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Spherical Homogeneous Cosmological Model with Conformally Invariant Scalar Field

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***Abstract:** Einstein's field equation for a spatially homogeneous four dimensional spherically symmetric space-time in presence of a conformally invariant scalar field has been discussed. The cosmological solutions for the cosmic parameters 'λ' and 'μ' are obtained in two particular cases using the various field equations and new cosmological models of the universe has been designed . The kinematical properties are discussed later followed by the concluding remarks.*
***Key words:** Cosmology, Spherically Symmetric, Space-time, conformally invariant Scalar field.*

I. INTRODUCTION

No branch of science can claim to have a bigger area of interest than cosmology. It is a study of large scale structure of the universe. Different fields of gravitation in form of tensor equations are applied to various form of space-times or model of the universe and a new form of it is constructed with varied physical and geometrical properties. There are two types of scalar fields viz. zero rest mass scalar field and massive scalar field, which describe long range and short range interactions respectively. The study of zero-rest-mass scalar fields in general relativity has been initiated to provide an understanding of the space-time and the gravitational field associated with neutral elementary particles of zero spin. Bergmann and Leipnik [1] studied solutions of Einstein's field equations in static, spherically symmetric space-time with an energy-momentum tensor generated from a neutral massless scalar fields. R. N.Patra et. al. [2] have studied in a New Scalar tensor theory of gravitation in five dimensional spherically symmetric space-time. R. N.Patra and R.R.Swain [3] have studied identification of scalar meson field with perfect fluid in Bimetric theory of gravitation. Accioly et. al. [4] obtained solution of field equations to the conformally invariant scalar field with trace free energy momentum tensor as source in Bianchi type-I spacetime. Innaiah and Reddy [5] have discussed a flat Robertson-Walkar type solution treating the conformally invariant scalar fields. Venkateswarlu and Reddy (6) have obtained Bianchi type-II, VIII and IX cosmological solutions in conformally invariant scalar field. Shri Ram [7] has obtained a exact solution of the Einstein equations corresponding to a conformally invariant scalar-field with trace-free energy momentum tensor. In the present paper, we are interested to construct a four dimensional spherically symmetric cosmological model with conformally invariant scalar field. The metric and the field equations with the exact solutions are given in Section-2 and Section-3 respectively. The various physical and geometrical properties has been studied in Section-4 followed by the concluding remarks in Section-5.

II. METRIC AND FIELD EQUATIONS

We consider a spherically symmetric metric of the form

$$ds^2 = -e^{\lambda}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^{\mu}dt^2, \quad (1)$$

where $\mu(t)$ and $\lambda(t)$ are cosmic scale factors. The field equations for the conformally invariant scalar field and the gravitational field from a conformally invariant action integral are

$$R_{\mu\nu}\, f(u) = g_{\mu\nu}\, u_{,\alpha}u^{,\alpha} - 4u_{,\mu}u_{,\nu} + 2uu_{,\mu\nu} \quad (2)$$

$$R = 0 \quad (3)$$

and $$u^{;\nu}_{;\mu} = 0 \quad (4)$$

where comma ',' denotes ordinary derivative and semicolon ',' denotes covariant derivative and

$$f(u) = 1 - u^2, \quad (5)$$

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where $$u = \left(\frac{k}{6}\right)^{\frac{1}{2}} \phi, \tag{6}$$

here ϕ being the mass less scalar field and other symbols have their usual meaning.

The field equations (2) –(6) for metric (1) in co-moving coordinates lead to

$$4\lambda_{44} + 3\lambda_4^2 = \frac{-2\dot{u}^2 e^{\mu} - 8u\dot{u}\dot{\lambda}e^{\mu}}{1-u^2} \tag{7}$$

$$\bar{e}^{\lambda} - 1 = \frac{r^2\dot{u}^2}{1-u^2} \tag{8}$$

and $$2\lambda_{44} + \lambda_4^2 - \lambda_4\mu_4 = \frac{4\left(e^{\mu}\dot{u}^2 - 4\dot{u}^2 + u\dot{u}\dot{\mu}\right) + 2u\ddot{u}}{1-u^2}, \tag{9}$$

here, '.' denotes ordinary differentiation w. r. t 't'.
For the metric (1), equation (3) becomes

$$4\lambda_{44} + 4\lambda_4^2 - \lambda_4\mu_4 = \frac{8}{r^2}e^{\mu-\lambda} - \frac{8}{r^2}e^{\mu} \tag{10}$$

and equation(4) becomes

$$\frac{\lambda_4}{2}\dot{u} + \frac{\mu_4}{2}\dot{u} + \ddot{u} = 0$$

$$\Rightarrow \lambda_4 + \mu_4 = \frac{-2\ddot{u}}{\dot{u}}. \tag{11}$$

A. *Solution to field Equations*

Equation(8) yields

$$\lambda = \ln\left(\frac{1-u^2}{1-u^2+r^2\dot{u}^2}\right). \tag{12}$$

Integrating both sides of equation (11) w.r.t. 't', we get,

$$u_4 = e^{-\left(\frac{\lambda+\mu}{2}\right)}. \tag{13}$$

We try particular solution from equation (9) to equation (13) in two ways:

B. *Solution-1*

Let us tak

$$\left.\begin{aligned} \lambda &= 1-t \\ \mu &= 3t-1 \end{aligned}\right\}. \tag{14}$$

Putting these values in equation (13), we get

$$u = -e^{-t} \tag{15}$$

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The scalar potential is a function of cosmic time which can be shown in the $u \sim t$ graph-I Putting the values of 'λ' and 'μ' from equation (14) in equation (1), the model of the universe becomes

$$ds^2 = -e^{(1-t)}\, dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 + e^{(3t-1)}\, dt^2. \tag{16}$$

C. Solution-2

Putting equation (13) in equation (12), we get

$$\lambda = \ln \frac{1-u^2}{1-u^2+r^2 e^{-(\lambda+\mu)}} \tag{17}$$

Let us take $\lambda = -\mu.$ (18)

So that equation (17) becomes

$$\lambda = \ln\left(\frac{1-u^2}{1-u^2+r^2}\right) = -\mu. \tag{19}$$

Equation (19) yields

$$u = \left(1 - \frac{r^2}{e^{-\lambda}-1}\right)^{\frac{1}{2}} = \left(1 - \frac{r^2}{e^{\mu}-1}\right)^{\frac{1}{2}}. \tag{20}$$

Thus 'u' is found to be a function of 't' as λ and μ are function of 't' and by taking

$$\lambda = -\mu = t, \tag{21}$$

equation (20) yields

$$u = \left(1 - \frac{r^2}{e^{-t}-1}\right)^{\frac{1}{2}}. \tag{22}$$

From which we study the variation of the scalar potential with time in $u \sim t$ graph-II, characteristics.

Putting the value of 'u' from equation (21) in equation (1), we get the form of our metric as

$$ds^2 = -e^t\, dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^{-t}\, dt^2. \tag{23}$$

D. Study of Kinematical Properties

The average scale factor 'R' of the metric(1) is defined as

$$R = (\lambda\mu)^{\frac{1}{2}}. \tag{24}$$

The volume scale factor

$$V = R^2 = \lambda\mu. \tag{25}$$

We define the generalized mean Hubble parameter

$$H = \frac{1}{2}(H_1 + H_2), \tag{26}$$

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where $H_1 = \frac{\dot{\lambda}}{\lambda}, H_2 = \frac{\dot{\mu}}{\mu}$ are directional Hubble parameter in the direction X and Y respectively.

The expansion scalar

$$\theta = 2H = H_1 + H_2. \tag{27}$$

The deceleration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{28}$$

The mean anisotropy parameter

$$A = \frac{1}{2}\left(\frac{\Delta H_1}{H} + \frac{\Delta H_2}{H}\right). \tag{29}$$

The above properties for the two different models of the universe are as follows:

1) *Volume (V) from equation (25)*

a) *Case-I* $(\lambda = 1 - t, \mu = 3t - 1)$

$V = -3t^2 + 4t - 1$, that means the volume of the universe decreases as time increases and which has shown in the v~t Graph-I.

b) *Case-II* $(\lambda = -\mu = t)$

$V = -t^2$, which shows that the spatial volume of the universe decrease rapidly with time which reflect in V~t Graph-II.

2) *Hubble parameter (H), from equation (26)*

a) *Case-I* $H = \frac{3t - 2}{-3t^2 + 4t + 1}$. $\quad H = \frac{1}{t}$.

The variation of the Hubble parameter with time for both the cases has been shown in the H~t Graph-I and II.

3) *Expansion Scalar* (θ), *from equation (27)*

a) *Case-I* $\theta = 2H = \frac{2(3t - 2)}{3t^2 - 4t + 1}$.

b) *Case-II:* $\theta = \frac{2}{t}$, it shows the decrease of volume with increase of 't' in $\theta \sim t$ graph I and II for both the cases respectively.

4) *Deceleration Parametric (q), From equation (28)*

a) *Case-I:* $q > 0$, it shows that the universe is contracting as time increases.

b) *Case-II:* $q = 0$, it shows the constant nature of the universe which is possible, since the volume decreases with 't' and after some time, it is constant.

5). *Mean anisotropy Parameter (A), from equation (29)*

b) *Case-I* $A = 0$, the universe shows isotropy.

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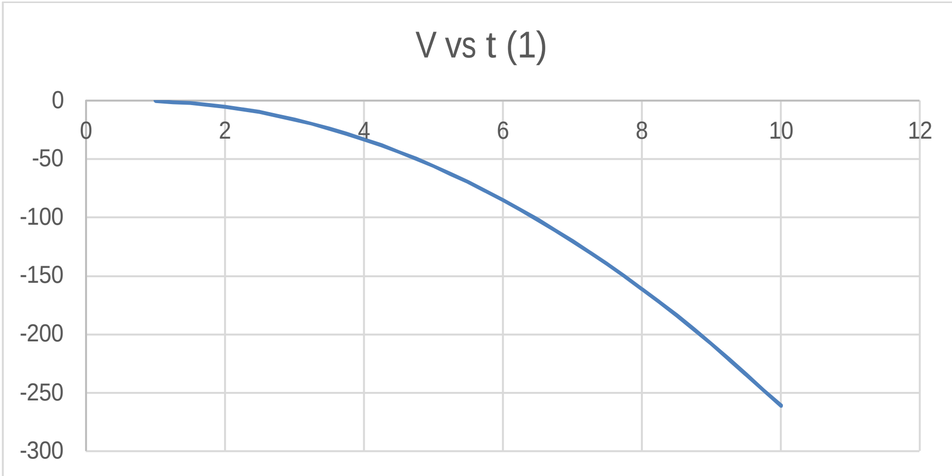
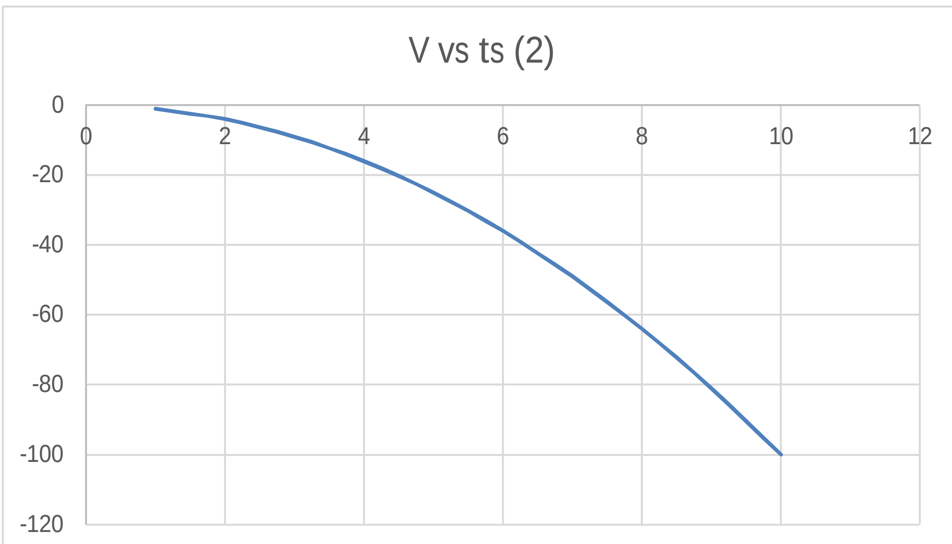
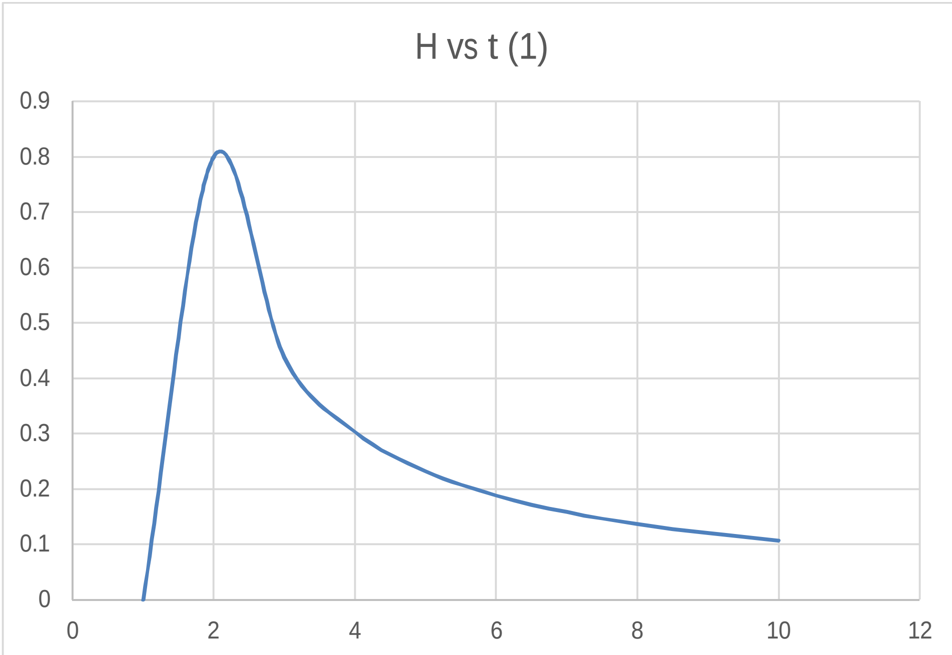
c) Case-II $A = 0$, the universe is isotropic by nature.



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0.4
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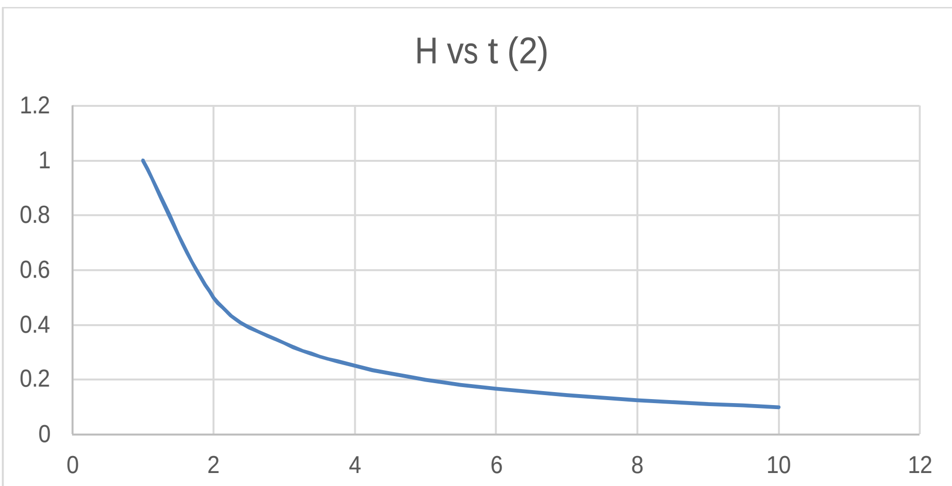
0.2
0.1
0
0
2
4
6
8
10
12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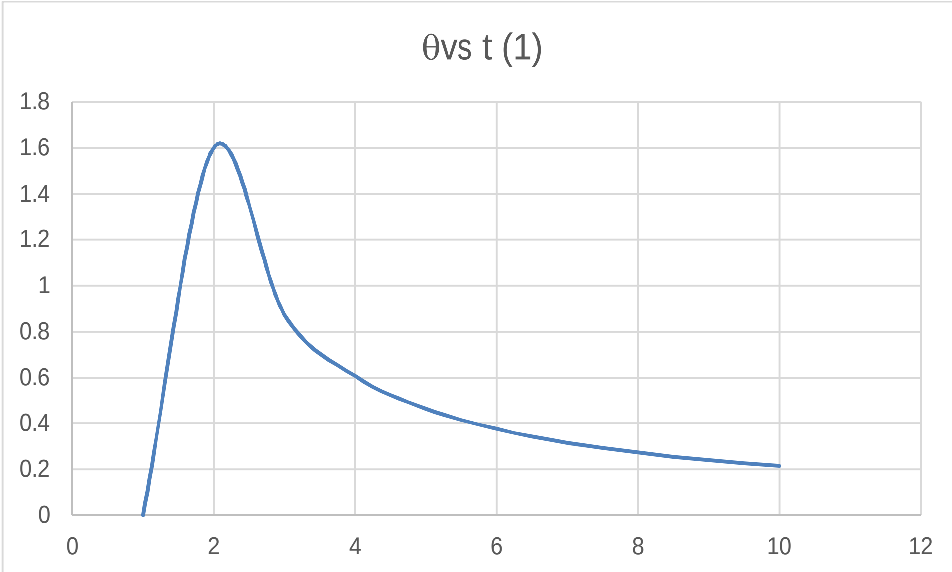
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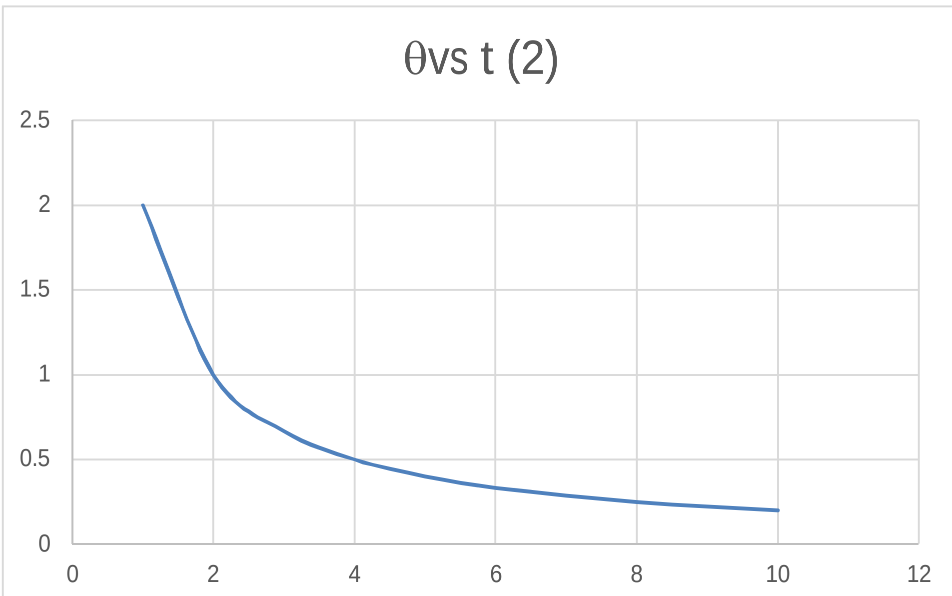
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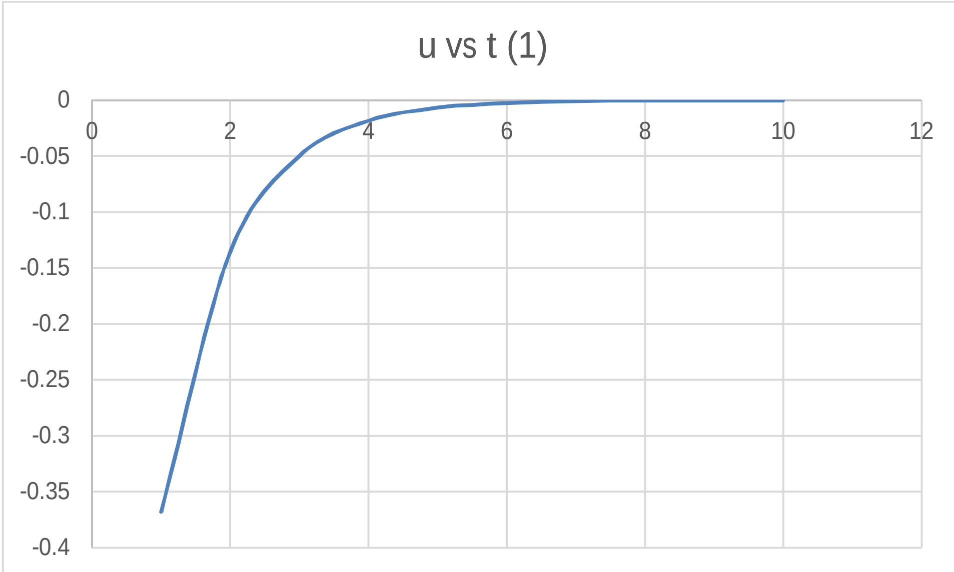

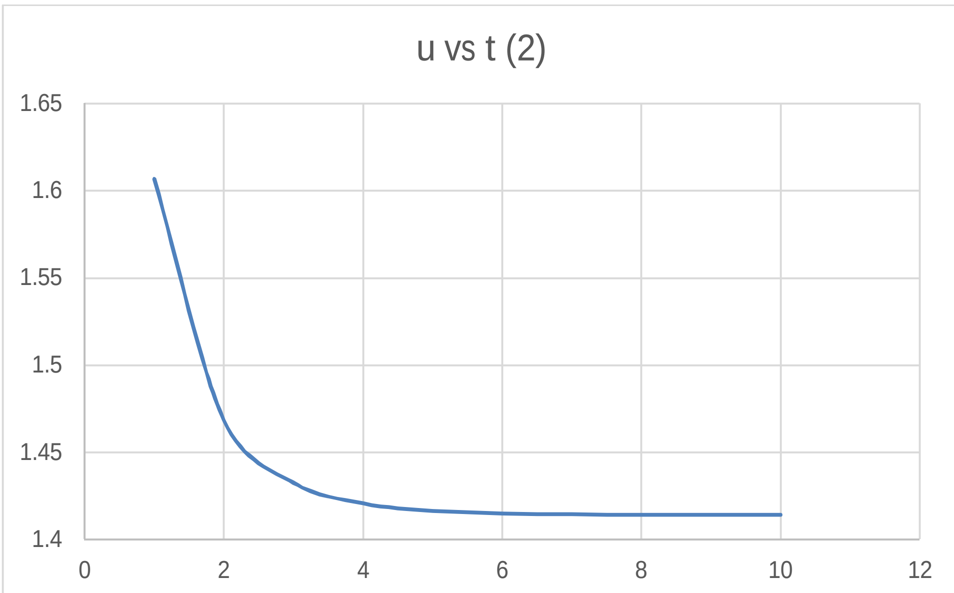

III. CONCLUSION

In this paper, we have taken a homogeneous spherically symmetric space-time with conformally invariant scalar field. We got solution of the field equation in two particular cases.

A. *Case:-I:* The scalar potential (u) and also the mass less scalar field (ϕ) are function of cosmic time when the cosmic parameters are $\lambda = 1 - t \text{ and } \mu = 3t - 1$. The universe is found to be isotropic $(\because A = 0)$ and $H = f(t)$. The spatial volume decreases with 't' which is found from the value of the expansion scalar (θ) and the deceleration parameter $q > 0$.

B. *Case-II:* It is found that $u = f(t)$ when $\lambda = -\mu = t$. The universe shows isotropy $(\because A = 0)$ and 'H' is found to be function of cosmic time. The volume of the universe decrease with time and after some time it becomes constant as $q = 0$. Lastly we conclude that for both the linear solutions of λ & μ, there exist two different metrices with same physical and geometrical properties except scalar potential.

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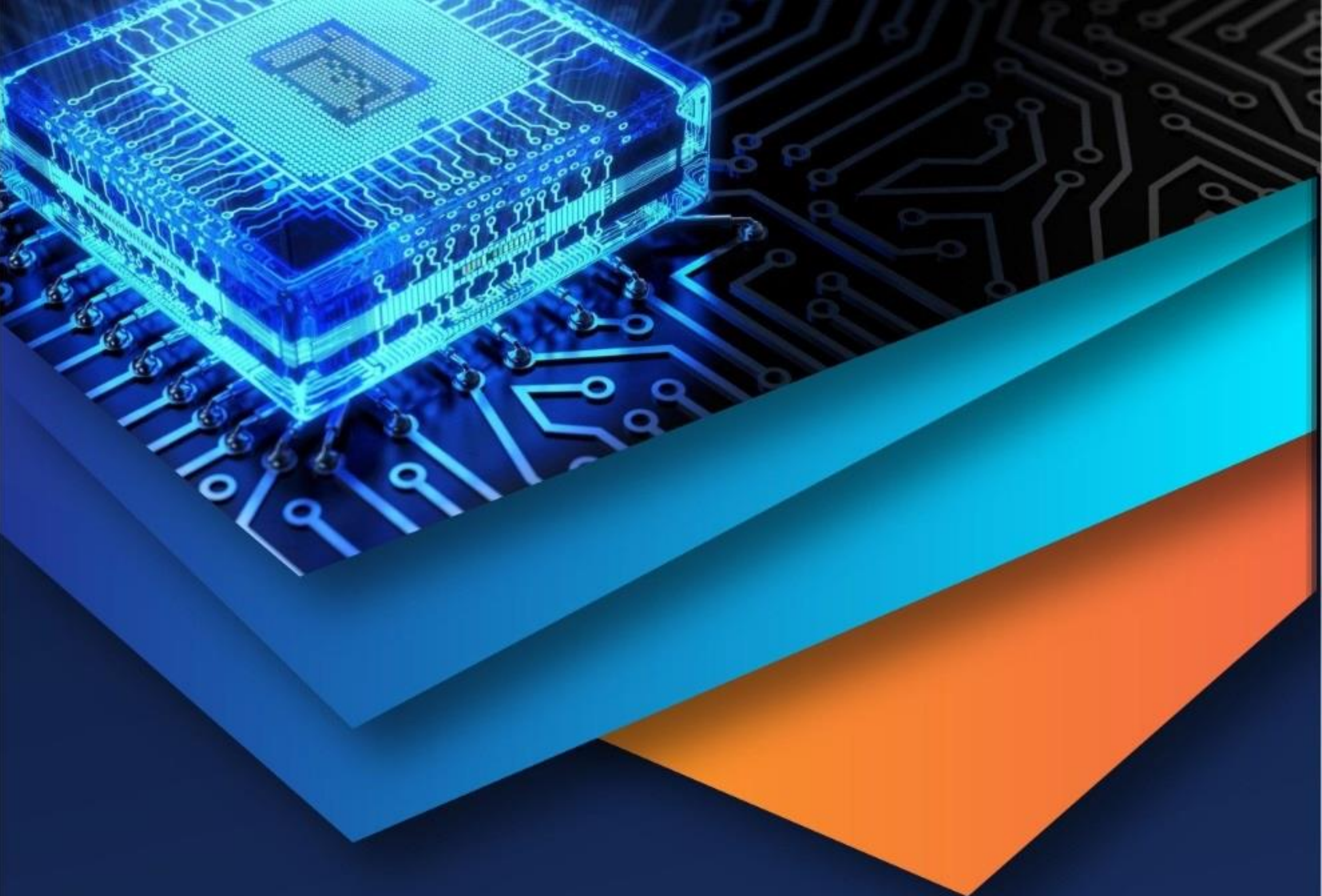

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