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Construction of the Diophantine Triple involving Pronic Number

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Abstract: We search for three distinct polynomials with integer coefficients such that the product of any two numbers increased by a non-zero integer (or polynomials with integer coefficients) is a perfect square.

Keywords: Diophantine triples, Pronic number, Perfect square.

Notations: Pronic Number of rank $n = n(n+1)$

I. INTRODUCTION

Let n be an integer. A set of positive integers $\{a_1, a_2, \dots, a_m\}$ is said to have the property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \leq i < j \leq m$; such a set is called a Diophantine m -tuple of size m . The problem of construction of such set was studied by Diophantus. Many mathematicians considered the problem of the existence of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n [1] and also for any linear polynomials n . Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-14]. In this communication, we present three sections where in each of which we find the Diophantine triples from Pronic number with different ranks. A few interesting relations among the numbers in each of the above Diophantine triples are presented.

II. METHOD OF ANALYSIS

A. Section A

Let $a = n^2 - n$ and $b = n^2 + n$ be Pronic number of rank $n - 1$ and n respectively such that $ab + (3n^2 + 1)$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (3n^2 + 1) = \beta^2 \tag{1}$$

$$bc + (3n^2 + 1) = \gamma^2 \tag{2}$$

Setting $\beta = a + \alpha$ and $\gamma = b + \alpha$, then subtracting (1) from (2), we get

$$\begin{aligned} c(b-a) &= \gamma^2 - \beta^2 = (\gamma + \beta)(\gamma - \beta) \\ &= (a+b+2\alpha)(b-a) \end{aligned}$$

Thus, we get $c = a + b + 2\alpha$

Similarly by choosing $\beta = a - \alpha$ and $\gamma = b - \alpha$, we obtain $c = a + b - 2\alpha$

Here we have $\alpha = n^2 + 1$ and thus two values of c are given by $c = 4n^2 + 2$ and $c = -2$.

Thus, we observe that $\{n^2 - n, n^2 + n, 4n^2 + 2\}$ and $\{n^2 - n, n^2 + n, -2\}$ are Diophantine triples with the property $D(3n^2 + 1)$.

Some numerical examples are given below in the following table.

Table 1

n	Diophantine Triples	$D(3n^2 + 1)$
1	(0,2,6) & (0,2,-2)	4
2	(2,6,18) & (2,6,-2)	13
3	(6,12,38) & (6,12,-2)	28

We present below, some of the Diophantine triples for Pronic number of rank mentioned above with suitable properties.

Table 2

a	b	c	$D(n)$
$n^2 - n$	$n^2 + n$	$4n^2 + 4$	$5n^2 + 4$
		-4	
$n^2 - n$	$n^2 + n$	$4n^2 + 6$	$7n^2 + 9$
		-6	
$n^2 - n$	$n^2 + n$	$4n^2 + 8$	$9n^2 + 16$
		-8	

B. Remarkable Observation

In general $\{n^2 - n, n^2 + n, 4n^2 + 2n\}$ and $\{n^2 - n, n^2 + n, -2n\}$ are Diophantine triples with the property $D((2n + 1)n^2 + n^2)$.

C. Section B

Let $a = n^2 - 3n + 2$ and $b = n^2 - n$ be Pronic number of rank $n - 2$ and $n - 1$ respectively such that $ab + (n^2 - 2n + 1)$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (n^2 - 2n + 1) = \beta^2 \tag{3}$$

$$bc + (n^2 - 2n + 1) = \gamma^2 \tag{4}$$

Applying the procedure as mentioned in section (A), we obtain

$$c = 4n^2 - 8n + 4 \text{ and } c = 0$$

Thus, we observe that $\{n^2 - 3n + 2, n^2 - n, 4n^2 - 8n + 4\}$ and $\{n^2 - 3n + 2, n^2 - n, 0\}$ are Diophantine triples with the property $D(n^2 - 2n + 1)$.

Some numerical examples are given below in the following table.

Table 3

n	Diophantine Triples	$D(n^2 - 2n + 1)$
2	(0,2,4) & (0,2,0)	1
3	(2,6,16) & (2,6,0)	4
4	(6,12,36) & (6,12,0)	9

We present below, some of the Diophantine triples for Pronic number of rank mentioned above with suitable properties.

Table 4

a	b	c	$D(n)$
$n^2 - 3n + 2$	$n^2 - n$	$4n^2 - 8n + 6$	$3n^2 - 6n + 4$
		-2	
$n^2 - 3n + 2$	$n^2 - n$	$4n^2 - 8n + 8$	$5n^2 - 10n + 9$
		-4	
$n^2 - 3n + 2$	$n^2 - n$	$4n^2 - 8n + 10$	$7n^2 - 14n + 16$
		-6	

D. Remarkable Observation

In general $\{n^2 - 3n + 2, n^2 - n, 4n^2 - 8n + 2n\}$ and $\{n^2 - 3n + 2, n^2 - n, -2t\}$ are Diophantine triples with the property $D((2k + 1)n^2 - 2pn + p^2)$ where $n = 2, 3, \dots$ $t = 0, 1, 2, \dots$

$$k = 0, 1, 2, \dots \quad p = 1, 2, \dots$$

E. Section C

Let $a = n^2 - 5n + 6$ and $b = n^2 - 3n + 2$ be Pronic number of rank $n - 3$ and $n - 2$ respectively such that $ab + (-5n^2 + 20n - 11)$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (-5n^2 + 20n - 11) = \beta^2 \tag{5}$$

$$bc + (-5n^2 + 20n - 11) = \gamma^2 \tag{6}$$

Applying the procedure as mentioned in section (A), we obtain

$$c = 4n^2 - 16n + 10 \text{ and } c = 6$$

Thus, we observe that $\{n^2 - 5n + 6, n^2 - 3n + 2, 4n^2 - 16n + 10\}$ and $\{n^2 - 5n + 6, n^2 - 3n + 2, 6\}$ are Diophantine triples with the property $D(-5n^2 + 20n - 11)$.

Some numerical examples are given below in the following table.

Table 5

n	Diophantine Triples	$D(-5n^2 + 20n - 11)$
1	(2,0,-2) & (2,0,6)	4
4	(2,6,10) & (2,6,6)	-11
5	(6,12,30) & (2,12,6)	-36

We present below, some of the Diophantine triples for Pronic number of rank mentioned above with suitable properties.

Table 6

a	b	c	$D(n)$
$n^2 - 5n + 6$	$n^2 - 3n + 2$	$4n^2 - 16n + 12$	$3n^2 + 12n - 8$
		4	
$n^2 - 5n + 6$	$n^2 - 3n + 2$	$4n^2 - 16n + 14$	$-n^2 + 4n - 3$

		2	
$n^2 - 5n + 6$	$n^2 - 3n + 2$	$4n^2 - 16n + 16$	$n^2 - 4n + 4$
		0	

III. CONCLUSION

In this paper we have presented a few examples of constructing a Diophantine triples for Pronic number of different rank with suitable properties. To conclude one may search for Diophantine triples for other special number with their corresponding suitable properties.

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