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On Ternary Quadratic Diophantine Equation

$$15x^2 + 15y^2 + 24xy = 438z^2$$

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Abstract: The ternary quadratic Diophantine equation $15x^2 + 15y^2 + 24xy = 438z^2$ is analyzed for its non-trivial distinct integral solutions. Six different patterns of integral solutions are obtained. A few interesting relations among the solutions and special polygonal numbers are presented.

Keywords: Ternary quadratic equation, Integral solutions.

Notations:

$$T_{3,n} = \frac{n(n+1)}{2} = \text{Triangular number of rank } n.$$

$$T_{7,n} = \frac{n(5n-3)}{2} = \text{Heptagonal number of rank } n.$$

$$T_{10,n} = n(4n-3) = \text{Decagonal number of rank } n.$$

$$T_{12,n} = n(5n-4) = \text{Dodecagonal number of rank } n.$$

$$T_{13,n} = \frac{n(11n-9)}{2} = \text{Tridecagonal number of rank } n.$$

$$T_{17,n} = \frac{n(15n-13)}{2} = \text{Heptadecagonal number of rank } n.$$

$$T_{18,n} = n(8n-7) = \text{Octadecagonal number of rank } n.$$

$$Gno_n = (2n-1) = \text{Gnomonic number of rank } n.$$

Mathematical Classification: 11D09.

I. INTRODUCTION

Ternary quadratic equations are rich in variety. For more detailed understanding one can see [1-7]. For the non-trivial integral solutions of ternary quadratic Diophantine equations [8-9] has been studied. [10-13] has been referred for various ternary quadratic Diophantine equations. In this communication, we consider yet another interesting ternary quadratic equation $15x^2 + 15y^2 + 24xy = 438z^2$ and obtain different patterns of non-trivial integral solutions. Also, a few interesting relations among the solutions and special polygonal, Centered, Gnomonic, Star numbers are presented.

II. METHOD OF ANALYSIS

The Quadratic Diophantine equation with four unknowns under consideration is

$$15x^2 + 15y^2 + 24xy = 438z^2 \quad (1)$$

The substitution of the linear transformations

$$x = u + v \quad \text{and} \quad y = u - v \quad (2)$$

in (1) leads to

$$9u^2 + v^2 = 73z^2 \quad (3)$$

Four different choices of solutions to (3) are presented below. Once the values of u and v are known, using (2), the corresponding values of X and Y are obtained.

A. Pattern 1

In (3), $9u^2 + v^2 = 73z^2$

Assume that

$$z = 9a^2 + b^2, \quad a, b \neq 0 \tag{4}$$

We can write $73 = (3 + 8i)(3 - 8i)$ (5)

Substituting (4) and (5) in (3), we get

$$(3u + iv)(3u - iv) = (3 + 8i)(3 - 8i)(3a + ib)^2(3a - ib)^2$$

Equating the positive and negative factors, we get

$$(3u + iv) = (3 + 8i)(3a + ib)^2 \tag{6}$$

$$(3u - iv) = (3 - 8i)(3a - ib)^2 \tag{7}$$

Equating the real and imaginary parts in either (6) or (7), we get

$$u = 9a^2 - b^2 - 16ab \tag{8}$$

$$v = 72a^2 - 8b^2 + 18ab \tag{9}$$

Substituting (8) and (9) in (2), we get

$$\left. \begin{aligned} x &= x(a, b) = 81a^2 - 9b^2 + 2ab \\ y &= y(a, b) = -63a^2 + 7b^2 - 34ab \\ z &= z(a, b) = 9a^2 + b^2 \end{aligned} \right\} \tag{10}$$

Thus (10) represent non-zero distinct integer solution to (1) in two parameters.

B. Observations

- 1) $x(a, a) - 74T_{4,a} = 0$
- 2) $y(a, a) + 90T_{4,a} = 0$
- 3) $x(2a, a) + y(2a, a) = 6a^2$, a nasty number.
- 4) $z(a(a + 1), b(b + 1)) - 9P_a^2 - P_b^2 = 0$
- 5) $x(a, a + 1) + y(a, a + 1) + z(1, 5a) + 29 = (3a - 6)^2$, a perfect square.
- 6) $x(a, a) + y(a, a) + z(a, a) = -6a^2$, a nasty number.

C. Pattern 2

In (3), $9u^2 + v^2 = 73z^2$

Consider the linear transformations

$$\left. \begin{aligned} z &= X + 9T \\ u &= X + 73T \end{aligned} \right\} \tag{11}$$

or $\left. \begin{aligned} z &= X - 9T \\ u &= X - 73T \end{aligned} \right\} \tag{12}$

Substituting (11) or (12) in (3), we get

$$v^2 = 64(X^2 - 657T^2) \tag{13}$$

Write $v = 8V$ (14)

Substituting (14) in (13), we get

$$\begin{aligned} V^2 &= X^2 - 657T^2 \\ X^2 &= 657T^2 + V^2 \end{aligned} \tag{15}$$

This is in the standard form $x^2 = Dy^2 + z^2$

The corresponding solutions to (15) are

$$\left. \begin{aligned} T &= 2ab \\ V &= 657a^2 - b^2 \\ X &= 657a^2 + b^2 \end{aligned} \right\} \tag{16}$$

Substituting (16) in (11) and (14), we get

$$\left. \begin{aligned} z &= 657a^2 + b^2 + 18ab \\ u &= 657a^2 + b^2 + 146ab \\ v &= 5256a^2 - 8b^2 \end{aligned} \right\} \tag{17}$$

Substituting (17) in (2), we get

$$\left. \begin{aligned} x &= x(a,b) = 5913a^2 - 7b^2 + 146ab \\ y &= y(a,b) = -4599a^2 + 9b^2 + 146ab \\ z &= z(a,b) = 657a^2 + b^2 + 18ab \end{aligned} \right\} \tag{18}$$

Thus (18) represent non-zero distinct integer solution to (1) in two parameters.

D. Observations

- 1) $x(a,a) - 6052T_{4,a} = 0$
- 2) $y(a,a) + 4444T_{4,a} = 0$
- 3) $z(a,a)$ is a perfect square.
- 4) $x(a,a+1) - y(a,a+1) \equiv 16 \pmod{32}$
- 5) $x(a,a+1) - y(a,a+1) - T_{68,a} \equiv -16 \pmod{10463}$
- 6) $y(1,B+3) = 100Gno_B + 9T_{4,B} - 3980$

E. Pattern 3

The Ternary quadratic equation (3) can be written as

$$9u^2 - 9z^2 = 64v^2 - v^2 \tag{19}$$

Factorizing (19) we have

$$\begin{aligned} (3u + 3z)(3u - 3z) &= (8z + v)(8z - v) \\ \frac{3u + 3z}{8z + v} &= \frac{8z - v}{3u - 3z} = \frac{A}{B}, B \neq 0 \end{aligned} \tag{20}$$

This is equivalent to the following two equations.

$$Av - 3Bu + z(8A - 3B) = 0 \tag{21}$$

$$Bv - 3uA - z(3A + 8B) = 0 \tag{22}$$

Applying the method of cross multiplication, we get

$$z = z(A, B) = 3A^2 + 3B^2 \tag{23}$$

$$\left. \begin{aligned} u = u(A, B) &= 3A^2 - 3B^2 + 16AB \\ v = v(A, B) &= -24A^2 + 24B^2 + 18AB \end{aligned} \right\} \tag{24}$$

Substituting (22) in (2), we get

$$\left. \begin{aligned} x = x(A, B) &= -21A^2 + 21B^2 + 34AB \\ y = y(A, B) &= 27A^2 - 27B^2 - 2AB \end{aligned} \right\} \tag{25}$$

Thus (23) and (25) represent non-zero distinct integer solution to (1) in two parameters.

F. Observations

1) $x(A, A) - y(A, A) = (6A)^2, a$ perfect square.

2) $z(A, A)$ is a nasty number.

3) $x(A, A+1) + y(A, A+1) + T_{44,A} \equiv -6 \pmod{53}$

4) $z(A, A(A+1)) = 3T_{4,A}^2 + 12P_A^5$

5) $x(A, 1) + Gno_{13n} + 8 = y(A, 1) + star_A$

6) $x(A, A(4A-3)) + y(A, A(4A-3)) + 96T_{4,A}^2 = T_{4,A} [Gno_{136A} - 143]$

G. Pattern 4

(15) can be written as

$$X^2 - V^2 = 657T^2$$

$$(X + V)(X - V) = (657T)T$$

Equating the positive and negative factors we get

$$X + V = 657T \tag{26}$$

$$X - V = T \tag{27}$$

Solving (26) and (27), we get

$$\left. \begin{aligned} X &= 329T \\ V &= 328T \end{aligned} \right\} \tag{28}$$

Substitute (28) in (14) we get

$$v = 2624T$$

For $T = A, v = 2624A$

Substitute the value of v in (11), we get

$$u = 402A$$

Substituting u and v in (2)

$$\left. \begin{aligned} x &= 3026A \\ y &= -2222A \\ z &= 338A \end{aligned} \right\} \quad (29)$$

Thus (29) represent non-zero distinct integer solution to (1) in one parameter.

H. Observations

- 1) $z(A^2) + z(A) - 338Pr_A = 0$
- 2) $2x(A^2) + 2y(A^2) - 4z(A^2) = (16A)^2$, is a perfect square.
- 3) $y(A^3) + y(A^2) + 4444P_A^5 = 0$
- 4) $3x(A^2) + 3y(A^2) + z(A^2) - 59T_{4,A} = (53A)^2$, is a perfect square.
- 5) $x(A^2) + x(A) - 6052P_A^2 = 0$

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