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Fuzzy τ*-Generalized Closed Sets in Fuzzy Topological Spaces

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***Abstract:** In this paper, we introduce a new class of sets called fuzzyτ*-generalized closed sets and fuzzy τ*-generalized open sets in fuzzy topological spaces and explore some of their properties.*
***Keywords:** fuzzy closed set fuzzy open set, fuzzyτ*-g-closed set, fuzzy τ*-g-open set.*

I. INTRODUCTION

Let X be a non-empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta (y) = \beta$ for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \leq B$ if and only if $\rceil(A_qB^c)$.A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy open subsets of A.

II. PRELIMINARIES

We recall the following definitions:

A. *Definition 2.1:* A subset A of a fuzzy topological space (X, τ) is called;

1) Fuzzy Generalized closed (briefly fuzzy g-closed) if cl(A) ⊆ G whenever A ⊆ G and G is fuzzy open in X.
2) Fuzzy Semi-generalized closed (briefly fuzzy sg-closed) if scl(A) ⊆ G whenever A ⊆ G and G is fuzzy semiopen in X.
3) Fuzzy Generalized semi closed (briefly fuzzy gs-closed) if scl(A) ⊆ G whenever A ⊆ G and G is fuzzy open in X.
4) Fuzzyα-closed[8] if cl(int(cl(A))) ⊆ A .
5) Fuzzyα-generalized closed (briefly fuzzy αg-closed) if clα(A)⊆ G whenever A ⊆ G and G is fuzzy open in X.
6) Fuzzy Generalized α-closed (briefly fuzzy gα-closed) if spcl (A) ⊆ G whenever A ⊆ G and G is fuzzy open in X.
7) Fuzzy Generalized semi-preclosed (briefly fuzzy gsp-closed) if scl(A) ⊆ G whenever A ⊆ G and G is fuzzy open in X.
8) Fuzzy Strongly generalized closed (briefly fuzzy strongly g-closed)if cl(A) ⊆ G whenever A ⊆ G and G is fuzzy g-open in X.
9) Fuzzy Preclosed if cl(int(A)) ⊆ A.
10) Fuzzy Semi-closed if int(cl(A)) ⊆ A.
11) Fuzzy Semi-preclosed (briefly fuzzy sp-closed) if int(cl(int(A))) ⊆ A.

The complements of the above mentioned sets are called their respective fuzzy open sets.

B. *Definition 2.2:*For the subset A of a fuzzy topological X, the fuzzy generalized closure operator cl*[5] is defined by the intersection of all fuzzy g-closed sets containing A.

C. *Definition 2.3:*For the subset A of a fuzzy topological X, the topology τ* is defined by $\tau^* = \{G : cl^*(G^c) = G^c \}$

D. *Definition 2.4:*For the fuzzy subset A of a fuzzy topological X,

1) Thefuzzy semi-closure of A (briefly scl(A)) is defined as the intersection of all fuzzy semi-closed sets containing A.
2) The fuzzy semi-preclosure of A (briefly fuzzy spcl(A)) is defined as the intersection of all fuzzy semi-preclosed sets containing A.
3) The fuzzy α-closure of A (briefly $cl_\alpha(A)$) is defined as the intersection of all fuzzy α-closed sets containing A.

III. FUZZY T*-GENERALIZED CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

In this section, we introduce the concept of fuzzy τ*-generalized closed sets in fuzzy topological spaces.

*A. Definition 3.1.*A fuzzy subset A of a fuzzy topological space X is called fuzzy τ*-generalized closed set (briefly fuzzy τ*-g-closed) if cl*(A)⊆ G whenever A ⊆ G and G is fuzzy τ*-open. The complement of fuzzy τ*-generalized closed set is called the fuzzy τ*-generalized open set (briefly fuzzy τ*-g-open)
.

*B. Theorem 3.1.*Every fuzzy closed set in X is fuzzy τ*-g-closed.

1) *Proof.* Let A be a fuzzy closed set. Let A ⊆ G. Since A is fuzzy closed, cl(A) = A ⊆ G. But cl*(A) ⊆ cl(A). Thus, we havecl*(A) ⊆ G whenever A ⊆ G and G is fuzzy τ*-open. Therefore A is fuzzy τ*-g-closed.

*C. Theorem 3.2.*Every fuzzy τ*-closed set in X is fuzzy τ*-g-closed.

1) *Proof.* Let A be a fuzzy τ*-closed set. Let A ⊆ G where G is fuzzy τ*-open. Since A is fuzzy τ*-closed, cl*(A) = A ⊆G. Thus, we have cl*(A) ⊆ G whenever A ⊆ G and G is fuzzy τ*-open. Therefore A is fuzzy τ*-g-closed.

*D. Theorem 3.3.*Every fuzzy g-closed set in X is a fuzzy τ*-g-closed set but not conversely.

1) *Proof :*Let A be a fuzzy g-closed set. Assume that A ⊆ G, G is fuzzy τ*-open in X. Then cl(A) ⊆ G, since A is fuzzy g-closed. But cl*(A) ⊆ cl(A). Therefore cl*(A) ⊆ G. Hence A is fuzzy τ*-g-closed. The converse of the above theorem need not be true .
2) *Remark 3.1.:*The following example shows that fuzzy τ*-g-closed sets are independent from fuzzy sp-closed set, fuzzy sg-closed set, fuzzy α-closed set, fuzzy preclosed set fuzzy gs-closed set, fuzzy gsp-closed set, fuzzy αg-closed set and fuzzy gα-closed set.

*E. Theorem 3.4.:*For any two fuzzy sets A and B, **c**l*(A ∪ B) = cl*(A) ∪ cl*(B)

1) *Proof :*Since A ⊆ A ∪ B , we have cl*(A) ⊆ cl*(A ∪ B) and since B ⊆ A ∪ B , we have cl*(B) ⊆ cl*(A ∪ B).Therefore cl*(A) ∪ cl*(B) ⊆ cl*(A ∪ B). Also , cl*(A) and cl*(B) are the fuzzy closed sets Therefore cl*(A) ∪ cl*(B) is also a fuzzy closed set. Again, A ⊆ cl*(A) and B ⊆ cl*(B) implies A∪B ⊆ cl*(A) ∪ cl*(B).Thus, cl*(A) ∪ cl*(B) is a closed set containing A ∪ B. Since cl*(A ∪ B) is the fuzzy smallest closed set containing A∪B we have cl*(A ∪ B) ⊆ cl*(A) ∪ cl*(B). Thus, cl*(A ∪B) = cl*(A) ∪ cl*(B

*F. Theorem 3.5.:*Union of two fuzzy τ* g-closed sets in X is a fuzzy τ*- g-closed set in X.

1) *Proof :*Let A and B be two fuzzy τ* g-closed sets. Let A ∪ B ⊆ G, where G is fuzzy τ*-open. Since A and B are fuzzy τ*-g-closed sets, cl*(A) ∪ cl*(B) ⊆ G. But cl*(A) ∪ cl*(B) = **c**l*(A ∪ B). Therefore **c**l*(A ∪ B) ⊆ G. Hence A ∪ B is a fuzzy τ*- g-closed set.

*G. Theorem 3.6.:*A subset A of X is fuzzy τ*-g-closed if and only if cl*(A) – A contains no non-empty fuzzy τ* -closed set in X.

1) *Proof:* Let A be a fuzzy τ*-g-closed set. Suppose that F is a nonempty fuzzy τ*-closed subset of cl*(A) – A.Now F ⊆ cl*(A) – A. Then F ⊆ cl*(A) ∩ Ac, since cl*(A) – A = cl*(A) ∩ A^c. Therefore F ⊆ cl*(A) and F ⊆ A^c. Since Fc is a fuzzy τ*-open set and A is a fuzzy τ*-g-closed, cl*(A) ⊆ F^c. That is F ⊆ $[cl^*(A)]^c$. Hence F ⊆ cl*(A) ∩ $[cl^*(A)]^c$ = φ. That is F = φ, a contradiction. Thus cl*(A) – A contain no non-empty fuzzy τ*-closed set in X. Conversely, assume that cl*(A) – A contains no nonempty fuzzy τ*-closed set. Let A ⊆ G, G is fuzzy τ*-open. Suppose that cl*(A) is not contained in G, then cl*(A) ∩ G^c is a non-empty fuzzy τ*-closed set of cl*(A) – A which is a contradiction. Therefore cl*(A) ⊆ G and hence A is fuzzy τ*-g-closed.
2) *Corollary 3.1.*A subset A of X is fuzzy τ* g-closed if and only if cl*(A) – A contain no non-empty fuzzy closed set in X.
3) *Proof:* Easy
4) *Corollary 3.2:*.A subset A of X is fuzzy τ*-g-closed if and only if cl*(A) – A contain no non-empty fuzzy open set in X.
5) *Proof:* The proof follows from the Theorem 3.10 and the fact that every open set is fuzzy τ*-open set in X.

*H. Theorem 3.7.*If a subset A of X is fuzzy τ*-g-closed and A ⊆ B ⊆ cl*(A), then B is fuzzy τ*-g-closed set in X.

1) *Proof:* A be a fuzzy τ*-g-closed set such that A ⊆ B ⊆ cl*(A). Let U be a fuzzy τ*-open set of X such that B ⊆ U. Since A is fuzzy τ*-g-closed, we have cl*(A) ⊆ U. Now cl*(A) ⊆ cl*(B) ⊆ cl*[cl*(A)] = cl*(A) ⊆ U. That is cl*(B) ⊆ U, U is fuzzy τ*-open. Therefore B is fuzzy τ*-g-closed set in X.The converse of the above theorem need not be true .

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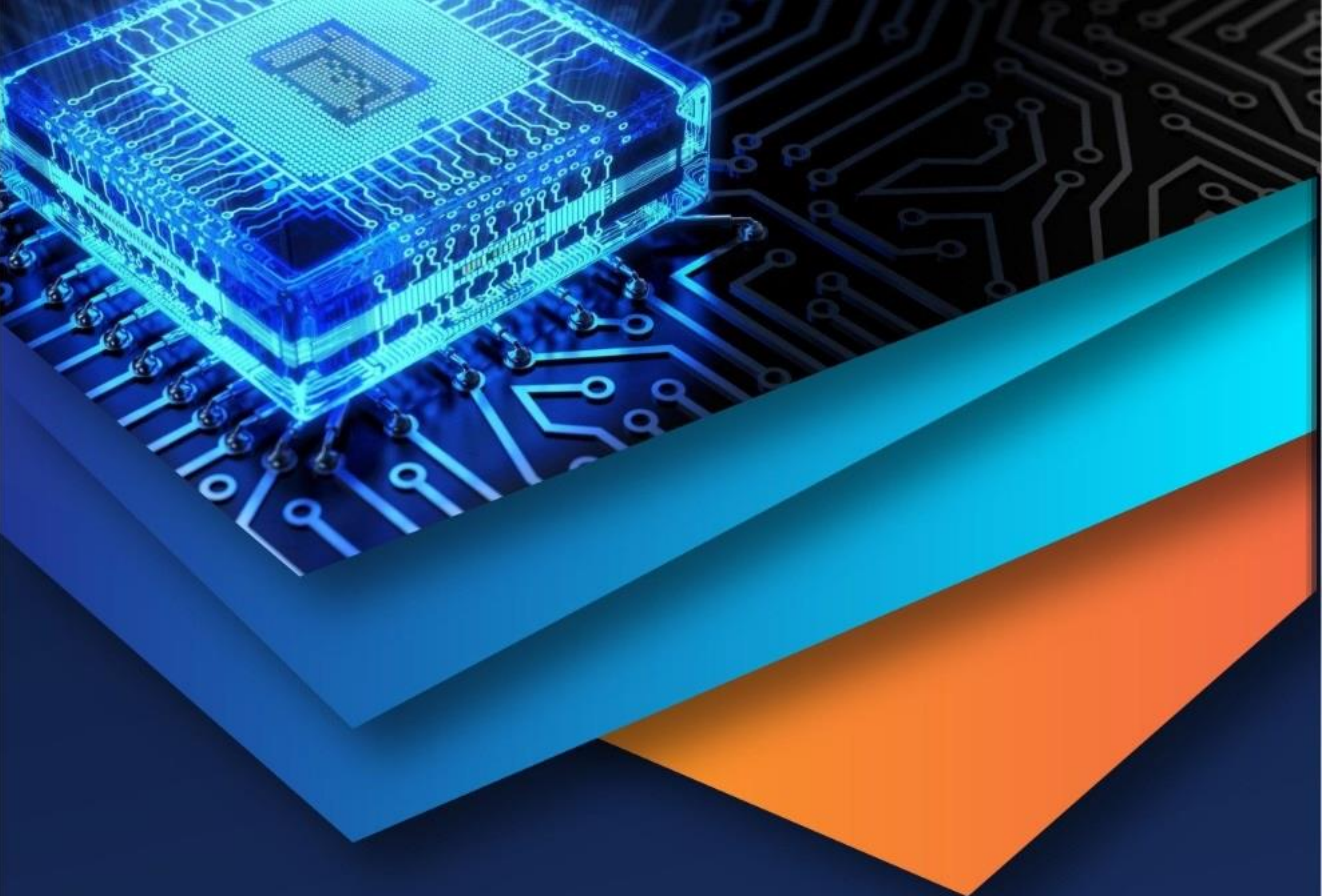

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