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Fuzzy Matrix with Application in Decision Making

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Abstract: As fuzzy decision making is a most important scientific, social and economic Endeavour, there exist several major approaches within the theories of fuzzy decision making. Here we have used the ranking order to deal with the vagueness in imprecise determination of preferences.
Keywords: Decision making, Relativity function, Comparison matrix and Ranking.

I. INTRODUCTION

The problem in making decisions is that the possible outcome, the value of new information, the way the conditions change with time, the utility of each outcome-action pair and our preferences for each action is typically vague, ambiguous and fuzzy.

II. PRE-REQUISITES

A. *Definition 2.1 Relativity function:*

Let x and y be variables defined on a set X. the relativity function denoted as $f(x/y)$ is defined as

$$f(x/y) = \frac{f_y(x)}{max\{f_y(x)\,,\,f_x(y)\}} \qquad (1)$$

Where $f_y(x)$ be the membership function of x with respect to y and $f_x(y)$ be the membership function of y with respect to x. Then the relativity function is a measurement of the membership value of preferring (or) choosing x over y.The relativity function $f(x/y)$ can be regarded as the membership of preferring variable x over the variable y. Equation (1) can be extended for many variables.

B. *Definition 2.2 Comparison Matrix:*

Let $A = \{x_1, x_2 \dots \dots . x_{i-1}, x_i, x_{i+1} \dots \dots \dots x_n\}$ be the set of n variables defined on universe X.From a matrix of relativity values $f(x_i/x_j)$ where x_i's for i=1 to n, are n variables defined on an universe X.The matrix $C = C_{ij}$ a square matrix of order n with $C_{ij=}\, f(x_i/x_j)$ is called the comparison matrix (or) C- matrix.

The C-matrix is used to rank different fuzzy sets. The smallest value in the ith row of the C- matrix, that is $C_i' = min\{f(x_i/X), i = 1\ to\ n\}$ is the membership value of the ithvariable. The minimum of$\{C_i'/\ i = 1\ to\ n\}$, that is the smallest value in each of the rows of the C – matrix will have the lowest weights for ranking purpose. Thus ranking, the variables $x_1, x_2 \dots \dots . x_n$ are determined by ordering the membership values C_1', $C_2' \dots \dots .\ C_n'$.

III. ILLUSTRATIVE EXAMPLE

A. *Example 1.*

A piece of property is evaluated so that it best suits a client's needs. Different available pieces of properties may have different benefits when compared to each other and to the needs of the client. Assume that four pieces of the property are available and the client compares from criteria p_1, p_2, p_3 and $p_4$with each other and to his needs.

The pair wise function as follows:

$f_{p_1}(p_1) = 1$,$f_{p_1}(p_2) = 0.5$, $f_{p_1}(p_3) = 0.3$ and $f_{p_1}(P_4) = 0.2$

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$f_{p_2}(p_1) = 0.8$, $f_{p_2}(p_2) = 1$, $f_{p_2}(p_3) = 0.4$ and $f_{p_2}(p_4) = 0.6$

$f_{p_3}(p_1) = 0.5$, $f_{p_3}(p_2) = 0.9$, $f_{p_3}(p_3) = 1$ and $f_{p_3}(p_4) = 0.95$

$f_{p_4}(p_1) = 0.7$, $f_{p_4}(p_2) = 0.4$, $f_{p_4}(p_3) = 0.2$ and $f_{p_4}(p_4) = 1$

Develop a comparison matrix based on this information and determine the overall ranking.

B. Solution

The relativity function

$$f(x/y) = \frac{f_y(x)}{max\,\{f_y(x),\ f_x(y)\}}$$

To find the comparison matrix and ranking:

$$f(p_1/p_1) = 1\ ;\ f(p_2/p_2) = 1;\ f(p_3/p_3) = 1\ ;\ f(p_4/p_4) = 1$$

$$f(p_1/p_2) = \frac{f_{p_2}(p_1)}{max\{f_{p_2}(p_1), f_{p_1}(p_2)\}} = \frac{0.8}{max\{0.8, 0.5\}} = 1$$

$$f(p_1/p_3) = \frac{f_{p_3}(p_1)}{max\{f_{p_3}(p_1), f_{p_1}(p_3)\}} = \frac{0.5}{max\{0.5, 0.3\}} = 1$$

$$f(p_1/p_4) = \frac{f_{p_4}(p_1)}{max\{f_{p_4}(p_1), f_{p_1}(p_4)\}} = \frac{0.7}{max\{0.7, 0.2\}} = 1$$

$$f(p_2/p_1) = \frac{f_{p_1}(p_2)}{max\{f_{p_1}(p_2), f_{p_2}(p_1)\}} = \frac{0.5}{max\{0.5, 0.8\}} = 0.625$$

$$f(p_2/p_3) = \frac{f_{p_3}(p_2)}{max\{f_{p_3}(p_2), f_{p_2}(p_3)\}} = \frac{0.9}{max\{0.9, 0.4\}} = 1$$

$$f(p_2/p_4) = \frac{f_{p_4}(p_2)}{max\{f_{p_4}(p_2), f_{p_2}(p_4)\}} = \frac{0.4}{max\{0.4, 0.6\}} = 0.667$$

$$f(p_3/p_1) = \frac{f_{p_1}(p_3)}{max\{f_{p_1}(p_3), f_{p_3}(p_1)\}} = \frac{0.3}{max\{0.3, 0.5\}} = 0.6$$

$$f(p_3/p_2) = \frac{f_{p_2}(p_3)}{max\{f_{p_2}(p_3), f_{p_3}(p_2)\}} = \frac{0.4}{max\{0.4, 0.9\}} = 0.444$$

$$f(p_3/p_4) = \frac{f_{p_4}(p_3)}{max\{f_{p_4}(p_3), f_{p_3}(p_4)\}} = \frac{0.2}{max\{0.2, 0.95\}} = 0.211$$

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$$f(p_4/p_1) = \frac{f_{p_1}(p_4)}{max\{f_{p_1}(p_4), f_{p_4}(p_1)\}} = \frac{0.2}{max\{0.2, 0.7\}} = 0.286$$

$$f(p_4/p_2) = \frac{f_{p_2}(p_4)}{max\{f_{p_2}(p_4), f_{p_4}(p_2)\}} = \frac{0.6}{max\{0.6, 0.4\}} = 1$$

$$f(p_4/p_3) = \frac{f_{p_3}(p_4)}{max\{f_{p_3}(p_4), f_{p_4}(p_3)\}} = \frac{0.95}{max\{0.95, 0.2\}} = 1$$

The comparison matrix $C = C_{ij} = \left(f(x_i/x_j)\right)$ is given by

$$C = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{cccc} p_1 & p_2 & p_3 & p_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.625 & 1 & 1 & 0.667 \\ 0.6 & 0.444 & 1 & 0.211 \\ 0.286 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} c_i' = \min of\ the\ i^{th}\ row \\ 1 \\ 0.625 \\ 0.211 \\ 0.286 \end{array}$$

The extra column to the right of the comparison matrix C is the minimum value for each of the rows.
The ranking is$p_1, p_2, p_3 and\ p_4$. The best suits a client is p_1.

C. Example 2.

Consider a comparison of four chemical with respect to human toxicity, Assume that c_1 is highly toxic, c_2 is very toxic. c_3 is moderately toxic and c_4 is slightly toxic. Hence on a pair wise comparison basis c_2 resembles c_1 with membership value is 0.75; c_3 resembles c_1 with a fuzzy membership value is 0.5 and $c_4$resembles c_1 with membership value of 0.25.
The remainder of the pair wise comparisons as follows:

$$f_{c_1}(c_1) = 1 f_{c_1}(c_2) = 0.75\ f_{c_1}(c_3) = 0.5 f_{c_1}(c_4) = 0.25$$

$$f_{c_2}(c_1) = 0.75 f_{c_2}(c_2) = 1 f_{c_2}(c_3) = 0.5 f_{c_2}(c_4) = 0.25$$

$$f_{c_3}(c_1) = 0.5 f_{c_3}(c_2) = 0.25 f_{c_3}(c_3) = 1 f_{c_3}(c_4) = 0.5$$

$$f_{c_4}(c_1) = 0.25 f_{c_4}(c_2) = 0.25 f_{c_4}(c_3) = 0.5 f_{c_4}(c_4) = 1$$

Develop a comparison matrix and determine the overall ranking of toxicity.

The relativity function

$$f(x/y) = \frac{f_y(x)}{max\{f_y(x), f_x(y)\}}$$

To find the comparison matrix and ranking:

$f(c_1/c_1) = 1\,;\, f(c_2/c_2) = 1$

$f(c_3/c_3) = 1\,;\, f(c_4/c_4) = 1$

$$f(c_1/c_2) = \frac{f_{c_2}(c_1)}{max\{f_{c_2}(c_1), f_{c_1}(c_2)\}} = \frac{0.75}{max\{0.75, 0.75\}} = 1$$

$$f(c_1/c_3) = \frac{f_{c_3}(c_1)}{max\{f_{c_3}(c_1), f_{c_1}(c_3)\}} = \frac{0.5}{max\{0.5, 0.5\}} = 1$$

$$f(c_1/c_4) = \frac{f_{c_4}(c_1)}{max\{f_{c_4}(c_1), f_{c_1}(c_4)\}} = \frac{0.25}{max\{0.25, 0.25\}} = 1$$

$$f(c_2/c_1) = \frac{f_{c_1}(c_2)}{max\{f_{c_1}(c_2), f_{c_2}(c_1)\}} = \frac{0.75}{max\{0.75, 0.75\}} = 1$$

$$f(c_2/c_3) = \frac{f_{c_3}(c_2)}{max\{f_{c_3}(c_2), f_{c_2}(c_3)\}} = \frac{0.25}{max\{0.25, 0.5\}} = 0.5$$

$$f(c_2/c_4) = \frac{f_{c_4}(c_2)}{max\{f_{c_4}(c_2), f_{c_2}(c_4)\}} = \frac{0.25}{max\{0.25, 0.25\}} = 1$$

$$f(c_3/c_1) = \frac{f_{c_1}(c_3)}{max\{f_{c_1}(c_3), f_{c_3}(c_1)\}} = \frac{0.5}{max\{0.5, 0.5\}} = 1$$

$$f(c_3/c_2) = \frac{f_{c_2}(c_3)}{max\{f_{c_2}(c_3), f_{c_3}(c_2)\}} = \frac{0.5}{max\{0.5, 0.25\} = 1}$$

$$f(c_3/c_4) = \frac{f_{c_4}(c_3)}{max\{f_{c_4}(c_3), f_{c_3}(c_4)\}} = \frac{0.25}{max\{0.25, 0.25\}} = 1$$

$$f(c_4/c_1) = \frac{f_{c_1}(c_4)}{max\{f_{c_1}(c_4), f_{c_4}(c_1)\}} = \frac{0.25}{max\{0.25, 0.25\}} = 1$$

$$f(c_4/c_2) = \frac{f_{c_2}(c_4)}{max\{f_{c_2}(c_4), f_{c_4}(c_2)\}} = \frac{0.25}{max\{0.25, 0.25\}} = 1$$

$$f(c_4/c_3) = \frac{f_{c_3}(c_4)}{max\{f_{c_3}(c_4), f_{c_4}(c_3)\}} = \frac{0.5}{max\{0.5, 0.5\}} = 1$$

The comparison matrix $C = C_{ij} = \left(f\left(x_i/x_j\right)\right)$ is given by

$$\begin{array}{c c c} & \begin{matrix} c_1 & c_2 & c_3 & c_4 \end{matrix} & C_i' = \min of the\ i^{th}\ row \\ C = \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0.5 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} & \begin{matrix} 1 \\ 0.5 \\ 1 \\ 1 \end{matrix} \end{array}$$

The extra column to the right of the comparison matrix C is the minimum value for each of the rows.

IV. CONCLUSION

The fuzzy decision model in which overall ranking (or) ordering of different fuzzy sets are determined by using comparison matrix. When we compare objects that are fuzzy or vague, we may have a situation where there is a contradiction of transitivity in the ranking. This form of non transitive ranking can be accommodated by means of relativity function which is defined as a measurement of the membership value of choosing one variable over the other. Hence in this paper, Fuzzy Matrix with Applications in Decision Makingmainly deals with fuzzy matrix.

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