto regressive representation

$$(1-\Pi_1 B - \Pi_2 B^2 ....), (Z_t - \mu) = a_t$$

The Π weights are given by

$$\Pi\,(B) \;=\; \theta\text{-}1\,(B) \qquad (3.2)$$

where $\theta(B) = 1 - \theta_1 B - ...... - \theta_q B^q$

IV. ARMA (P,Q) FORECASTING MODEL

Box and Jenkins (1976) introduced the ARMA model as a general way of explaining a variable in terms of its own past. The Box and Jenkins methodology was producing short term forecasts. Wold (1954) proved that any covariance stationary stochastic process Yt with Mcan μ and variance σ2 can be written in the form.

$$Y_t - \mu \;=\; \varphi_o \varepsilon_t + \varphi_1 \varepsilon_{t-1} + \varphi_2 \varepsilon_{t-2} + ... \qquad (4.1)$$

$$= \sum_{j=0}^{\infty} \varphi_t \varepsilon_{t-j}$$

Where εt is a sequence of uncorrelated random variables with Mean 0 and constant variance σ2.The moving average coefficients are subject to the condition that they are absolutely summable.

$$\sum_{j=0}^{\infty} / \varphi_t / < \infty$$

Using the lag operator L, equation (4.1) can be written as

$$Y_t - \mu = (1 + \varphi_1 L + \varphi_2 L^2 + ...)\varepsilon_t = \varphi(L)\varepsilon_t \qquad (4.2)$$

whereφ (L) is a polynomial function in the lag operator.

The Polynomial φ (L) can be factorized as the product of its roots

$$\varphi(L) \;=\; \prod_{j=1}^{\infty}(1+\beta_j L) = (1+\beta_1 L)(1+\beta_2 L)........ \qquad (4.3)$$

With roots given by $\dfrac{-1}{\beta_1}\,,\ \dfrac{-1}{\beta_2}\,,\ ....\ \ etc.,$

These moving average roots must satisfy the condition of identifiability that

$$\left\|\beta_j\right\| \le 1 \;,\;\; \forall\, j$$

V. AUTO REGRESSIVE INTEGRATED MOVING AVERAGE (ARIMA) MODEL

ARIMA model is a generalization of an Autoregressive Moving Average (ARMA) model. This ARIMA (p,d,q) model, where p, d and q are non – negative integers that refer to the order of the auto regressive, integrated and moving average parts of the model respectively. ARIMA models form an important part of the Box – Jenkins (1976) approach to time – series modeling. A time series data Xt , where t is an integer index and Xt are real numbers.

Then an ARMA (p,q) model is given by

$$\left(1-\sum_{i=1}^{p}\alpha_i L^i\right)X_t = \left(1+\sum_{i=1}^{q}\theta_i L^i\right)\varepsilon_t \qquad (5.1)$$

where L is the Lag Operator

αi are the parameters of the autoregressive part of the model.

θi are the parameters of the moving average part and

εt are error terms.

Assume that the polynomial $\left(1-\sum_{i=1}^{p}\alpha_i L^i\right)$ has a unitary root of multiplicity d, then it can be rewritten as

$$\left(1-\sum_{i=1}^{p}\alpha_i L^i\right) = \left(1+\sum_{i=1}^{p-d}\phi_i L^i\right)(1-L)^d \qquad (5.2)$$

An ARIMA (p,d,q) process expresses this polynomial factorization property, and is given by

$$\left(1-\sum_{i=1}^{p}\phi_i L^i\right)(1-L)^d X_t = \left(1+\sum_{i=1}^{q}\theta_i L^i\right)\varepsilon_t \qquad (5.3)$$

VI. AUTO REGRESSIVE DISTRIBUTED FORECASTING MODEL

The Auto Regressive Distributed Lag Model (ADL) of Order (p,q) :

$$y_t = \sum_{m=1}^{P}\beta_m y_{t-m} + \sum_{n=0}^{q}\gamma_n{}^1 x_{t-n} + \zeta Z_t + \varepsilon_t \qquad (6.1)$$

whereyt is the dependent variable

xt is a dx1 vector of key explanatory variable.

Zt is other explanatory variable (S) potentially containing a constant,and ε_t ~ N (o, σ^2) .The Bayesian prior is set to

$$\beta_m \sim N\left({}_{X\{1\}}(m), \sigma_m^2\right) \qquad (6.2)$$

$$\gamma_{in} \sim N\left(0, \sigma_{in}^2\right)$$

where ${}_{X\{1\}}(m)$ is an indicator function .

From Doan, Litterman and Sims (1964), the specification of the standard deviation of the prior is :

σm = θk m-ϕ

$$\sigma in = \theta l\,(1+n)\text{-}\phi \left(\frac{\hat{\sigma}_{u,i}}{\hat{\sigma}_{u,y}}\right) \qquad (6.3)$$

where $\hat{\sigma}_{u,i}$ and $\hat{\sigma}_{u,y}$ are the standard errors .drop Zt from (6.1) and rewrite it as

Y = Xβ + ε (6.4)

Such that E(ε) = 0 (6.5)

and E (εε1) = σ2 ITxT

The Bayesian Prior is included in

r = Rβ + υ (6.6)

Such that E (υ)= 0 (6.7)

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

and E (υυ1) is a diagonal matrix. The sample and the independent extraneous information may be combined,

$$\begin{bmatrix} y \\ r \end{bmatrix} = \begin{bmatrix} X \\ R \end{bmatrix} \beta + \begin{bmatrix} u \\ \upsilon \end{bmatrix} \qquad (6.8)$$

and $E \begin{bmatrix} u \\ v \end{bmatrix} = 0$

and $E\left(\begin{bmatrix} u \\ v \end{bmatrix} \begin{bmatrix} u^1 & v^1 \end{bmatrix} \right) = \begin{bmatrix} \Sigma & 0 \\ 0 & \Omega \end{bmatrix}$

An application of generalized least squares (GLS) procedure leads to estimating βas :

$$\hat{\beta} = \left([X^1 \; R^1] \begin{bmatrix} \Sigma & 0 \\ 0 & \Omega \end{bmatrix}^{-1} \begin{bmatrix} X \\ R \end{bmatrix} \right)^{-1} [X^1 \; R^1] \begin{bmatrix} \Sigma & 0 \\ 0 & \Omega \end{bmatrix}^{-1} \begin{bmatrix} y \\ r \end{bmatrix} \qquad (6.9)$$

0r $\hat{\beta} = \left[X^1 \Sigma^{-1} X + R^1 \Omega^{-1} R \right]^{-1} \left[X^1 \Sigma^{-1} y + R^1 \Omega^{-1} r \right]$ (6.10)

And the GLS estimator in (6.10) reduces to an ordinary least squares estimator :

$$\hat{\beta} = \left[X^1 X + \tilde{R}^1 \tilde{R} \right]^{-1} \left[X^1 y + \tilde{R}^1 \tilde{r} \right].$$

VII. AUTOREGRESSIVE MOVING AVERAGE MODEL WITH EXOGENEOUS INPUTS MODEL (ARMAX MODEL)

The notation ARMAX (p,q,b) refers to the model with P autoregressive terms, q moving average terms and b exogeneousterms. This model contains the AR (p) and MA (q) models and a Linear combination of the last b terms of a known and external time series dt. It is given by

$$X_t = \varepsilon_t + \sum_{i=1}^{P} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \sum_{i=1}^{b} \eta_i d_{t-i} \qquad (7.1)$$

where $\eta_1, \eta_2, ..., \eta_b$ are the parameters of the exogenous input d_t.

The ARMAX Model is implemented through the use of “exogeneous” or “independent” variables. The estimated parameters usually refer to the regression :

$$X_t - m_t = \varepsilon_t + \sum_{i=1}^{P} \varphi_i (X_{t-i} - m_{t-i}) + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \qquad (7.2)$$

wheremt incorporates all exogeneous (or independent) variables:

$$m_t = C + \sum_{i=1}^{b} \eta_i d_{t-i}.$$

VIII. NON–LINEAR AUTOREGRESSIVE EXOGENEOUS FORECASTING MODEL

In time series modeling, a non – linear Autoregressive exogenous forecasting model (NARX) is a non – linear autoregressive model which has exogenous inputs. This means that the model relates the current value of a time series which one would like to explain or predict to both :

Past values of the same series : and Current and past values of the driving (exogeneous) series.

The NARX model is stated algebraically as

$$y_t = F\left(y_{t-1}, y_{t-2}, y_{t-3}, \; ...., u_t, u_{t-1}, u_{t-2}, u_{t-3} .... \right) + \varepsilon_t \qquad (8.1)$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 5 Issue XII December 2017- Available at www.ijraset.com

where y is the variables of interest ,u is the externally determined variable and ε is the error term. The function F is some non-linear function , such as a polynomial. F can be a neural network, a wavelet network, a sigmoid network and so on. To test the non – linearity in a time–series, use the BDS – test (Brock – Dechert – Scheinman test).

IX. ADAPTIVE EXPONENTIAL SMOOTHING FORECASTING MODEL

Trigg and Leach (1967) introduced a forecasting model, Adaptive Exponential smoothing forecasting model, for to react faster to sudden changes in the level or a shift in the demand pattern. The smoothing equation is given by

$$S_t = \alpha_t Z_t + (1-\alpha_t)S_{t-1} = S_{t-1} + \alpha_t e_t \qquad (9.1)$$

where et = Zt – St-1 , is the one- step ahead forecast error and αt is a smoothing constant that changes with time.Trigg and leach (1967) recommend

$$\alpha_t = \left| \frac{SE_t}{\hat{\Delta}_t} \right| \qquad (9.2)$$

whereSEt is the smoothed forecast error $\hat{\Delta}_t$ is the smoothed absolute forecast error. Why bark (1973), recommends changing the smoothing constant only if the errors exceed certain specified control limits.

This scheme uses three different smoothing constants ,If δ_t denotes the indicator variable, $0 \le \alpha_B \le \alpha_M \le \alpha_H$

$$\delta_t = \begin{cases} 1, & if\ |e_t| > 4\sigma\ or\ |e_{t-1}| > 1.2\sigma\ and\ |e_t| > 1.2\sigma \\ & and\ both\ errors\ with\ the\ same\ sign \\ 0, & otherwise \end{cases}$$

then the smoothing constant is adjusted according to

$$\alpha_t = \begin{cases} \alpha_H\ , & if\quad \delta_t = 1 \\ \alpha_M\ , & if\quad \delta_t = 0\quad and\quad \delta_{t-1} = 1 \\ \alpha_B\ , & Otherwise \end{cases}$$

A pth order VAR, denoted by VAR (P), is

$$y_t = C + A_1 y_{t-1} + A_2 y_{t-2} + \ldots\ldots\ldots\ldots + A_p y_{t-p} + e_t \qquad (10.1)$$

where C is a (Kx l) vector of constants (intercept),

Ai is a kxk matrix, (for every i = 1,2 …P)

andet is a (Kxl) vector of error terms satisfying

E (et) = 0, every error term has Mean Zero;

E (et etl) = Ω , the contemporaneous covariance matrix of error terms is Ω

E (etelt-k) = 0, for any non – zero K

Thus the pth order VAR is called a VAR with Plags.One can write a VAR (P) with a concise matrix notation:

Y = BZ + U (10.2)

A VAR with P lags can always be equivalently rewritten as a VAR with only one lag by appropriately redefining the dependent variable.

The transformation amounts to merely stacking the lags of the VAR (p) variable in the new VAR (1) dependent variable.

The structural VAR with P lags (SVAR), is

Boyt =Co+Blyt-l + B2yt-2 + ……… + BP yt-p + ϵt (10.3)

where Co is a (kxl) vector of constants,

Bi is a (kxk) matrix,

andϵt is a (kxl) vector of error terms.

By pre multiplying the SVAR with the inverse of Bo ,

yt = $B_o^{-1}C_o + B_0^{-1}B_1 y_{t-1} + B_o^{-1}B_2 y_{t-2} + \ldots\ldots + B_0^{-1}B_P y_{t-P} + B_o^{-1}\varepsilon_t$ (10.4)

And denoting B0-1 C0 = C ,

Bo-1 Bi = Ai for i=1, …………P , and B0-1 ϵt = et

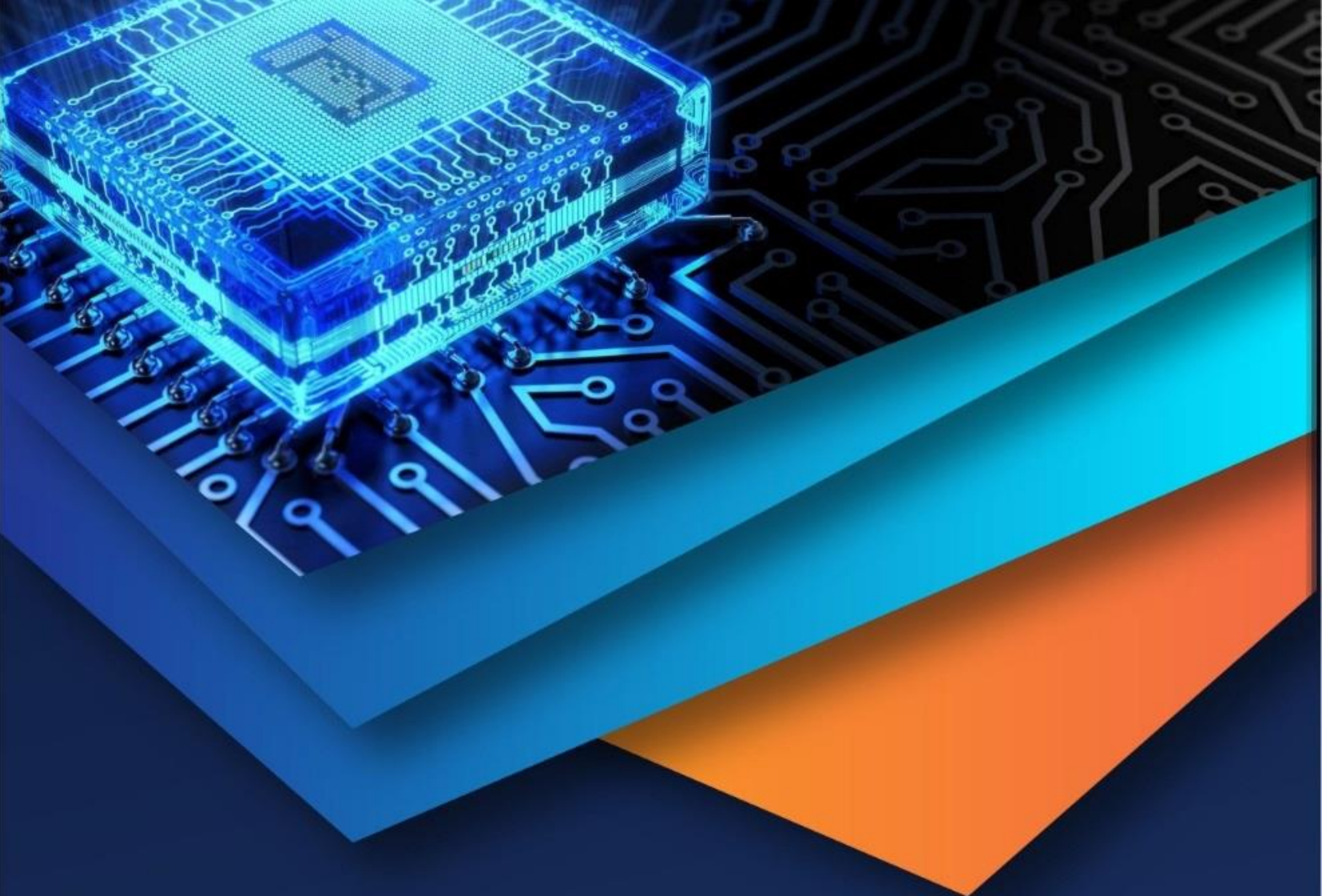
One obtain the Pth order reduced VAR $y_t = C + A_1 y_{t-1} + A_2 y_{t-2} + \ldots\ldots\ldots + A_p y_{t-p} + e_t$.

X. CONCLUSIONS

Autoregressive models have been an integral part of time series analysis for a long time. Structured regression models for time series have been around for many years and have figured prominently in the econometrics and business analysis these structural models have been used for years in forecasting and decomposition of time series in to trend, seasonal, cyclic and irregular components. he class of autoregressive integrated moving average (ARIMA) models that came to be associated with Box and Jenkins (1976) but has its roots in the Poineering work of Slutski and Yule in year 1920 and wold in year 1930. Most of the work deals with linear models for time series assuming continuous values. In the Applied regression analysis, the autoregressive models, moving average models and combined autoregressive and moving average models have a wide number applications. The study on autoregressive process/models is considered to be essential to both the theoretical and applied statisticians

REFERENCES

[1] Abraham, Bovas and William W.S. Wei, (1979), "Inferences in a Switching Time Series," American Statistical Association Proceedings of the Business and Economic Statistics Section, 354-358.
[2] Akaike, H. (1969), "Fitting Autoregressions for Prediction", Annals of the Institute of Statistical Mathematics, 21, 243-247.
[3] Berenblut, I.I., and Webb, G.I. (1973), "A New Test for Autocorrelated Errors in the Linear Regression Model," The City University, London, 33-50.
[4] Berndt, E.R., and Savin, N.E. (1977), "Conflict Among Criteria for Testing Hypotheses in Multivariate Linear Regression Model," Econometrika, 45, 1263-127.
[5] Durbin, J., and Watson, G.S. (1951), "Testing for Serial Correlation in Least Squares Regression II", Biometrica, 38, 159-178.
[6] Engle, R.F. (1982), "Autoregressive Conditional Heteroscedasticity with estimates of the variance of United Kingdom Inflation", Econometrica, 50: 987-1007.
[7] Engle, R.F. (1982), "Autoregressive conditional Heteroscedaticity with estimates of the variance of united kingdom inflation", Econometrica, Vol.50, pp. 987-1007.
[8] Hamilton, J.D. (1994), " TinaeSereis Analysis", Princeton university press, Princeton, New Jersey.
[9] Harvey, A.C. (1990), "Forecasting, Structural Time Series Models, and the Kalman Filter", Cambridge, U.K: Cambridge University Press.
[10] Magee, L. (1985), "Efficiency of Iterative Estimators in the Regression Model with AR(1) Disturbances, Journal of Econometrics, 29, 275-287.
[11] McCullagh, P., and Nelder, J.A. (1989), "Generalized Linear Models (2nd ed.)," London: Chapman and Hall.
[12] Williams, R.M. (1952). "Experimental Designs for Serially Correlated Observations," Biometrika, 39, 151-167.
[13] Yale, C., and Forsythe, A.B. (1976), "Winsorized Regression," Technometrics, 81, 291-300.
[14] Zellner, A. (1971), "An Introduction to Bayesian inference in Econometrics," Wiley, New York.



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY