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Three Phase Lag : Bioheat Transfer Equation

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Abstract: In this work, a new Pennes' bioheat transfer equation is derived based on three-phase-lag (TPL) model. The phase lag times for the heat flux, the temperature gradient and the thermal displacement gradient are introduced in the heat conduction law to capture microscale responses more accurately. The problem is solved by using the normal mode analysis technique.
Keywords: ***Bioheat transfer; Three-phase-lag; Normal Mode Analysis***.

I. INTRODUCTION

Temperature predictions for living tissues have great attentions due to its significance in clinical, basic, and environmental sciences. Especially, understanding the heat transfer in biological tissues involving either the raising or lowering of temperature is a necessity for many therapeutic practices such as cancer hyperthermia, burn injury, disease diagnostics, thermal comfort analysis, cryosurgery and cryopreservation. To analyse such problems, the most widely used bioheat model was introduced by Pennes [1]. He used the classical Fourier's law in the conduction term

$$\mathbf{q}(\mathbf{r},t) = -k\nabla T(\mathbf{r},t), \tag{1}$$

which assume that the thermal disturbance propagates with an infinite speed. In fact, heat is always found to propagate with a finite speed within living biological tissues as they have highly non-homogeneous inner structure.

To remove this paradox, thermal wave model of bioheat transfer has been proposed based on single phase lagging constitutive relation given by Cattaneo and Vernotte [2-3].

$$\mathbf{q}(\mathbf{r},t+\tau_q) = -k\nabla T(\mathbf{r},t), \tag{2}$$

which capture microscale responses in time but does not capture microscale responses in space. Due to this, it produces some unusual thermal behaviour. In order to consider the effect of micro-structural interactions along with the fast transient effects, a phase lag of temperature gradient τ_t has introduced in single phase lag constitutive relation.

Tzou [4-5] proposed this model that allows either the temperature gradient (cause) to precede heat flux vector (effect) or the heat flux vector (cause) to precede the temperature gradient (effect) i.e.

$$\mathbf{q}(\mathbf{r},t+\tau_q) = -k\nabla T(\mathbf{r},t+\tau_t). \tag{3}$$

Mitra at el. [6] presented experimentally the wave nature of heat propagation in processed meat. Antaki [7] has employed the dual-phase-lag heat conduction model to interpret the non-Fourier heat conduction phenomena in processed meats. Askarizadeh et al. [8] utilized the dual phase lag model in the transient heat transfer problems in skin tissue and studied the effects of temperature gradient relaxation time on the tissue temperature, damage and on the blood perfusion in the skin tissue. Poor et al. [9] explored an analytical solution to obtain thermal responses of biological tissues during laser irradiation.

Closed form analytical solutions to the bioheat transfer problems with generalized spatial heating inside the biological bodies are investigated by Othman et al. [10].

Choudhuri et al. [11] established the three-phase-lag (TPL) constitutive model by introducing the phase-lag of heat flux, temperature gradient, and thermal displacement gradient in heat conduction equation. The introduction of TPL model provides a general theoretical heat conduction model with different microstructural considerations in order to predict accurately the thermal behavior of structures.

The Fourier law is replaced by an approximation to a modification of the Fourier's law with three different translations for the heat flux vector, the temperature gradient and also for the thermal displacement gradient. i.e.

$$\mathbf{q}(\mathbf{r},t+\tau_q) = -[k\nabla T(\mathbf{r},t+\tau_t) + k^*\nabla v(\mathbf{r},t+\tau_v)]. \tag{4}$$

In the present study, we have obtained the solution of Pennes's bioheat transfer equation based on three phage lag model by the normal mode analysis.

II. MATHEMATICAL FORMULATION

In three phase constitutive relation

$$\mathbf{q}(\mathbf{r},t+\tau_q) = -[k\nabla T(\mathbf{r},t+\tau_t) + k^*\nabla v(\mathbf{r},t+\tau_v)], \tag{5}$$

where q, k, k^*, T, v, τ_q, τ_t, τ_v, and t are the heat flux vector, thermal conductivity, rate of thermal conductivity, tissue temperature, thermal displacement which satisfies $\dot{v} = T$, phase-lag of heat flux, phase-lag of temperature gradient, phase-lag of thermal displacement gradient, and time, respectively; $\mathbf{r}$ is the position vector and ∇ is the gradient operator. Retaining terms of the order of τ_q^2 in the Taylor's expansion of the generalized conduction law (5), we have

$$\mathbf{q} + \tau_q \frac{\partial \mathbf{q}}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2 \mathbf{q}}{\partial t^2} \cong -[\tau_v^* \nabla T + k\tau_t \frac{\partial}{\partial t}\nabla T + k^* \nabla v], \tag{6}$$

where

$$\tau_v^* = k + k^*\tau_v.$$

Taking divergence of both sides of equation (6) and then the time derivative, we obtain

$$\nabla.\dot{\mathbf{q}} + \tau_q \frac{\partial \nabla.\dot{\mathbf{q}}}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2 \nabla.\dot{\mathbf{q}}}{\partial t^2} = -[\tau_v^* \nabla^2 \dot{T} + k\tau_t \frac{\partial}{\partial t}\nabla^2 \ddot{T} + k^* \nabla^2 T]. \tag{7}$$

Pennes' bioheat equation is

$$\rho c \frac{\partial T}{\partial t} = -\nabla.q + w_b \rho_b c_b (T_a - T) + q_m, \tag{8}$$

where ρ, c, T, ω_b, c_b, T_a, q_m, t and x are the tissue density, tissue specific heat, tissue temperature, blood perfusion rate, blood specific heat, blood temperature, metabolic heat generation, time and space coordinate, respectively.

Equation (7) and (8) yields the following form of bioheat transfer equation

$$\left(1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2}\right)\left(\rho c_p \ddot{T} - w_b \rho_b c_b \frac{\partial}{\partial t}(T_a - T) - \dot{q}_m\right) = [\tau_v^* \nabla^2 \dot{T} + k\tau_t \nabla^2 \ddot{T} + k^* \nabla^2 T]. \tag{9}$$

Boundary Condition

$$\theta(x, y, t) = \theta_0, at\ x = 0. \tag{10}$$

III. SOLUTION OF THE PROBLEM

Dimensionless parameters are defined as

$$\begin{aligned} \theta &= \frac{T - T_0}{q_{m0} L^2/k}, x' = \frac{x}{L}, y' = \frac{y}{L}, \\ t' &= \frac{\alpha t}{L^2}, \tau_{q'} = \frac{\alpha \tau_q}{L^2}, \tau_{t'} = \frac{\alpha \tau_t}{L^2}, \\ \tau_{v'} &= \frac{\alpha \tau_v}{L^2},, G = \frac{q_m}{q_{m0}}. \end{aligned} \tag{11}$$

where, $\alpha = \dfrac{k}{\rho c}$ and L is the length of tissue.

The dimensionless form of equation (9) by using (11) after removing primes is

$$\left(1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2}\right)\left[\frac{\partial^2 \theta}{\partial t^2} + \left(\frac{w_b c_b \rho_b L^2}{k}\right)\frac{\partial \theta}{\partial t} + \frac{\partial G}{\partial t}\right] = \left(\frac{\tau_v^*}{k}\nabla^2 \dot{\theta} + \tau_t \nabla^2 \ddot{\theta} + \frac{k^*}{k^2}(L^2 \rho c)\nabla^2 \theta\right), \tag{12}$$

where

$$\tau_v^* = k + k^* \frac{L^2}{\alpha} \tau_v. \tag{13}$$

The solution of equation (12) can be decompose in terms of normal mode as the following form

$$\begin{aligned} \theta(x,y,t) &= \theta^*(x) e^{i(by-\omega t)} \\ G(y,t) &= G^* e^{i(by-\omega t)}, \end{aligned} \tag{14}$$

where, b is the wave number in the y-direction, ω is the angular frequency, θ^* and G^* are the amplitude of the function θ and G respectively. Using equation (14) in equation (12), the following equation is obtained

$$\frac{d^2\theta^*}{dx^2} - m\theta^* = b_1 G^*. \tag{15}$$

The solution of equation (15) has the form

$$\theta^*(x) = c_1 e^{-\sqrt{m}x} - \frac{b_1}{m} G^*. \tag{16}$$

Therefore, solution becomes

$$\theta(x,y,t) = [c_1 e^{-\sqrt{m}x} - \frac{b_1}{m} G^*] e^{i(by-\omega t)}, \tag{17}$$

where m, b_1, and, c_1 are given in appendix.

IV. CONCLUSIONS

In this work, we have derived the analytical solution of the Pennes's bioheat equation based on three phase lag model of heat transfer by the normal mode analysis technique to capture microscale responses more accurately. This three-phase-lag based bioheat transfer equation may have significant application in skin burn injuries. This model can be used to study the neuro-physiological behavior of skin tissue under different thermomechanical loadings.

V. APPENDIX

$$\begin{aligned}
m &= b^2 + \frac{a_4}{a_2},\ b_1 = \frac{a_3}{a_2},\ a_1 = \frac{\omega_b c_b \rho_b L^2}{k}, \\
a_2 &= \frac{\tau_v^*(-i\omega)}{k} + \tau_t(-i\omega)^2 + \frac{k^*}{k}(L^2\rho c), \\
a_3 &= (-i\omega) + \tau_q(-i\omega)^2 + \frac{1}{2}\tau_q^2(-i\omega)^3, \\
a_4 &= (-i\omega + a_1)[-i\omega + \tau_q(-i\omega)^2 + \frac{1}{2}\tau_q^2(-i\omega)^3], \\
m_1 &= \tau_q(-iw)^2 + (1 + a_1\tau_q)(-iw) + a_1 + b^2 b_1, \\
b_1 &= 1 + \tau_t(-iw),\ c_1 = \theta_0^* + \frac{b_1}{m} G^*.
\end{aligned}$$

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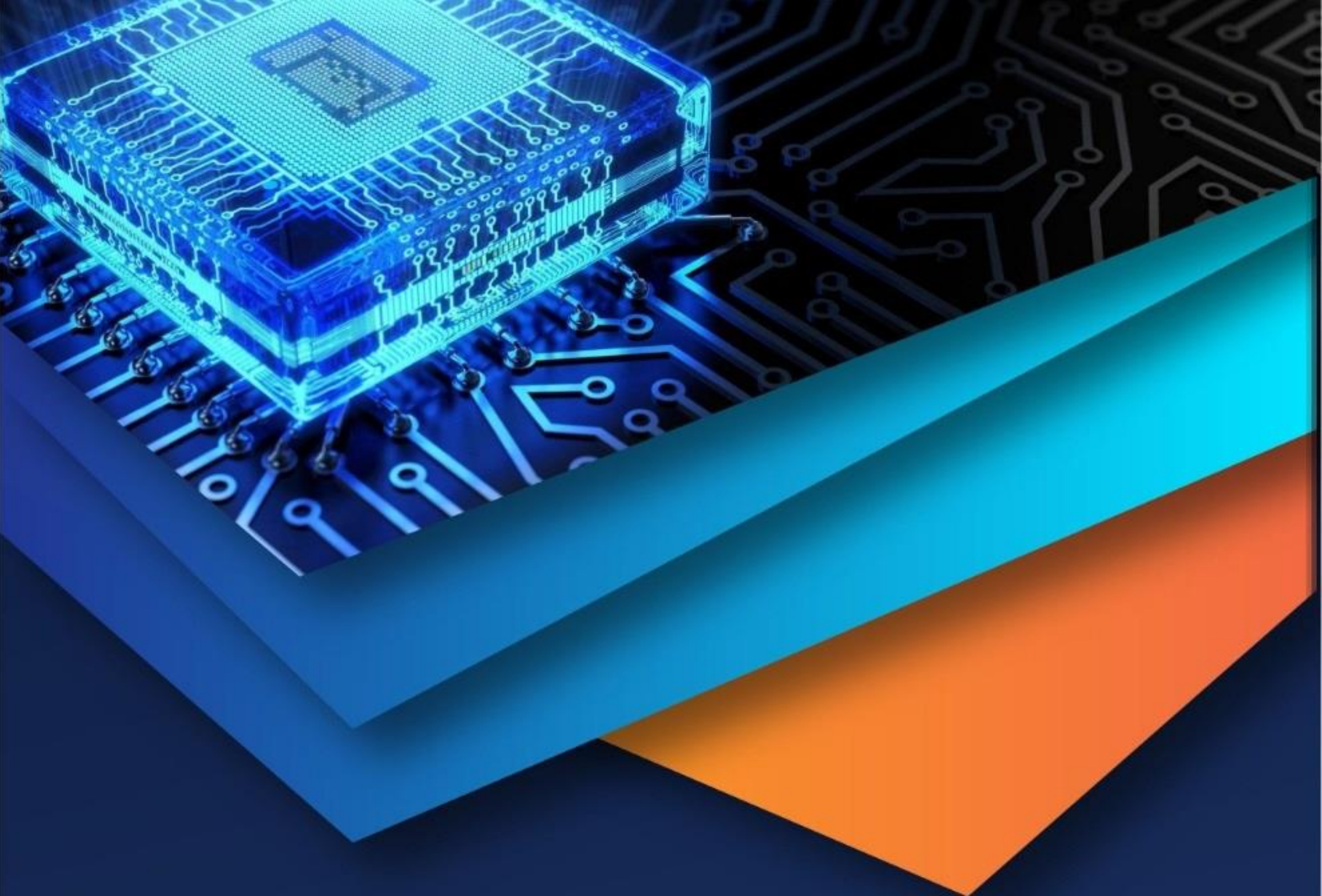

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