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Border and Exterior of Soft Semi $\#g\alpha$ Closed Sets in Soft Bi-Topological Spaces

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Abstract: In this paper, we introduce a new class of closed sets via, soft semi $\#g\alpha$ -closed sets in bi-topological spaces. And also we study the concepts of border and exterior of soft semi $\#g\alpha$ -closed sets in bi-topological spaces which are denoted by $(1, 2)^*$ soft semi $\#g\alpha$ -bd (F,A) and $(1, 2)^*$ soft semi $\#g\alpha$ -ext (F,A) , where (F,A) is any soft set of (X,E) and also investigate their basic properties.

Keywords: $(1, 2)^*$ soft semi $\#g\alpha$ -closed set, $(1, 2)^*$ soft semi $\#g\alpha$ -open set, $(1, 2)^*$ soft semi $\#g\alpha$ -interior, $(1, 2)^*$ soft semi $\#g\alpha$ -closure, $(1, 2)^*$ soft semi $\#g\alpha$ -border and $(1, 2)^*$ soft semi $\#g\alpha$ -exterior.

I. INTRODUCTION

In 1963, the concepts of bi-topological spaces was originally initiated by J.C. Kelly[3]. The theory of generalized closed sets in topological spaces which was found by Levine[8] in 1970. The concepts of generalized and semi generalized closed sets was introduced and studied by Lellis[7] in classical topology. He defined a bi-topological space (X, τ_1, τ_2) to be a set X with two topologies τ_1 and τ_2 on X and initiated the systematic study of bi-topological spaces. The soft theory is rapidly processing in different field of mathematics. It was first proposed by Russian researcher Molodtsov[9] in 1999. Muhammad Shabir and Manazza Naz [10] introduced soft topological spaces in 2011. It was defined over an initial universe with a fixed set of parameters. N. Cagman and S. Karatas[2] introduced topology on a set called “soft topology” and initiated the theory of soft topological spaces in 2013. In this paper we defined and examined the basic properties of $(1, 2)^*$ soft semi $\#g\alpha$ -border and $(1, 2)^*$ soft semi $\#g\alpha$ -exterior in soft bi-topological spaces and study their properties.

II. PRELIMINARIES

In this section we have presented some of the basic definitions and results of soft set, soft topological space, bi-topological space to use in the sequel. Throughout this paper, X is an initial universe, E is the set of parameters, $P(X)$ is the power set of X , and $A \subseteq X$.

A. Definition 2.1.

Let $\tilde{\tau}$ be the collection of soft sets over X , then $\tilde{\tau}$ is called a soft topology on X if $\tilde{\tau}$ satisfies the following axioms:
 \emptyset, \tilde{X} belongs to $\tilde{\tau}$.
 The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
 The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
 The triplet $(X, \tilde{\tau}, E)$ is called a soft topological space over X . For simplicity, we can take the soft topological space $(X, \tilde{\tau}, E)$ as X throughout the work.

B. Definition 2.2.

A set X together with two different topologies is called bi-topological space. It is denoted by (X, τ_1, τ_2) .

C. Definition 2.3.

A soft set (F,A) of a soft topological space $(X, \tilde{\tau}, E)$ is called
 1) soft α - closed [4] if $\tilde{sc}l(\tilde{int}(\tilde{sc}l(F,A))) \subseteq (F,A)$. The complement of soft α -closed set is called soft α -open.
 2) soft semi - closed [2] if $\tilde{sc}l(\tilde{int}(\tilde{sc}l(F,A))) \subseteq (F,A)$. The complement of soft semi - closed set is called soft semi-open.
 3) soft g -closed [5] if $\tilde{sc}l(F,A) \subseteq (U,E)$, whenever $(F,A) \subseteq (U,E)$ and (U,E) is soft open in $(X, \tilde{\tau}, E)$. The complement of soft g -closed set is called soft g -open.

- 4) soft $g^\# \alpha$ - closed [6] if $\tilde{s}acl(F,A) \subseteq (U,E)$, whenever $(F,A) \subseteq (U,E)$ and (U,E) is soft g -open in $(X, \tilde{\tau}, E)$. The complement of soft $g^\# \alpha$ -closed set is called soft $g^\# \alpha$ -open. soft $\#g\alpha$ - closed [8] if $\tilde{s}acl(F,A) \subseteq (U,E)$, whenever $(F,A) \subseteq (U,E)$ and (U,E) is soft $g^\# \alpha$ - open in $(X, \tilde{\tau}, E)$. The complement of soft $\#g\alpha$ -closed set is called soft $\#g\alpha$ -open.
- 5) soft semi $\#g\alpha$ -closed [9] if $\tilde{s} scl(F,A) \subseteq (U,E)$, whenever $(F,A) \subseteq (U,E)$ and (U,E) is soft $\#g\alpha$ - open in $(X, \tilde{\tau}, E)$. The complement of soft semi $\#g\alpha$ -closed set is called soft semi $\#g\alpha$ -open
- 6) The union of all soft semi $\#g\alpha$ open sets [10] each contained in a set (F,A) of $(X, \tilde{\tau}, E)$ is called soft semi $\#g\alpha$ interior of (F,A) which is denoted by \tilde{s} semi $\#g\alpha$ -int (F,A)
- 7) The intersection of all soft semi $\#g\alpha$ - closed sets [10], each containing a set (F,A) of $(X, \tilde{\tau}, E)$ is called soft semi $\#g\alpha$ -closure of (F,A) , which is denoted by \tilde{s} semi $\#g\alpha$ -closure of (F,A) .

D. Definition 2.4.

Let X be a non-empty soft set on the universe $X, \tilde{\tau}_1, \tilde{\tau}_2$ are different soft topologies on \tilde{X} . Then $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ is called a soft bi-topological space.

E. Definition 2.5.

Let $F_A \in S(U)$. Power soft set of F_A is defined by, $\tilde{P}(F_A) = \{F_{Ai} \subseteq F_A : i \in I\}$

And its cardinality is defined by $|\tilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$ where $|f_A(X)|$ is cardinality of $f_A(X)$.

F. Example 2.6.

Let $U = \{u_1, u_2, u_3\}, E = \{x_1, x_2\}$ and $F_E = X = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$. And let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bi-topological space, where $\tilde{\tau}_1 = \{\emptyset, \tau_2, \tau_3, \tau_5, X\}, \tilde{\tau}_2 = \{\tau_1, \tau_2, \tau_8, \tau_{14}, X\}$, then $\tilde{\tau}_{1,2}$ soft open sets are $\{\tau_1, \tau_2, \tau_3, \tau_5, \tau_8, \tau_{14}, \tau_{17}, \tau_{32}, X\}$ and $\tilde{\tau}_{1,2}$ soft closed sets are $\{\tau_4, \tau_6, \tau_7, \tau_{12}, \tau_{31}, \tau_{44}, \tau_{46}, X\}$. Then,

- $\tau_1 = \{(x_1, \{u_1, u_2, u_3\})\}$
- $\tau_2 = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_3 = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$
- $\tau_4 = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$
- $\tau_5 = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_6 = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_7 = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_8 = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_9 = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{10} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{11} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{12} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{13} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{14} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{15} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{16} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{17} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{18} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{19} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{20} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{21} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$
- $\tau_{22} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$

- $$23 = \{(1, \{3\}), (2, \{3, 1\})\}$$
- $$24 = \{(1, \{1\}), (2, \{3\})\}$$
- $$25 = \{(1, \{1\}), (2, \{2, 3\})\}$$
- $$26 = \{(1, \{2\}), (2, \{3, 1\})\}$$
- $$27 = \{(1, \{2\}), (2, \{3\})\}$$
- $$28 = \{(1, \{2\}), (2, \{2, 3\})\}$$
- $$29 = \{(1, \{1\}), (2, \{3, 1\})\}$$
- $$30 = \{(1, \{3\}), (2, \{3\})\}$$
- $$31 = \{(1, \{3\}), (2, \{2, 3\})\}$$
- $$32 = \{(1, \{1, 2\}), (2, \{1\})\}$$
- $$33 = \{(1, \{1, 2\}), (2, \{2\})\}$$
- $$34 = \{(1, \{1, 2\}), (2, \{1, 2\})\}$$
- $$35 = \{(1, \{2, 3\}), (2, \{1\})\}$$
- $$36 = \{(1, \{2, 3\}), (2, \{2\})\}$$
- $$37 = \{(1, \{2, 3\}), (2, \{1, 2\})\}$$
- $$38 = \{(1, \{3, 1\}), (2, \{1\})\}$$
- $$39 = \{(1, \{3, 1\}), (2, \{2\})\}$$
- $$40 = \{(1, \{3, 1\}), (2, \{1, 2\})\}$$
- $$41 = \{(1, \{1, 2\}), (2, \{3\})\}$$
- $$42 = \{(1, \{1, 2\}), (2, \{2, 3\})\}$$
- $$43 = \{(1, \{2, 3\}), (2, \{3\})\}$$
- $$44 = \{(1, \{2, 3\}), (2, \{2, 3\})\}$$
- $$45 = \{(1, \{3, 1\}), (2, \{3\})\}$$
- $$46 = \{(1, \{3, 1\}), (2, \{2, 3\})\}$$
- $$47 = \{(1, \{1, 2, 3\})\}$$
- $$48 = \{(1, \{1, 2, 3\}), (2, \{1\})\}$$
- $$49 = \{(1, \{1, 2, 3\}), (2, \{2\})\}$$
- $$50 = \{(1, \{1, 2, 3\}), (2, \{1, 2\})\}$$
- $$51 = \{(1, \{1, 2, 3\}), (2, \{3\})\}$$
- $$52 = \{(1, \{1, 2, 3\}), (2, \{2, 3\})\}$$
- $$53 = \{(1, \{1, 2, 3\}), (2, \{3, 1\})\}$$
- $$54 = \{(1, \{1\}), (2, \{1, 2, 3\})\}$$
- $$55 = \{(1, \{2\}), (2, \{1, 2, 3\})\}$$
- $$56 = \{(1, \{1, 2\}), (2, \{1, 2, u_3\})\}$$
- $$57 = \{(1, \{3\}), (2, \{1, 2, 3\})\}$$
- $$58 = \{(1, \{2, 3\}), (2, \{1, 2, 3\})\}$$
- $$59 = \{(1, \{1, 3\}), (2, \{1, 2, 3\})\}$$
- $$60 = \{(1, \{1, 3\}), (2, \{1, 3\})\}$$
- $$61 = \{(1, \{1, 2\}), (2, \{1, 3\})\}$$
- $$62 = \{(1, \{2, 3\}), (2, \{1, 3\})\}$$
- $$63 = \{(2, \{1, 2, 3\})\}$$

$\mathcal{S}_6 = \{ \{ \{ 1, 2, 3 \}, \{ 1, 2, 3 \} \} = X$. Are all soft subsets of X . So $|\mathcal{S}_6| = 2^6 = 64$

G. Definition 2.7.

A soft set (F, A) of a soft bi-topological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called $(I, 2)^*$ soft semi #g α -closed if $\tilde{scl}(F, A) \subseteq (U, E)$, whenever $(F, A) \subseteq (U, E)$ and (U, E) is $(I, 2)^*$ soft #g α -open in $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$. The complement of $(I, 2)^*$ soft semi #g α -closed set is called $(I, 2)^*$ soft semi #g α -open.

H. Theorem 2.8.

If (F, A) and (G, B) are soft subset of (X, E) , then

- 1) (F, A) is $(I, 2)^*$ soft semi #g α -open iff $(I, 2)^*$ soft semi #g α -int $(F, A) \cong (F, A)$.
- 2) $(I, 2)^*$ soft semi #g α -int (F, A) is $(I, 2)^*$ soft semi #g α -open
- 3) (F, A) is $(I, 2)^*$ soft semi #g α -closed iff $(I, 2)^*$ soft semi #g α -cl $(F, A) \cong (F, A)$
- 4) $(I, 2)^*$ soft semi #g α -cl (F, A) is $(I, 2)^*$ soft semi #g α -closed.
- 5) $(I, 2)^*$ soft semi #g α -cl $((X, E) \setminus (F, A)) \cong (X, E) \setminus (I, 2)^*$ soft semi #g α -int (F, A) .
- 6) $(I, 2)^*$ soft semi #g α -int $((X, E) \setminus (F, A)) \cong (X, E) \setminus (I, 2)^*$ soft semi #g α -cl (F, A) .
- 7) If (F, A) is $(I, 2)^*$ soft semi #g α -open in $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ and (G, B) is $(I, 2)^*$ soft semi #g α -open in $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$, then $(F, A) \cap (G, B)$ is $(I, 2)^*$ soft semi #g α -open in $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$
- 8) A point $x \in (I, 2)^*$ soft semi #g α -cl (F, A) iff every $(I, 2)^*$ soft semi #g α -open set in (X, E) containing x intersects (F, A) .
- 9) Arbitrary intersection of $(I, 2)^*$ soft semi #g α -closed sets in $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is also $(I, 2)^*$ soft semi #g α -closed in $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$.

Proof We know that $(I, 2)^*$ soft union of all $(I, 2)^*$ soft open sets contained in (F, A) is called $\tilde{\text{int}}(F, A)$. So $(I, 2)^*$ soft semi #g α -open sets also is in (F, A) .

Since $(I, 2)^*$ soft semi #g α -int (F, A) are $(I, 2)^*$ soft open sets. Therefore $(I, 2)^*$ soft semi #g α -int (F, A) is $(I, 2)^*$ soft semi #g α -open sets.

Let the $(I, 2)^*$ soft intersection of all $(I, 2)^*$ soft closed sets containing (F, A) is called $\tilde{\text{cl}}(F, A)$. So $(I, 2)^*$ soft semi #g α -closed sets is in (F, A) . (iv) is similar to (iii).

$(I, 2)^*$ soft complement of (X, E) and (F, A) is equal to $(I, 2)^*$ soft complement of (X, E) and $(I, 2)^*$ soft semi #g α -int (F, A) .

Similarly (vi) can be proved. (vii) and (viii) are follow from the definition of $(I, 2)^*$ soft interior. (ix) obvious from the definition of $(I, 2)^*$ soft closure.

I. Definition 2.9.

For any soft subset (F, A) of (\tilde{X}, E) ,

The soft border of (F, A) is defined by soft bd $(F, A) \cong (F, A) \setminus (I, 2)^*$ soft int (F, A) .

The soft exterior of (F, A) is defined by soft ext $(F, A) \cong (I, 2)^*$ soft int $((X, E) \setminus (F, A))$.

III. SOFT SEMI #G-ALPHA BORDER AND EXTERIOR OF A SET IN BI-TOPOLOGICAL SPACES

In this section, we introduce and study the concepts of border and exterior of soft semi #g α -closed sets in soft bi-topological spaces.

A. Definition 3.1.

For any soft subset (F, A) of (\tilde{X}, E) , $(I, 2)^*$ soft semi #g α -border of (F, A) is defined by

$$(I, 2)^* \text{ soft semi #g}\alpha\text{-bd } (F, A) \cong (F, A) \setminus (I, 2)^* \text{ soft semi #g}\alpha\text{-int}(F, A).$$

B. Theorem 3.2.

In a soft bi-topological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$, for any soft subset (F, A) of (X, E) , the following statements hold.

$$(I, 2)^* \text{ soft semi #g}\alpha\text{-bd } (\emptyset) \cong (I, 2)^* \text{ soft semi #g}\alpha\text{-bd } (X, E) \cong \emptyset.$$

$$(I, 2)^* \text{ soft semi #g}\alpha\text{-bd } (F, A) \subseteq (F, A).$$

$$(F, A) \cong (I, 2)^* \text{ soft semi #g}\alpha\text{-int } (F, A) \cup (I, 2)^* \text{ soft semi #g}\alpha\text{-bd } (F, A).$$

$$(I, 2)^* \text{ soft semi #g}\alpha\text{-int } (F, A) \cap (I, 2)^* \text{ soft semi #g}\alpha\text{-bd } (F, A) \cong \emptyset.$$

$$\text{soft semi #g}\alpha\text{-int } (F, A) \cong (F, A) \setminus (I, 2)^* \text{ soft semi #g}\alpha\text{-bd } (F, A).$$

Proof Let us take $(I, 2)^*$ soft semi #g α -bd (\emptyset) is in $(I, 2)^*$ soft semi #g α -bd (X, E) and is an empty set.

$(I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) it should be any subset of (F,A) .

Since soft union of $(I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) and $(I,2)^*$ soft semi $\#g\alpha$ -int (F,A) are for any subset of (F,A) .

The soft intersection of $(I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) and $(I,2)^*$ soft semi $\#g\alpha$ -int (F,A) are empty set.

The soft complement of $(I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -int (F,A) .

C. Theorem 3.3.

$(I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A)) \cong \square.(F,A)$ is $(I,2)^*$ soft semi $\#g\alpha$ -open if and only if $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \cong \square$.

soft semi $\#g\alpha$ -bd $((I,2)^*$ soft semi $\#g\alpha$ -int $(F,A)) \cong \square$.

$(I,2)^*$ soft semi $\#g\alpha$ -bd $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A)) \cong (I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) .

soft semi $\#g\alpha$ -bd $(F,A) \cong (F,A) \tilde{\cap} (I,2)^*$ soft semi $\#g\alpha$ -cl $((X,E) \setminus (F,A))$.

Proof:Let $x \in (I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A))$. Then $x \in (I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) , since $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \subseteq (F,A)$, $x \in (I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A)) \subseteq (I,2)^*$ soft semi $\#g\alpha$ -int (F,A) .

Therefore $x \in (I,2)^*$ soft semi $\#g\alpha$ -int $(F,A) \tilde{\cap} (I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) which is contradiction to the above theorem (iv). Thus (i) is proved.

(F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -open iff $(I,2)^*$ soft semi $\#g\alpha$ -int $(F,A) \cong (F,A)$ [Theorem 2.8(i)]. But $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \cong (F,A) \setminus (I,2)^*$ soft semi $\#g\alpha$ -int (F,A) implies $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \cong \square$. This proves (ii) and (iii).

And when $(F,A) \cong (I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) Definition 3.1 becomes $(I,2)^*$ soft semi $\#g\alpha$ -bd $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A)) \cong (I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \setminus (I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A))$. Using (iii), we will get (iv).

(v) $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \cong (F,A) \setminus (I,2)^*$ soft semi $\#g\alpha$ -int $(F,A) \cong (F,A) \tilde{\cap} (\widetilde{(I,2)^*}(X,E) \setminus (I,2)^*$ soft semi $\#g\alpha$ -int $(F,A)) \cong (F,A) \tilde{\cap} (I,2)^*$ soft semi $\#g\alpha$ -cl $((X,E) \setminus (F,A))$ [Theorem 2.8 (v)]. Hence (v) is also proved.

D. Definition.3.4.

For any soft subset (F,A) of $\widetilde{(I,2)^*}(X,E)$, its $(I,2)^*$ soft semi $\#g\alpha$ -exterior is defined by, $(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong (I,2)^*$ soft semi $\#g\alpha$ -int $((X,E)\setminus(F,A))$.

E. Theorem 3.5.

For any $\cong (I,2)^*$ soft subsets (F,A) and $\widetilde{(I,2)^*}(G,B)$ of $\widetilde{(I,2)^*}(X,E)$, in soft bi-topological space $(X, \widetilde{(I,2)^*}(X,E))$, the following statements hold.

$(I,2)^*$ soft semi $\#g\alpha$ -ext $(\square) \cong (I,2)^*$ soft semi $\#g\alpha$ -ext $(X,E) \cong \sim$.

If $(F,A) \subseteq \widetilde{(I,2)^*}(G,B)$, then $(I,2)^*$ soft semi $\#g\alpha$ -ext $(G,B) \subseteq (I,2)^*$ soft semi $\#g\alpha$ -ext (F,A) .

$(I,2)^*$ soft semi $\#g\alpha$ -ext (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -open.

(F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -closed if and only if $(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong \widetilde{(I,2)^*}(X,E) \setminus (F,A)$.

$(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong (X,E) \setminus (I,2)^*$ soft semi $\#g\alpha$ -cl (F,A) .

Proof:Let us take (i) $(I,2)^*$ soft semi $\#g\alpha$ -ext (\square) is in $(I,2)^*$ soft semi $\#g\alpha$ -ext (X,E) and is an empty set.

And (ii) if any soft subset of (F,A) is contained in $\widetilde{(I,2)^*}(G,B)$ then, $(I,2)^*$ soft semi $\#g\alpha$ -ext (F,A) is always contained in $(I,2)^*$ soft semi $\#g\alpha$ -ext (G,B) .

Since $(I,2)^*$ soft semi $\#g\alpha$ -int (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -open, proof of (iii) is follow from the definition 3.4.

is $(I,2)^*$ soft semi $\#g\alpha$ -cl (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -closed.

Since $(I,2)^*$ soft semi $\#g\alpha$ -int $((X,E) \setminus (F,A)) \cong (X,E) \setminus (I,2)^*$ soft semi $\#g\alpha$ -cl (F,A) , (v) follows from definition 3.4.

F. Theorem 3.6.

$(I,2)^*$ soft semi $\#g\alpha$ -ext $((I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A)) \cong (I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -cl $(F,A))$.

(F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -regular, then $(I,2)^*$ soft semi $\#g\alpha$ -ext $((I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A)) \cong (F,A)$.

$(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong (I,2)^*$ soft semi $\#g\alpha$ -ext $((X,E) \setminus ((I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A)))$.

$(I,2)^*$ soft semi $\#g\alpha$ -int $(F,A) \subseteq (I,2)^*$ soft semi $\#g\alpha$ -ext $((I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A))$.

Proof:Since $(I,2)^*$ soft semi $\#g\alpha$ - int $(X,E)\setminus(F,A) \cong (X,E) \setminus (I,2)^*$ soft semi $\#g\alpha$ -ext (F,A) , (i) follows from definition 3.4. Similarly (ii) can be proved. If (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -regular, from the above theorem (iv), we have $(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong (X,E)\setminus(F,A)$ which is also $(I,2)^*$ soft semi $\#g\alpha$ -regular.

Thus $(I,2)^*$ soft semi $\#ga$ -ext(soft semi $\#ga$ -ext (F,A)) \cong (F,A), (ii) is proved. (iii) $(I,2)^*$ soft semi $\#ga$ -ext((X,E) \ $(I,2)^*$ soft semi $\#ga$ -ext (F,A)) \cong $(I,2)^*$ soft semi $\#ga$ -ext ((X,E)\ $(I,2)^*$ soft semi $\#ga$ -int ((X,E)\(F,A))) \cong $(I,2)^*$ soft semi $\#ga$ -int ((X,E)\(X,E)\ $(I,2)^*$ soft semi $\#ga$ -int((X,E)\(F,A))) \cong $(I,2)^*$ soft semi $\#ga$ -int($(I,2)^*$ soft semi $\#ga$ -int(X,E)\(F,A)) \cong ($(I,2)^*$ soft semi $\#ga$ -int (X,E)\(F,A)) \cong $(I,2)^*$ soft semi $\#ga$ -ext(F,A). Hence (iii) is proved.

Since (F,A) \subseteq $(I,2)^*$ soft semi $\#ga$ -cl(F,A), using (i) (iv) can be proved.

G. Theorem 3.7.

$(X,E) \cong (I,2)^*$ soft semi $\#ga$ -int(F,A) \cup $(I,2)^*$ soft semi $\#ga$ -ext (F,A) \cup $(I,2)^*$ soft semi $\#ga$ -fr (B,E).

$(I,2)^*$ soft semi $\#ga$ -ext ((F,A) \cup $\widetilde{(I,2)}(G,B)) \subseteq (I,2)^*$ soft semi $\#ga$ -ext(F,A) \cap $(I,2)^*$ soft semi $\#ga$ -ext (B,E).

$(I,2)^*$ soft semi $\#ga$ -ext ((F,A) \cap $\widetilde{(I,2)}(G,B)) \subseteq (I,2)^*$ soft semi $\#ga$ -ext(F,A) \cup $(I,2)^*$ soft semi $\#ga$ -ext (B,E).

Proof If we know that complement of $(I,2)^*$ soft semi $\#ga$ -int (X,E) is in (F,A) and union of $(I,2)^*$ soft semi $\#ga$ -int(F,A) is $\widetilde{(I,2)}(X,E)$.

And then any subset of (F,A) of $\widetilde{(I,2)}(,)$, its exterior is complement of $\widetilde{(I,2)}(X,E)$, its exterior is complement of $\widetilde{(I,2)}(X,E)$ and (F,A). So union of all $(I,2)^*$ soft semi $\#ga$ -interior, exterior and frontier is in $\widetilde{(I,2)}(X,E)$. Hence (i) is proved.

Proof of (ii) union of all exterior of (F,A) and $\widetilde{(I,2)}(G,B)$ is contained in intersection of $(I,2)^*$ soft semi $\#ga$ -ext(F,A) and $(I,2)^*$ soft semi $\#ga$ -ext (B,E). Hence (ii) is proved.

And next, proof of (iii) intersection of all exterior of (F,A) and $\widetilde{(I,2)}(G,B)$ is contained in union of all $(I,2)^*$ soft semi $\#ga$ -ext(F,A) and $(I,2)^*$ soft semi $\#ga$ -ext (B,E). Hence (ii) is proved.

Example 3.8.

Let $U = \{ 1, 2, 3 \}$, $E = \{ 1, 2 \}$ and $X = \{ (1, \{ 1, 2, 3 \}), (2, \{ 1, 2, 3 \}) \}$. And let $(X, \widetilde{(I,2)}_1, \widetilde{(I,2)}_2)$ be a soft bi-topological space, where $\widetilde{(I,2)}_1 = \{ , 2, 3, 5, X \}$, $\widetilde{(I,2)}_2 = \{ , 2, 8, 14, X \}$, then $\widetilde{(I,2)}$ soft open sets are $\{ , 2, 3, 5, 8, 14, 17, 32, X \}$ and $\widetilde{(I,2)}$ soft closed sets are $\{ , 4, 6, 7, 12, 31, 44, 46, X \}$. Consider a $(I,2)^*$ soft semi $\#ga$ -closed set $\sim = \{ (1, \{ 1, 2, 3 \}), (2, \{ 1 \}) \}$

$(I,2)^*$ soft semi $\#ga$ -border :

$\{ ((1, \{ 1, 2, 3 \}), (2, \{ 1 \})) \}$

$(I,2)^*$ soft semi $\#ga$ -exterior :

$\{ ((1, \{ 1, 2, 3 \}), (2, \{ 1 \})) \}$

IV. CONCLUSIONS

In this paper ,Border and Exterior of soft semi $\#ga$ -closed sets in soft bi-topological spaces were introduced and studied with already existing sets in soft bi-topological spaces. The scope for further research can be focused on the applications of soft bi-topological spaces.

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