INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Adaptive Parameter estimation based Multimodal Medical Image Fusion Frame work in SWT domain

Shweta Goel[1], Sumit Budhiraja[2]
[1, 2] ECE Department, UIET Punjab University, Chandigarh, India (160015)

Abstract: Multimodal medical image fusion is extracting the necessary information from the source images into a single image which could be more informative for resourceful clinical study. This paper presents multimodal medical image fusion framework using the stationary wavelet transform (SWT) for medical images (i.e., magnetic resonance imaging and computed tomography scan) acquired using two distinct medical imaging sensors. The main objectives of the proposed approach are to improve the quality of the image from the point of view of clinical diagnosis, to optimize the performance and reduce complexity. The main advantage of proposed methodology is improvement upon the directionality, contrast, and phase information in the fused image. In the proposed fusion methodology, principal component analysis is employed in SWT domain, to improve upon redundancy. Maximum fusion rule is also applied to enhance the contrast of the image features. Detailed study of fused images is carried out using different wavelet families and various fusion metrics. The comparative analysis of different fusion quality parameters of the improved approach with other state-of-the-art fusion methods is carried out.
Keywords: CT scan, MRI Scan, DWT, SWT, PCA.

I. INTRODUCTION

In the past two decades, large number of developments in the field of Image Fusion has been done, in a wide range of disciplines that includes Image Processing, Computer Artificial Intelligence, and Signal Processing [1]. The basis of Image fusion is to combine two images- might be of different modalities, taken from different sources, view or time, into a single image in order to get more fine description of the image. Advancements in Image Fusion for Multimodal medical images like CT scan, MRI scan, PET, Ultrasound, X-ray etc. have led to detailed diagnosis of patient. As multimodal medical mages contains plenty of useful information, so the main goal of fusion is to extract this relevant data from medical images for the better analysis and detection of diseases in patient. In this paper we deal with the fusion of CT and MRI images [2]. The main idea behind fusion is, these medical images contain complementing information i.e. the response obtained from sensors is such that particular sensor modality is deprived of information carried by other. For example, CT scan of a patient deals with the details of denser tissues, while MRI scan contains soft tissues. Therefore, radiologists always prefer diagnosis for both modalities to analyze patient's disease. Further, enhancement of the fused image can be done with the help of large number of image processing techniques like de-noising, de-blurring, contrast, sharpness or brightness adjustments etc. Thus it will be advantageous on the part of radiologist that the enhancement techniques will be applied to fused image only, instead of applying it to each of the sensor modalities. Another benefit is that, there will be no need to store both CT and MRI scans for later references, as single compiled fused image can serve the purpose.

II. REVIEW ON FUSION APPROACHES

Image fusion can be done in two domains, namely transform domain and spatial domain [3]. In spatial domain, the values of pixels are manipulated directly i.e., all the operations are performed on original image pixels. Methodologies in this domain includes, Averaging of pixel values, Selecting maximum or minimum values, Principal component analysis (PCA), Intensity Hue saturation (IHS) [4-6].Fusion technique in spatial domain introduces spatial distortions in the finally fused image. Another is transform domain, in this, first the image is transformed to another domain and then the fusion algorithms are applied [3]. The input image is first decomposed into sub-bands using transform, and these coefficients are operated instead of operating the original image components thereby preserving image spatial information content. Numbers of transforms are present viz. Laplacian pyramid, wavelet, curvelet, Contourlet, Shearlet, etc [7-12]. Wavelets can be categorized as Discrete wavelet transforms (DWT), Stationary or Redundant wavelet transform (SWT or RWT) or Continuous wavelet transform (CWT). Wavelet transform basically is a multi-resolution analysis in which the source image is decomposed into different levels and
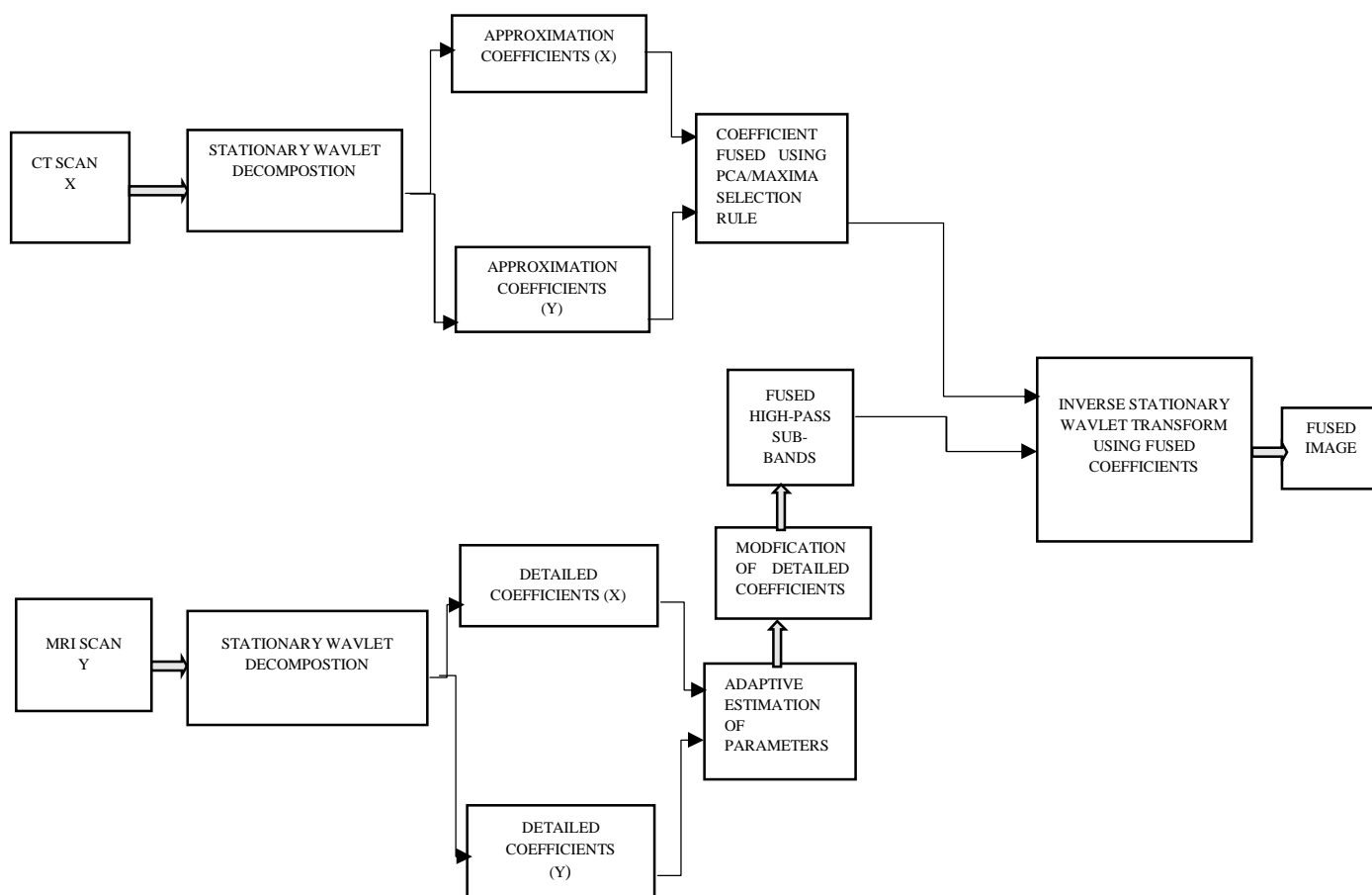
Figure 1: Block diagram for Adaptive Parameter Estimation based Multimodal Medical Image Fusion in Stationary Wavelet Domain

analyzed so that the features missing at one level can be restored at other [5]. After decomposition, different fusion rules can be applied to fuse these low and high band image coefficients.

With the above review of existing image fusion approaches, it can be concluded SWT not only provided optimal time and frequency localization; but also discovers the curved structures more accurately than DWT [13]. However, transforming via SWT leads to a high level of non-directionality in the decomposed coefficients. To overcome this drawback, PCA based fusion rule can be applied [14]. PCA being a highly directional fusion rule not only overcomes the non-directionality limitation of SWT, but also enhances the key features which makes it more appropriate for fusion of medical images. Use of adaptive framework or estimation of high band coefficients accounts for the remarkable reconstruction of the finest details in the fused image [10]. Also, the maximum selection fusion rule has been applied to enhance upon the contrast and morphological details (in the fused image) [4].

III. PROPOSED ALGORITHM

Image fusion using Stationary wavelet transform improves upon translational invariance. This section discusses decomposition approach along with the fusion algorithm employed to fuse the sub-band decomposed coefficients. In this research work enhanced multimodal medical image sensor fusion methodology is proposed. Figure 1, represents the block diagram for the proposed algorithm.

The input images are CT denoted by X, and MRI denoted by Y. The methodology involves pre-processing of the source multimodal images. This involves image adjustment and sharpening by applying appropriate threshold and sharpening amount intensity. This is followed by decomposition using stationary or redundant wavelet transform as shown in figure1. The detailed stepwise procedure for figure 1, is discussed below.

A. Stationary Wavelet Transform

Stationary Wavelet Transform (SWT) or Redundant Wavelet Transform (RDWT) [13] is also referred to as a "trous"Algorithm [15]. SWT is a wavelet transform which is basically intended to overcome the problem of translational invariance as in case of Discrete wavelets transform (DWT). In case of discrete wavelets, [16] we have down-samplers and up-samplers which causes lack of translational invariance, thus by removing these and up-sampling the filter coefficients by a factor of $2^{(j-1)}$, for j^{th} level, this can be overcome. Stationary wavelet transforms are also known as redundant wavelet transforms [15] as the number of coefficients before and after decompositions remain same at each level. This means for N-level decomposition, the SWT will have N redundant coefficients. For this redundancy, it was also named as 'trous', which means inserting zeros in the filters. The main applications of SWT include, Signal de noising, Brain image classification, Pattern recognition.

Wavelets are described by two functions viz. the scaling function, also known as, "father wavelet" and the wavelet function or "mother wavelet". The transformation of mother wavelet gives daughter wavelets which exhibits the

Properties of mother wavelet.

The daughter wavelets are given by Equation:

$$\psi_{a,b}(t)= 1/\sqrt{a}\ (\psi[\ t-b/a]\ )\text{for }(a,\ b \in R)\ ,\ a >0 \qquad (1)$$

where: a and b are the dilation and the translation factor as given in Eq. 2

$$a = a_o^i\ ,\ b = na_o^i b_o \qquad \text{for (i,n} \in Z) \qquad (2)$$

Hereby, we can represent the wavelet family by the Eq.3

$$\psi_{i,n}(t) = a_o^{-i/2}\psi(a_o^{-i}t - nb_o)\text{for (i,n} \in Z) \qquad (3)$$

SWT can also be mathematically expressed as by dyadic discretization of wavelet:

$$\sqrt{S_\psi R(a,b)} = \frac{1}{2\pi a}\int rt\psi(\frac{t-b}{a})\,dt \qquad (4)$$

SWT decompose the input image into two sub-bands coefficients-low and high [17]. Low band coefficients are the approximation coefficients, and the high band coefficients. These high band coefficients, in turn consist of three plane coefficients- horizontal, vertical and diagonal. These low can be checked by evaluating performance parameters for and high band SWT coefficients are fused based on the algorithm discussed below

B. Evaluation of low frequency wavelet coefficients

The decomposed approximation wavelet coefficients are fused by following algorithm:

Firstly, the low-band coefficients of CT and MRI images are compared and selected on the basis of maximum selection rule. This is done, so as to select the coefficients that has more information content and eliminate the values that contains noise or less information.

Secondly, the approximation coefficients are undergone fusion through PCA algorithm [14].The fusion algorithm based on PCA is discussed in algorithm below. PCA helps to reduce the redundancy present in the input images due to SWT and also improves upon non-directionality problem in SWT.

It is an orthogonal transform, it transforms a number of correlated variables into uncorrelated ones, called principal components. The first principal component is taken along the direction with maximum variance and so accounts for maximum possible variance in the data. Each following component accounts for as much of the residual variance possible and are taken in the direction perpendicular to the leading two and so on. The PCA is also called Karhunen-Loève transform or the Hotelling transform.

Finally, the value from both the above methodologies are compared and one with maximum value is chosen.

C. Algorithm: Principal Component Analysis

Step1: Input: Stationary wavelet approximation coefficients (CT and MRI)

$$\varphi_m(x,y) = \varphi(x)\psi(y) \qquad (5)$$

where, $\varphi(x)$:wavelet function,$\psi(y)$: scaling function, and 'm' accounts for modality- CT or MRI.

Step2: Compute Column vectors from the above low-band coefficients.

Step3: Construct a Covariance matrix using these column vectors and store the values of diagonal elements of this matrix.

Step4: Evaluate Eigen vectors and Eigen values from the covariance matrix.

Step5: Select the Column vector corresponding to maximum Eigen value.

Step6: Multiply normalized Eigen vector values by each approximation coefficient.

D. Evaluation Of High Frequency Wavelet Coefficients

In this novel fusion scheme, Adaptive estimation of parameters is done for fusing high-pass Stationary wavelet coefficients [10,19].

Let $H^l_m(x,y)$, is the high pass SWT coefficient, where (x,y) denote the current pixel value at location (x,y), 'l' denotes the level viz horizontal(h), vertical(v) or diagonal(d) and 'm' shows the modality-CT scan or MRI scan.

Now as after SWT decomposition, we have three levels for high pass band coefficients – horizontal, vertical and diagonal. Fusion at each stage using the proposed framework can be explained as follows:

$S^h_m(x,y)$ indicates the summation of all the horizontal high pass wavelet coefficients.

It is calculated as:

$$S^h_m(x,y) = \sum H^h_m(x,y) \qquad (6)$$

Similarly, $S^v_m(x,y)$ and $S^d_m(x,y)$ values are calculated by the summation of all the vertical and diagonal high pass wavelet coefficients.

Next step is to calculate a relationship parameter $N^l_m(x,y)$, which is evaluated as-

$$N^h_m(x,y) = \frac{S^h(x,y)}{S^h(x,y)+S^v(x,y)+S^d(x,y)} \qquad (7)$$

The parameter $N^h_m(x,y)$, derives the relationship between the horizontal coefficients and all other neighbors in the same horizontal plane.

Using the same set of calculations, $N^v_m(x,y)$ and $N^d_m(x,y)$ parameters are evaluated.

Now, the final step is to calculate new high-pass coefficients for each of the sensor modalities. Each of the new coefficients for all three levels is calculated as:

$$H^l_{m,new}(x,y)=H^l_{m,old}(x,y)*\sqrt{1 + N^h_m{}^2 + N^h_m{}^2 + N^h_m{}^2} \qquad (8)$$

The fused high pass wavelet coefficient is obtained by choosing the $H_{m,new}(x,y)$ from CT and MRI images by maximum selection rule. This can be explained as below:

$$H_f(x,y) = \begin{cases} H_{CT}(x,y), & \text{if } H_{CT}(x,y) > H_{MRI}(x,y) \\ H_{MRI}(x,y), & \text{otherwise} \end{cases} \qquad (9)$$

E. Evaluation of Fusion Metrics

The expected requirement of a fusion process is all the information of the source image is preserved and any artifacts or unwanted noise must not be introduced in these the resulting fused image. Entropy (E), Standard deviation (SD), Mutual Information (I_{CF} & I_{MF}) Fusion factor (FF) [17] accounts to the extend fusion has taken place and how accurately the information has been restored during reconstruction process. Other parameters based on estimation of error that evaluates the error introduced during the process by comparing it with the original input image are such as Peak signal to noise ratio (PSNR), Root mean square error (RMSE). All these performance parameters [20] are listed in Table 1, with all the necessary details about how these parameters are calculated and their significance.

Table 1: Image Quality Metrics of Fused Image

Image Quality Metrics	Formulae
Entropy	Indicates amount of information carried by the image
Mutual Information	I_{CF} and I_{MF}: Mutual information between fused image and each of the CT and MRI images
Fusion Factor	$FF = I_{CF} + I_{MF}$ Higher value of FF indicates adequate quality of fusion
Structural Content	Higher value indicates higher degree of edge preservation. (SC)
Root Mean Square Error	$RMSE=\sqrt{\frac{1}{mn}\sum_{i=1}^{M}\sum_{j=1}^{N}(Aij - Bij)^2}$ where, A - the perfect image, B - the fused image i,j – pixel row and column index, M,N- total no. of rows and columns
Peal signal to noise ratio	Higher value shows better signal strength in the finally fused image

Standard Deviation	$\sigma = \sqrt{E[I- \mu]^2}$
	E[I] : expected value of I (Image intensity)
	μ : Mean value
	Measures the amount of variance of each pixel value from mean.

Table 2: Study of different wavelet families on Test Image Set-1 for different Performance parameters

Wavelet	E	FF	SC	SD	RMSE	PSNR
Dmey	7.7077	3.0869	15.2879	82.8718	0.5638	53.1427
Db	7.3630	2.1062	14.4696	73.2495	0.5545	53.2872
Haar	7.1568	2.8568	15.1370	88.0476	0.5586	53.2234
Rbio	7.5098	2.9466	15.1609	85.9354	0.5601	53.1991
Bior	7.5443	2.8605	14.8838	85.5871	0.5555	53.2713
Coif	7.6671	1.9329	14.3629	75.2363	0.5499	53.3598

Table 3: Analysis of different levels of decomposition of Stationary Wavelets

LEVEL	E	FF	SC	SD	RMSE	PSNR
Two	6.6256	3.251	14.6427	79.374	0.4358	51.256
Three	7.3453	3.1621	15.6789	83.077	0.5738	52.989
Four	7.3630	3.0869	15.2879	82.8718	0.5638	53.1427

IV. SIMULATION RESULTS AND DISCUSSIONS

All simulations for Adaptive Parameter estimation based Multimodal Medical Image Fusion Framework in SWT domain are performed on MATLAB (2013).The test images include: two different sets of images namely Image Set-1 and Image Set-2 as shown in Figure 2 Both the test sets used for simulations consist of: CT and MRI images, where CT scan images represents bone & hard tissue details whereas MRI represents soft tissue details. The main aim is that while transformation of multimodal medical images into different domains and performing fusion, should not intrude the spectral and the spatial features in these images.

At our very first step, decomposition of images of both modalities is done using SWT transforms. This calls for analysis of different wavelets [21-25] available to select the one which gives best results comparatively. Appropriate selection of wavelet family is necessary as wavelets play a very important role in the decomposition and accurate reconstruction of the fused image. Different types of wavelets are available like, biorthogonal, daubechies, coiflet, symlets, haar wavelets, each have their diverse features and so the one which suits best to our requirement is preferred. So using SWT simulations are performed on different wavelet families on both the above mentioned test image sets.
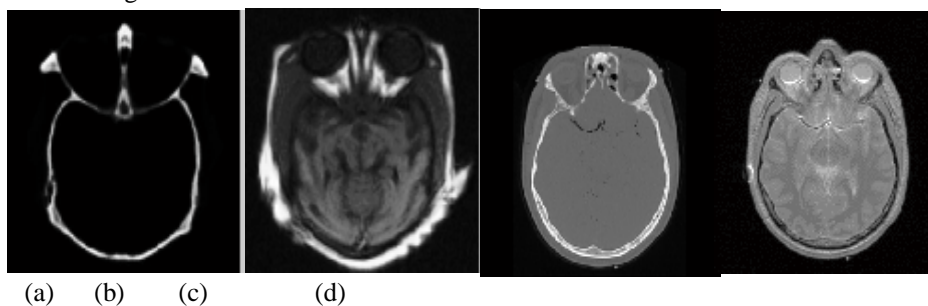

(a)   (b)   (c)   (d)

Figure 2: Test Image Sets used for Multimodal medical image fusion using proposed technology and comparison with existing approaches. (a)-(b): Test Image Set-1, (c)-(d): Test Image Set-2. (a)CT Scan-1, (b) MRI Scan-1, (c) CT Scan-2 (d) MRI Scan-2

Comparison of wavelets is done on the basis of fusion metrics discussed earlier. Table 2 depicts responses of fusion metrics [26]: E, FF, SC, SD, RMSE and PSNR. As per the values, it is clearly depicted that Dmeyer shows better performance in terms of Entropy, Fusion factor, and RMSE. Other factors are also in comparable range. Now, based on same criteria, from the various metrics are

graphically represented. Figure 3 gives the graphical representation of the fusion responses for all the wavelet families. From the figure 3, we can clearly portray the superiority of Dmeyer wavelet.

After selection of wavelet, further detailed analysis of different levels of selected wavelet family for suitable decomostion of source images is required.
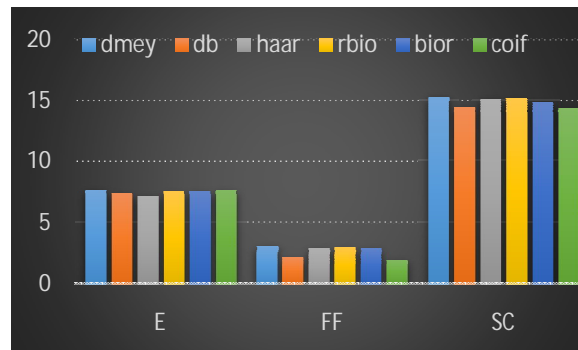

Figure 3: Graphical Analysis of Fusion Metrics using Proposed Fusion algorithm for different wavelet families

In case of Dmeyer wavelets, no further decomposition is available into different levels, so we move on to next analysis.Moreover, the level of decomposition of selected wavelet is also an essential feature, as there is loss of information during fusion or reconstruction as the level of decomposition changes. After selecting the suitable wavelet family, SWT decomposition is done respectively based on selected wavelets for proposed fusion technique and comparison with other existing approached has also been performed on the basis of same performance parameters.

table 3, represents different decomposition for Stationary Wavelet. The value for decomposition level-'N' must be positive and the value must be such that 2^N must divide the size of image. In table the value of 'N' is taken to be three and four. As we can see the values for Fusion metrics is better if the value of 'N' is more.

Comparative analysis of proposed technique with existing by evaluating Performance Parameters- Entropy, Fusion Factor, PSNR, and Root Mean Square Error is carried out. Table 4, 5 represents various fusion approaches and their comparison with implemented fusion framework for different test image sets. Different fusion algorithm includes fusion in sparse domain, discrete wavelet domain and stationary wavelet domain.

Table 4: Comparative analysis of Implemented fusion technique with existing on the basis of Fusion metrics. (Image Set-1)

Fusion Methodology	E	PSNR	RMSE	FF
Sparse+PCA Innovative	4.3156	46.5510	0.9042	1.7462
Sparse+PCA	5.6864	53.4747	0.5426	3.2228
PCA	5.6579	55.1829	0.4458	1.7568
SWT+PCA	7.0604	36.5441	0.7742	1.9343
Proposed	7.5098	53.991	0.5672	3.9892

For the purpose of validation of results, comparison for values of Entropy and PSNR is carried out. In Table 4, 5 first technique involves transformation of source images into Discrete wavelet domain and thereby fusion using Principal component analysis for contrast enhancement. From the visual results, in Figure 5(a), the fused image have missing edges and curved shapes, even though the quality is not that poor. In Second technique, transformation to Stationary wavelet domain is done in combination with PCA and further noise reduction is done using wiener filter. Finally fused image in stationary wavelet domain gives better response, from Figure 5(b) it can be observed that the image carries the information of both modalities. For third, extraction of innovative features from the complementary images is done and thereby the process conducts fusion. From table it can be easily concluded that this technique gives large fall in the entropy of the fused image although the measure of PSNR is comparable. It is clearly indicated from the table that proposed technique shows superiority in comparison to others

From graphical analysis shown in Figure 4, for Table 4 (Image set-1) it can be clearly seen that the improved methodology shows peaks for E and FF values giving an edge over existing approaches. For the proposed technique, both the test image sets shows marginal change in the values of performance parameters. From the visual inspection of the finally fused images, in Figure 4, for different approaches it is ascertained the preservation of edges and better contrast for the implemented approach.

The qualitative (visual response) and quantitative (performance parameters) comparison discussed illustrates that the proposed research methodology has been shown to yield better response in comparison to other approaches. The reproduction of complementary features along with preservation of structural and morphological details with better contrast is the combined result of the implemented fusion approaches.
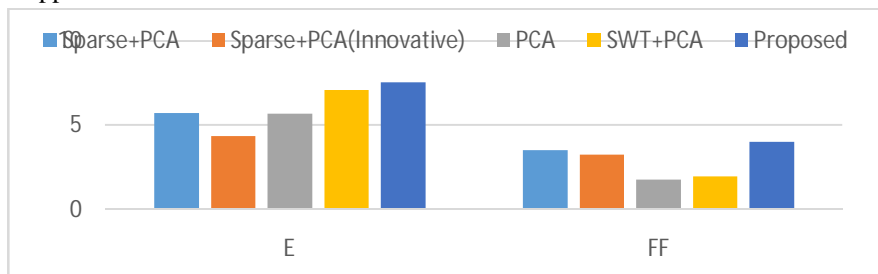

Figure 4: Comparative Analysis of Fusion Metrics for different state-of-art fusion methodologies

Table 5: Comparative analysis of Implemented fusion technique with existing on the basis of Fusion metrics. (Image Set-2)

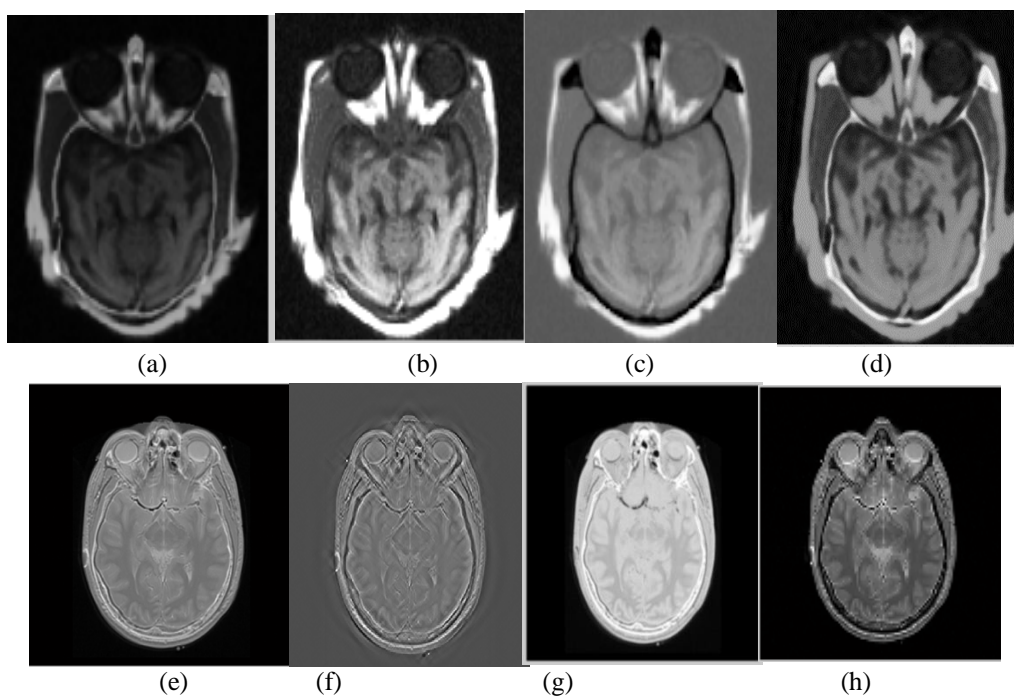| Fusion Methodology	E	PSNR	RMSE	FF
Sparse+PCA Innovative	4.7853	51.6744	0.6676	2.3522
Sparse+PCA	5.3646	59.9905	0.2563	2.7642
PCA	5.3753	60.1911	0.2504	1.9580
SWT+PCA	5.5833	52.071	0.410	1.7439
Proposed	5.8595	61.8192	0.2291	3.3915

(a)　　　　(b)　　　　(c)　　　　(d)

(e)　　　(f)　　　(g)　　　(h)

Figure 5: Qualitative comparison of existing state of art fusion approaches with the proposed approach on test image Set-1 & 2 using various Fusion metrics.

(a) DWT+PCA using test image set-1, (b)Sparse+PCA using test image set-1, (c) Innovative Sparse+PCA, using test image set-1 (d) Proposed Approach, using test image set-1 (e) DWT+PCA, using test image set-2 (f) Sparse+PCA, using test image set-2 (g)Innovative Sparse+PCA, using test image set-2 (h) Proposed Approach, using test image set-2.

V.   CONCLUSION

The sole aim of image fusion is that the transformation of multimodal medical images by undergoing image processing algorithms should not intrude the spectral and the spatial features of the multimodal images. SWT when used for decomposition into low and high sub-bands results in the image with increased frequency and time localization. This research work explores the key potential in restoring the aforesaid features of CT and MRI images with fusion of complementary structures. PCA and maximum fusion rule augments to the performance of the fusion approach in terms of minimization of redundancy, and adaptive estimation of parameters accounts for the restoration of finest details for high band coefficients. Appropriateness of chosen stationary wavelet parameters can be easily inferred from the results on the basis of improved values of E, PSNR, and Q values. The visual results of the fused images shows consistency with the human perception. In fusion metrics, from the values, it can be concluded that the proposed framework shows superiority as compared to other existing state of art fusion approaches, thereby confirming the appropriateness of the proposed fusion methodology for efficient clinical diagnosis

REFERENCES

[1] S. Casciaro, P. Pisani, G. Soloperto, A. Greco, A. Lay Ekuakille, and F. Conversano, "An innovative ultrasound signal processing technique to selectively detect nanosized contrast agents in echographic images," IEEE Trans. Instrum. Meas., vol. 64, no. 8, pp. 2136–2145, Aug. 2015.

[2] A. Alkan, S. A. Tuncer, and M. Gunay, "Comparative MR Image analysis for thyroid nodule detection and quantification," Measurement, vol. 47, pp. 861–868, Jan. 2014

[3] Deepak Kumar Sahu, M.P.Parsai, "Different Image Fusion Techniques-A Critical Review", IJMER2012

[4] DeepaliA.Godse, Dattatraya S. Bormane (2011) "Wavelet based image fusion using pixel based maximum selection rule" International Journal of Engineering Science and Technology (IJEST), Vol. 3 No. 7 July 2011, ISSN: 0975-5462.

[5] Himanshi, V. Bhateja, A. Krishn, And A. Sahu, "An Improved Medical Image Fusion Approach Using PCA And Complex Wavelets," In Proc. Int. Conf. Med. Imag., M-Health Emerg. Commun. Syst. (Medcom), Nov. 2014, Pp. 442–447.

[6] N. Cvejic, D. Bull, And N. Canagarajah, "Region-Based Multimodal Image Fusion Using ICA Bases," IEEE Sensors J., Vol. 7, No. 5, Pp. 743–751, May 2007

[7] Himanshi, V. Bhateja A. Krishn, And A. Sahu, "Medical Image Fusion InCurvelet Domain Employing PCA And Maximum Selection Rule," In Proc. IC3TAISC, Vol. 379. Jul. 2015, Pp. 1–8.

[8] BiswajitBiswas, AmlanChakrabarti, "Medical Image Fusion By Combining SVD And Shearlet Transform", (SPIN) 2015.

[9] Emmanuel J. Candès And Michael B. Wakin, "Compressive Sampling", IEEE SIGNAL PROCESSING MAGAZINE, March-2008.

[10] BiswajitBiswas, AmlanChakrabarti, "Medical Image Fusion By Combining SVD And Shearlet Transform", (SPIN) 2015.

[11] J. Ma and G. Plonka, "The curvelet transform," IEEE Signal Process.Mag., vol. 27, no. 2, pp. 118–133, Mar. 2010.

[12] 12, pp. 2091–2106, Dec. 2005.

[13] R. Singh and A. Khare, "Redundant discrete wavelet transform based medical image fusion," in Advances in Signal Processing and Intelligent Recognition Systems, vol. 264. Berlin, Germany: Springer-Verlag, 2014, pp. 505–515.

[14] V. P. S. Naidu and J. R. Raol, "Pixel-level image fusion using wavelets and principal component analysis," Defence Sci. J., vol. 58, no. 3, pp. 338–352, May 2008.

[15] J. E. Fowler, "The redundant discrete wavelet transform and additive noise," IEEE Signal Process. Lett., vol. 12, no. 9, pp. 629–632, Sep. 2005.

[16] R. Singh, R. Srivastava, O. Prakash, and A. Khare, "DTCWT based multimodal medical image fusion," in Proc. Int. Conf. Signal, Image Video Process., Jan. 2012, pp. 403–407.

[17] S. S. Agaian, K. Panetta, and A. M. Grigoryan, "Transform-based image enhancement algorithms with performance measure," IEEE Trans. Image Process., vol. 10, no. 3, pp. 367–382, Mar. 2001.

[18] A. Sahu, V. Bhateja, A. Krishn, and Himanshi, "Medical image fusion with Laplacian pyramids," in Proc. Int. Conf. Med. Imag., m-Health Emerg. Commun. Syst. (MedCom), Nov. 2014, pp. 448–453.

[19] X. Xiao and Z.Wu, "Image fusion based on lifting wavelet transform," in Proc. Int. Symp. Intell. Inf. Process. Trusted Comput. (IPTC), Oct. 2010, pp. 659–662.

[20] G. Bhatnagar, Q. M. J. Wu, and Z. Liu, "A new contrast based multimodal medical image fusion framework," Neurocomputing, vol. 157, pp. 143–152, Jun. 2015

[21] I. Daubechies, "The wavelets transform, time frequency localization and signal analysis," IEEE Trans. Inform. Theory, vol. 36, pp. 961–1005, 1990.

[22] G. Kaiser, A Friendly Guide toWavelets. Berlin, Germany: Birkhäuser, 1994.

[23] L. Rebollo-Neira, "Frames of cross-ambiguity functions," IEEE Signal Processing Lett., vol. 7, pp. 293–296, Oct. 2000.

[24] Proc. R. Soc. Lond. "Frames in two dimensions arising from wavelet transforms,"B, vol. 457, pp. 2079–2091, 2001.

[25] R. R. Coifman, Y. Meyer, and M. V. Wickerhauseret al., "Wavelets analysis and signal processing," in Wavelets and its Applications, M. B. Ruskaiet al., Eds. Boston, MA: Jones and Bartlett, 1992, pp. 153–178.

[26] L. Rebollo-Neira, J. Fernandez-Rubio, and A. Plastino, "Frames a maximum entropy statistical estimate of the inverse problem," J. Math. Phys., vol. 38, no. 9, pp. 4863–4871, 1997.

INTERNATIONAL JOURNAL
FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089   (24*7 Support on Whatsapp)