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Combined Harvesting in a Prey-Predator Fishery with Time Delay

Dipankar Sadhukhan[1], Biswajit Mondal[2]
[1, 2] Haldia Government College, Haldia-721657, W.B., India,[*]Raja N.L.K. Women's College, Midnapore, W. B., India.

Abstract-In this paper, the combined harvesting policies of fishes have for been investigated as a prey- predator model with Beddington-DeAngelis ([1], [2]) response function considering time delay in both prey and predator in different situations. We have also discussed the behaviour of the system by con- trolling the harvesting effort. Finally, the model has been illustrated with some numerical examples and stability of the system under different conditions is depicted.

Keywords: Prey-predator, Fishery, Time-Delay, Beddington-DeAngelis response, Combined Harvesting.

I. INTRODUCTION

Predation, a complex natural phenomenon, exists almost everywhere in the nature. But the pioneer work on this phenomenon was first done by Lotka[3] and Volterra[4].After that, a number of research works for the existence of the system has been reported in the literature. Also, there are various papers studying the effect of harvesting of competition species, specially the effect of constant rate harvesting on the prey-predator systems [*cf.* Brauer and Soudack [5,6], Dai and Tang [7], Myers cough *et al.* [8]and Xiao and Ruan[9] and Clark [10] etc].They investigated stability, Hopf bifurcation, limit cycles of the systems. After that, researchers have introduced the delay in biological systems mainly on prey-predator system with or without harvesting. Work on general delay at the beginning were studied by Cushing([11], [12]), Gopalsamy[13],Kuang([14], [15])and MacDonald[14]etc. Beretta and Kuang[17],May[18]and Gail *et.al*[19] have developed some models with time-delay on prey-predator system. Also in recent time, Martin and Ruan[20]and Kar[21] have constructed some models on combined effects of constant rate harvesting and time-delay on the dynamics of predator-prey systems. Population dynamics of prey- predator system is normally effected by harvesting. This effort depends on the accepted strategic which may vary from rapid depletion to complete preservation of a population. Mathematical bio economics and also optimal management of renewable resources encompasses the study of the population dynamic of prey-predator system with harvesting. The exploitation of biological resources and harvesting of population species are common phenomena in forestry, fisheries and wildlife management. The study of the population of multispecies system are very difficult both theoretically and practically. Brauer and Soudact ([5], [6]) and Dai and Tang[5] developed their models with constant rate of harvesting in both prey and predator systems. They also established asymptotic stability theoretically and global stability by simulation. Mesterton-Gibbons ([22]- [24]) developed a different technique to find out the solution of combined harvesting of competing fish species and this technique has a wide application in ecological modelling, Fan and Wang [25] developed a model with exploitation of single fish population with periodic coefficients. Pielou[26]developed a logistic fishery model with impulsive selective harvesting and time delay. Recently Kar[21]and Kar and Matsuda[27]have developed a general type prey-predator model with selective harvesting and time delay in harvesting.

It may be noted that Kar[21] and Kar and Matsuda[27] considered the models with time delay either in prey and predator separately. They did not consider the time delay in both prey and predator simultaneously. Moreover, with Holling type-II response function, the models with time delay become easier to tackle mathematically. Till now, none has considered the time delayed prey-predator model with other types of interaction functions and time delays in both spices.

In this paper, a prey-predator model has been studied with Beddington-DeAngelis functional response form, harvesting and time-delay on prey and predator considered separately and on both species simultaneously. Here, the effect of time-delay is considered. The models are illustrated with some numerical examples. The stability aspect of the model with time delays on both species, harvesting is depicted. As a particular case, the models of Kar[21]andKar and Matsuda[27] are derived from the present models.

II. MODEL FORMULATION AND SOLUTIONS

A. Model-1

Let $x(t)$ and $y(t)$ denote the prey and predators' population densities at any time t respectively. Then considering harvesting and

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logistic growth of prey, a general ecological model with Beddington-DeAngelis ([1], [2]) response function i.e. Model-1 can be written as

$$\left.\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{\alpha xy}{1+ax+by}-Eqx\\ \frac{dy}{dt} &= \frac{\beta xy}{1+ax+by}-Eqy-dy\end{aligned}\right\}(1)$$

where prey population grows logistically with k as the carrying capacity, and r is the growth rate of the prey population, α, β, a, b are positive constants, E is harvesting effort, q is the catch per unit effort and d is predators' death rate.

B. Model-2

Now, considering the time delay on the harvesting of predator population only, we get the Model-2 as

$$\left.\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{\alpha xy}{1+ax+by}-Eqx\\ \frac{dy}{dt} &= \frac{\beta xy}{1+ax+by}-Eqy(t-\tau)-dy\end{aligned}\right\}(2)$$

C. Model-3

Again, considering time delay on the harvesting of prey population only, we get Model-3 as

$$\left.\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{\alpha xy}{1+ax+by}-Eqx(t-\tau)\\ \frac{dy}{dt} &= \frac{\beta xy}{1+ax+by}-Eqy-dy\end{aligned}\right\}(3)$$

D. Model-4

Finally considering time delay on both prey and predator, we get another Model-4 as

$$\left.\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{\alpha xy}{1+ax+by}-Eqx(t-\tau)\\ \frac{dy}{dt} &= \frac{\beta x(t-\tau)y(t-\tau)}{1+ax(t-\tau)+by(t-\tau)}-Eqy-dy\end{aligned}\right\}(4)$$

The delay τ (≥ 0) is a constant representing the assumption that the harvesting begins to occur after a certain age or size. When τ = 0, this becomes a system of ordinary differential equations (Model-1).

Model-2, we now investigate the dynamics of the system (2). Let (x^*, y^*) be the only interior equilibrium point of the Model-2and let $X = x - x^*, Y = y - y^*$ be the perturbed values. After removing the nonlinear terms, we obtain the linear variational system, by using equilibria conditions as

$$\left.\begin{aligned}\frac{dX}{dt} &= \left(r-\frac{\alpha y^*}{1+ax^*+by^*}+\frac{a\alpha x^*y^*}{(1+ax^*+by^*)^2}-Eq\right)X+\left(\frac{b\alpha x^*y^*}{(1+ax^*+by^*)^2}-\frac{\alpha x^*}{1+ax^*+by^*}\right)Y\\ \frac{dY}{dt} &= \left(\frac{\beta y^*}{1+ax^*+by^*}-\frac{a\beta x^*y^*}{(1+ax^*+by^*)^2}\right)X+\left(\frac{\beta x^*}{1+ax^*+by^*}-\frac{b\beta x^*y^*}{(1+ax^*+by^*)^2}-d\right)Y-EqY(t-\tau)\end{aligned}\right\}(5)$$

From the linearized system, we obtain the characteristic equation

$$\Delta(\lambda,\tau) = \lambda^2 + l\lambda + m + (n\lambda + f)e^{-\lambda\tau} = 0 \quad (6)$$

where

$$l = -\left[\left(r-\frac{\alpha y^*}{1+ax^*+by^*}+\frac{a\alpha x^*y^*}{(1+ax^*+by^*)^2}-Eq\right)+\left(\frac{\beta x^*}{1+ax^*+by^*}-\frac{b\beta x^*y^*}{(1+ax^*+by^*)^2}-d\right)\right],$$

$$m = \left(r-\frac{\alpha y^*}{1+ax^*+by^*}+\frac{a\alpha x^*y^*}{(1+ax^*+by^*)^2}-Eq\right)\left(\frac{\beta x^*}{1+ax^*+by^*}-\frac{b\beta x^*y^*}{(1+ax^*+by^*)^2}-d\right) - \left(\frac{b\alpha x^*y^*}{(1+ax^*+by^*)^2}-\frac{\alpha x^*}{1+ax^*+by^*}\right)\left(\frac{\beta y^*}{1+ax^*+by^*}-\frac{a\beta x^*y^*}{(1+ax^*+by^*)^2}\right),$$

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$$n = Eq,$$

and

$$f = -\left(r - \frac{\alpha y^*}{1 + ax^* + by^*} + \frac{a\alpha x^* y^*}{(1 + ax^* + by^*)^2} - Eq\right) Eq.$$

For τ = 0, the characteristic equation (6) reduces to

$\lambda^2 + (l + n)\lambda + (m + f) = 0$ (7)

As sum of the roots = $-(l + n)$ and product of the roots = $(m + f)$, we conclude that both the roots of (7) are real and negative or complex conjugate with negative real parts if and only if

$l + n > 0 \; and \; m + f > 0$ (8)

Hence in the absence of time delay, the system is locally asymptotically stable if and only if $l + n > 0$ and $m + f > 0$ hold simultaneously.

Now for $\tau \neq 0$, if $\lambda = i\omega$ is a root of the equation (6), then we have

$-\omega^2 + fe^{-i\omega\tau} + il\omega + m + in\omega e^{-i\omega\tau} = 0$ (9)

Separating real and imaginary parts we get

$$\left.\begin{array}{l} m - \omega^2 + f\cos\omega\tau + m\omega\sin\omega\tau = 0 \\ l\omega - f\sin\omega\tau + n\omega\cos\omega\tau = 0 \end{array}\right\} (10)$$

From (10), we obtain the fourth-order equation for ω as

$\omega^4 + (l^2 - n^2 - 2m)\omega^2 + m^2 - f^2 = 0$ (11)

From (11) it follows that if

$l^2 - n^2 - 2m > 0 \; and \; m^2 - f^2 > 0$(12)

Then equation (12) does not have any real solution. To find the necessary and sufficient conditions for nonexistence of delay induced instability, we now use the following theorem.

1) *Theorem-1:* A set of necessary and sufficient conditions for (x^*, y^*) to be asymptotically stable for all τ ≥ 0 is the following.

a) *The real parts of all the roots of* $\Delta(\lambda, 0) = 0$ *are negative.*

b) *For all real m and* $\tau \geq 0, \Delta(im, \tau) \neq 0$, *where* $i = \sqrt{-1}$.

Proof of the theorem is obvious from earlier equations (6) and (7).

2) *Theorem-2.* If conditions (8) and (12) are satisfied, then (x^*, y^*) is locally asymptotically stable for all τ ≥ 0.

3) *Proof:* Proof is obvious from conditions (8), (12) and *Theorem-1.* From (11), it is observed that there is a unique positive solution ω_0^2 if

$m^2 - f^2 < 0$(13)

Substituting ω_0^2 into (9) and solving for τ,we get

$\tau_{0n} = \frac{1}{\omega_0} \, arc\tan\left(-\frac{\omega_0(lf - mn + n\omega_0^2)}{nl\omega_0^2 + (m - \omega_0^2)f}\right) + \frac{2n'\pi}{\omega_0}, \quad n' = 0, \pm1, \pm2, \cdots$ (14)

Again, if $m^2 - f^2 > 0$, $l^2 - n^2 - 2m > 0$ and

$(l^2 - n^2 - 2m)^2 > 4(m^2 - f^2)$(15)

hold, then there will be two positive solutions $\omega_\pm^2$. Substituting $\omega_\pm^2$ into (9) and solving for τ, we get

$\tau_\pm = \frac{1}{\omega_\pm} \, arc\tan\left(-\frac{\omega_\pm(lf - mn + n\omega_\pm^2)}{nl\omega_\pm^2 + (m - \omega_\pm^2)f}\right) + \frac{2k\pi}{\omega_\pm}, \quad k = 0, 1, 2, \cdots$(16)

Differentiating equation (6) with respect to τ, we have

$[2\lambda + l + ne^{-\lambda\tau} - \tau(n\lambda + f)e^{-\lambda\tau}]\frac{d\lambda}{d\tau} = \lambda e^{-\lambda\tau}(n\lambda + f)$(17)

Therefore,

$$\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{2\lambda+l}{-\lambda(\lambda^2+l\lambda+m)} + \frac{n}{\lambda(n\lambda+f)} - \frac{\tau}{\lambda} \quad (18)$$

by using $e^{-\lambda\tau} = -\left(\frac{\lambda^2+l\lambda+m}{n\lambda+f}\right)$

Thus,

$$sign\left\{\frac{d(\mathscr{R}e\lambda)}{d\lambda}\right\}_{\lambda=i\omega} = sign\left\{\mathscr{R}e\left(\frac{d\lambda}{d\tau}\right)^{-1}\right\}_{\lambda=i\omega} = sign\left[\frac{2(\omega^2-m)+l^2}{(-\omega^2+m)^2+l^2\omega^2} - \frac{n^2}{(f^2+n^2\omega^2)}\right] (19)$$

Theorem-3. If (7) and (12) hold, then the equilibrium (x^*, y^*) is asymptotically stable for $\tau < \tau_0$ and unstable for $\tau > \tau_0$. Further, as τ increases through τ_0, (x^*, y^*) bifurcates into small amplitude periodic solutions, where $\tau_0 = \tau_{0n}$ as $n = 0$.

Proof. For τ = 0, (x^*, y^*) is asymptotically stable if condition (7) holds. Hence, by Butler's lemma (x^*, y^*) remains stable for $\tau < \tau_0$. Now, it is to be shown that

$$\left[\frac{d(Re\lambda)}{d\tau}\right]_{\tau=\tau_0,\omega=\omega_0} > 0 \ (20)$$

This signifies that there exists at least one eigen value with positive real part for $\tau > \tau_0$. Moreover, the conditions of Hoff bifurcation are then satisfied yielding the required periodic solution.

From (17), it follows that

$$sign\left\{\frac{d(\mathscr{R}e\lambda)}{d\lambda}\right\}_{\lambda=i\omega_0} = sign\left[\frac{\sqrt{(l^2-n^2-2m)^2-4(m^2-f^2)}}{\{(-\omega^2+m)^2+l^2\omega^2\}(f^2+n^2\omega^2)}\right] (21)$$

Therefore,

$$\left[\frac{d(Re\lambda)}{d\tau}\right]_{\tau=\tau_0,\omega=\omega_0} > 0 (22)$$

Therefore, the transversability condition is satisfied and hence, Hopf bifurcation occurs at $\omega = \omega_0, \tau = \tau_0$. This completes the proof.

4) *Theorem-4.* Let $\tau_k^{\pm}$ be defined in (15). If (7) and (14) hold, then there exists a positive integer m such that there are m switches from stability to instability and then to stability. In other words, when $\tau \in [0, \tau_0^+), (\tau_0^-, \tau_1^+), \cdots, (\tau_{m-1}^-, \tau_m^+)$ equilibrium (x^*, y^*) is stable, and when $\tau \in [\tau_0^+, \tau_0^-), (\tau_1^+, \tau_1^-), \cdots, (\tau_{m-1}^+, \tau_{m-1}^-)$, (x^*, y^*) is unstable.

Therefore, there are bifurcations at (x^*, y^*) when $\tau = \tau_k^{\pm}, k = 0, 1, 2, \cdots$.

5) *Proof.* If conditions (7) and (14) hold, then the theorem will be proved if the following transversability conditions are satisfied.

$$\left[\frac{d(Re\lambda)}{d\tau}\right]_{\tau=\tau_k^+} > 0 \ \ and \ \left[\frac{d(Re\lambda)}{d\tau}\right]_{\tau=\tau_k^-} < 0$$

From (17), it follows that

$$sign\left\{\frac{d(\mathscr{R}e\lambda)}{d\lambda}\right\}_{\lambda=i\omega_+} = sign\left[\frac{\sqrt{(l^2-n^2-2m)^2-4(m^2-f^2)}}{\{(-\omega_+^2+m)^2+l^2\omega_+^2\}(f^2+n^2\omega_+^2)}\right]$$

Therefore,

$$\left[\frac{d(Re\lambda)}{d\tau}\right]_{\omega=\omega_+,\tau=\tau_k^+} > 0$$

Again,

$$sign\left\{\frac{d(\mathscr{R}e\lambda)}{d\lambda}\right\}_{\lambda=i\omega_-} = sign\left[\frac{\sqrt{(l^2-n^2-2m)^2-4(m^2-f^2)}}{\{(-\omega_-^2+m)^2+l^2\omega_-^2\}(f^2+n^2\omega_-^2)}\right]$$

Therefore,

$$\left[\frac{d(Re\lambda)}{d\tau}\right]_{\omega=\omega_-,\tau=\tau_k^-} < 0$$

Hence, the transversability conditions hold. This completes the proof.

6) *Model-3:* Also in Model-3, we now investigate the dynamics of the Model-3. Let (x^*, y^*)be the only interior equilibrium point of the Model-3 and let = − *, = − *be the perturbed values. After removing the nonlinear terms, we obtain the linear variational system, by using equilibria conditions as

$$\left.\begin{array}{l} - = (\ - \frac{*}{1+ \ *+ \ *} + \frac{* \ *}{(1+ \ *+ \ *)^2}) \ + (\frac{* \ *}{(1+ \ *+ \ *)^2} - \frac{*}{1+ \ *+ \ *}) \ - \ (\ - \ ) \\ \frac{}{d} = (\frac{*}{1+ \ *+ \ *} - \frac{* \ *}{(1+ \ *+ \ *)^2}) \ + (\frac{*}{1+ \ *+ \ *} - \frac{* \ *}{(1+ \ *+ \ *)^2} - \ - \ ) \end{array}\right\}(23)$$

From the linearized system, we obtain the characteristic equation for Model-3, as

$$\Delta(\ , \ ) = \ ^2 + \ _1 \ + \ _1 + (\ _1 \ + \ _1) \ ^- \ = 0 \quad (24)$$

where,

$$_1 = -\left[\left(\ - \frac{*}{1+ \ *+ \ *} + \frac{* \ *}{(1+ \ *+ \ *)^2}\right) + \left(\frac{*}{1+ \ *+ \ *} - \frac{* \ *}{(1+ \ *+ \ *)^2} - \ - \ \right)\right]$$

$$_1 = \left(\ - \frac{*}{1+ \ *+ \ *} + \frac{* \ *}{(1+ \ *+ \ *)^2}\right)\left(\frac{*}{1+ \ *+ \ *} - \frac{* \ *}{(1+ \ *+ \ *)^2} - \ - \ \right) - \left(\frac{* \ *}{(1+ \ *+ \ *)^2} - \frac{*}{1+ \ x^*+ \ *}\right)\left(\frac{*}{1+ \ *+ \ *} - \frac{* \ *}{(1+ \ *+ \ *)^2}\right)$$

$$_1 =$$

$$_1 = -\left(\ - \frac{*}{1+ \ *+ \ *} + \frac{* \ *}{(1+ \ *+ \ *)^2} - \ - \ \right)$$

The dynamics of Model-3 can also derived similar to Model-2.

Model-4: Let (*, *) be the only interior equilibrium of the Model-4 and let = − *, = − *be the perturbed values. After removing the nonlinear terms, we obtain the linear variational system, by using equilibria conditions as

$$\begin{array}{l} - = (\ - \frac{*}{1+ \ *+ \ *} + \frac{* \ *}{(1+ \ *+ \ *)^2}) \ + (\frac{* \ *}{(1+ \ *+ \ *)^2} - \frac{*}{1+ \ *+ \ *}) \ - \ (\ - \ ) \\ - = (\frac{*}{1+ \ *+ \ *} - \frac{* \ *}{(1+ \ *+ \ *)^2}) \ (\ - \ ) + (\frac{*}{1+ \ *+ \ *} - \frac{* \ *}{(1+ \ *+ \ *)^2}) \ (\ - \ ) - \ - \end{array} \quad (25)$$

From the linearized system, we obtain the characteristic equation

$$\Delta(\square,\square) = \square^2 + \square\square + \square + (\square\square + \square)\square^{-\square\square} + \square\square^{-2\square\square} = 0 \quad (26)$$

where,

$$\square = -\left(\square - \frac{\square\square^*}{1+\square\square^*+\square\square^*} + \frac{\square\square\square^*\square^*}{(1+\square\square^*+\square\square^*)^2} - \square\square - \square\right)$$
$$\square = \left(\square - \frac{\square\square^*}{1+\square\square^*+\square\square^*} + \frac{\square\square\square^*\square^*}{(1+\square\square^*+\square\square^*)^2} - \square\square\right)(\square\square + \square)$$
$$\square = -\left[\left(\frac{\square\square^*}{1+a\square^*+\square\square^*} - \frac{\square\square\square^*\square^*}{(1+\square\square^*+\square\square^*)^2}\right) - \square\square\right]$$
$$\square = -\left[\begin{array}{c}\left(\frac{\square\square\square^*\square^*}{(1+\square\square^*+\square\square^*)^2} - \frac{\square\square^*}{1+\square\square^*+\square\square^*}\right)\left(\frac{\square\square^*}{1+\square\square^*+\square\square^*} - \frac{\square\square\square^*\square^*}{(1+\square\square^*+\square\square^*)^2}\right) \\ -\left(\square - \frac{\square\square^*}{1+\square\square^*+\square\square^*} + \frac{\square\square\square^*\square^*}{(1+\square\square^*+\square\square^*)^2}\right)\left(\frac{\square\square^*}{1+\square\square^*+\square\square^*} - \frac{\square\square\square^*\square^*}{(1+\square\square^*+\square\square^*)^2}\right) - \square\square\square - \square^2\square^2\end{array}\right] \quad (27)$$
$$\square = -\left(\frac{\square\square^*}{1+\square\square^*+\square\square^*} - \frac{\square\square\square^*\square^*}{(1+\square\square^*+\square\square^*)^2}\right)\square\square$$

Now for $\square \neq 0$, if $\square = \square\square$ is a root of the equation (26), then we have

$$-\square^2 + \square\square\square + \square + (\square\square\square + \square)\square^{-\square\square\square} + \square\square^{-2\square\square\square} = 0 \quad (28)$$

Separating the real and imaginary parts we get

$$\begin{array}{l}\square\cos\square\square + \square\sin\square\square + \square\cos 2\square\square = \square^2 - \square \\ \square\square\cos\square\square - \square\sin\square\square - \square\sin 2\square\square = -\square\square\end{array} \quad (29)$$

It is quite difficult to find out the condition for stability and bifurcation from Model-4 so we illustrate the model-4 by numerical simulation.

III. NUMERICAL EXPERIMENTS

A. Simulation

For numerical simulation of Model-4, we consider the parametric values as $\square = 0.45, \square = 100, \square = 0.35, \square = 0.05, \square = 0.3, \square = 0.07, \square = 0.001, \square = 10\ \square\square\square\square = 0.6$.With this data, we simulate the Model-4(equation (4)) and the population of a species are depicted against the time, t for different values of τ, time delay. It is observed that up to τ = 17.65 the system is stable and for τ > 17.65, it oscillates. Here, with τ = 17and18, phase-space trajectory of the system has been presented in Fig-1 and -2 respectively.

It is observed that Fig-1 portrays the stability of the system whereas Fig-2 gives the divergent figures for both prey predator with the increase of time.

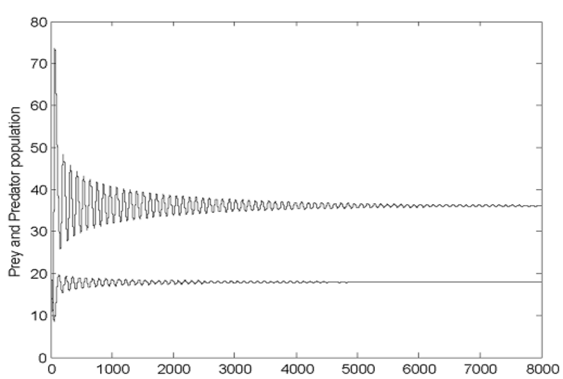

Fig.-1: Phase-space trajectory for τ =17

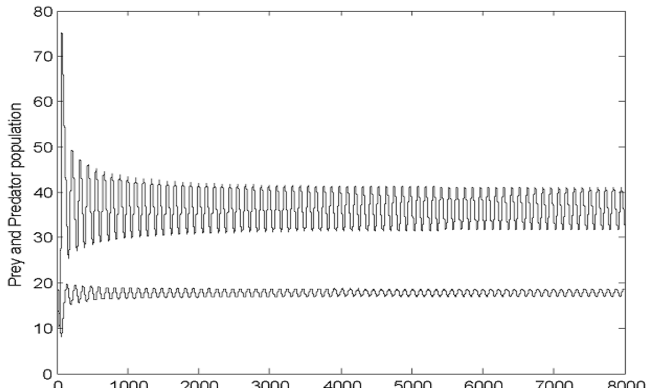

Fig.-2: Phase-space trajectory for τ =18

B. *Particular Case*

With□ = *1.8*, □ = *100*, □ = *0.1*, □ = *0.19*, □ = *0.114*, □ = *0.5*, □ = *0*, □□ = *0.5* and without considering the harvesting for prey in Model-2, we can get the first example of Kar[16]. Also taking□ = *2*, □ = *100*, □ = *0.05*, □ = *0.05*, □ = *0.035*, □ = *0.5*, □ = *0*, □□ = *0.224*and not considering the harvesting for predator in Model-3, we get the second example of Kar[16].

IV. CONCLUSION

In this model, we considered four prey-predator models with harvesting and using Beddington- DeAngelis response function. In each model, we considered combined harvesting of both populations. In model-2, selective harvesting of predator has been considered using time delay and in Model-3, time delay is considered on prey harvesting function and on response function.

From ecological point of view, to make a system stable for long duration, time delay in the harvesting is very useful. Though many researchers have worked with harvesting but till now, none considered combined harvesting with time delay. Also, none considered Beddington-DeAngelis response function for this type of models

For numerical illustration, we have studied four models with some chosen data. Also, some figures are depicted with different values of τ. In particular, some previous results have been derived.

This paper has been developed in a general form with a particular type of response function. In future, this can be extended with other types of interaction functions such as Holling type-III, Holling type-IV, etc.

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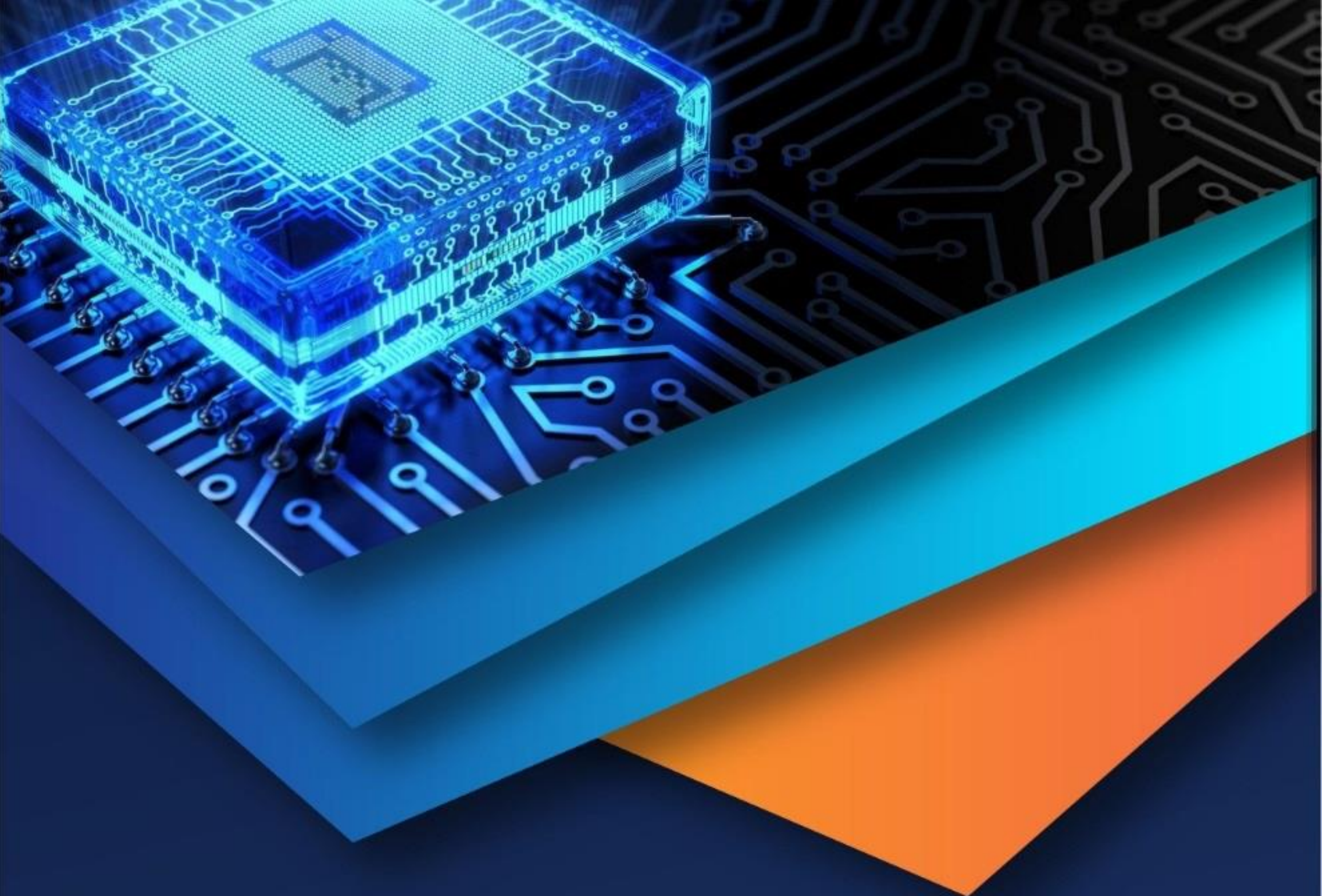

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