iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 **Issue:** II **Month of publication:** February 2018

DOI:

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor : 6.887
Volume 6 Issue II, February 2018- Available at www.ijraset.com

Independence of Mean Squares in Unbalanced Nested Designs

Dr.W.S.Brar[1]
[1]PG Department of Mathematics Baba Farid college Baba Farid Group of Institutions Bathinda -151001 , Punjab, India

Abstract: In this paper, the distributional properties of independence of mean squares in three stage unbalanced nested designs with random effects have been investigated. The correlation coefficients between mean squares of main classes and mean squares of subclasses of different design structures have been calculated. Also the condones have been derived under which mean squares of main classes and mean squares of sub classes are independently distributed
Keywords: unbalanced, nested, mean squares, independence, correlations , designs

I. INTRODUCTION AND LITERATURE SURVEY

In balanced three stage nested classifications with random effects, the mean squares in the analysis of variance are independent and are chi-squares distributed under the usual normality assumptions. The variance component estimators are linear functions of mean squares and their variances can be derived accordingly. When the classification data are not balanced, the mean squares , in general , are no longer independent and also they do not follow chi-square type distributions. Similarly the distributions of variance component estimators of classes and subclasses are not linear combinations of chi-squares in unbalanced classification. An exact test does not always exist even in balanced designs for random effects model. The ANOVA testing procedures were extended by [1], [2] and [3] for the construction of approximate F-tests in unbalanced designs. These tests make use of the fact that the mean squares are distributed independently and have chi-square type distributions but that do not generally hold for unbalanced designs. Ignoring all these factors, onle appromate tests developed for balanced designs have also been applied to unbalanced designs. [4] have discussed approximate F-tests for testing the variance components in which correlation of mean squares of numerator and synthesized denominator of F- test are calculated. Further, they have calculated correlations of mean squares in some special cases. There is a need to study the dependence of mean squares on all types of imbalance in nested designs. Further, [6] has mentioned that zero covariance of mean squares always implies independence under the normality assumptions. Therefore , an empirical study of twenty designs of different structures is undertaken and calculated the correlations and derived the conditions under which the mean squares are independently distributed. Consider the three stage unbalanced nested random effects model :

$$y_{ijk} = \mu + a_i + b_{ij} + e_{ijk}$$

i=1,2,…..a , j=1,2,….......$b_{i.}$ k= 1,2,………….n_{ij}

Where y_{ijk} is the observation, µ is the general mean, a_i and b_{ij} are the random effects, $e_{ijk}$the random error associated with y_{ijk}. As usual, it is assumed that $a_i \sim N(0, \sigma_a^2)$, $b_{ij} \sim N(0, \sigma_b^2)$, $e_{ijk} \sim N(0, \sigma_e^2)$

The ANOVA for three stage unbalanced nested design with random effects is given in Table 1 below :

Table 1
Analysis of Variance (ANOVA)

Source of variation	d.f	Mean Squares	Expected Mean Squares
Main classes	f_3	$y'Q_3y/f_3 = V_3$	$\sigma_e^2 + q_1\sigma_b^2 + q_2\sigma_a^2$
Sub classes	f_2	$y'Q_2y/f_2 = V_2$	$\sigma_e^2 + q_0\sigma_b^2$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor : 6.887
Volume 6 Issue II, February 2018- Available at www.ijraset.com

Within subclasses f_1 $y'Q_1y/f_1 = V_1$ σ_e^2

Where

$Q_3 = \sum_i^+ \frac{J_{n_i}}{n_i} - J_n/n$, $Q_2 = \sum_i \sum_j^+ J_{n_{ij}}/n_{ij} - \sum_i^+ \frac{J_{n_i}}{n_i}$, $Q_1 = I - \sum_i \sum_j^+ J_{n_{ij}}/n_{ij}$

$q_0 = \sum_i \sum_j n_{ij}^2 (1/n_{ij} - 1/n_i)/f_2$, $q_1 = \sum_i \sum_j n_{ij}^2 (1/n_i - 1/n)/f_3$, $q_2 = \sum_i n_i^2 (1/n_i - 1/n)/f_3$

also $\sum_i b_i = b$, $\sum_j n_{ij} = n_i$, $\sum_i \sum_j n_{ij} = \sum_i n_i = n$

$f_1 = n - \sum_i b_i$ $f_2 = \sum_i b_i - a$ $f_3 = a - 1$

The row vector y' of y_{ijk} ' s is $y' = [y_{111}, y_{112}, \ldots \ldots \ldots \ldots . y_{11n_{11}}; y_{121}, y_{122} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots]$

And J_n is the $n \times n$ matrix of 1's and $\sum_{i=1}^{k+} A_i$ is the direct sum of matrices A_1 , A_2 ……….A_k as defined in [6].

The designs are given below in Table 1.

Table 1

		1	2	3	4	5	6	7	8	
Design1	b_i	2	2	2	2					a=4
	n_{ij}	2,1	2,1	2,1	2,1					
Design2	b_i	2	2	2	2	2	2	2	2	a=8
	n_{ij}	2,1	2,1	2,1	2,1	2,1	2,1	2,1	2,1	
Design3	b_i	2	2	2	2					a=4
	n_{ij}	20,1	20,1	20,1	20,1					
Design4	b_i	2	3	2	1					a=4
	n_{ij}	2,2	2,2,2	2,2	2					
Design5	b_i	2	3	2	1					a=4
	n_{ij}	3,3	3,3,3	3,3	3					
Design6	b_i	5	4	2	5					a=4
	n_{ij}	2,2,2,2,2	2,2,2,2	2,2	2,2,2,2,2,2,2					
Design7	b_i	15	2	1						a=3
	n_{ij}	2,2,2,2,2,2,2,2, 2,2,2,2,2,2,2	2,2	2						
Design8	b_i	2	1	1	1	1				a=5
	n_{ij}	4,4	2	2	2	2				
Design9	b_i	1	2	3	4					a=4
	n_{ij}	4	3,3	2,2,2	1,1,1,1					
Design10	b_i	3	4	2	3	2				a=5
	n_{ij}	4,4,4	2,2,2,2	1,1	3,3,3	5,5				
Design11	b_i	2	2	2	2	1	1			a=6
	n_{ij}	2,2	1,1	1,1	1,1	1	1			
Design12	b_i	2	2	2	2	1	1			a=6
	n_{ij}	1,1	1,1	1,1	1,1	2	2			

Design13	b_i	2	2	1	1	1	1			a=6
	n_{ij}	1,1	1,1	2	2	2	2			
Design14	b_i	1	1	3						a=3
	n_{ij}	1	2	4,4,4						
Design15	b_i	1	1	3						a=3
	n_{ij}	5	4	2,2,2						
Design16	b_i	1	1	3						a=3
	n_{ij}	1	5	3,3,3						
Design17	b_i	1	2	2						a=3
	n_{ij}	7	3,3	1,1						
Design18	b_i	2	1	1	1	1	1			a=6
	n_{ij}	8,8	2	2	2	2	2			
Design19	b_i	4	4	4						a=3
	n_{ij}	1,8,12,2	7,1,15,3	10,2,4,6						
Design20	b_i	2	4	6	1					a=4
	n_{ij}	1,5	2,7,1,3	1,8,12,3,20,2	5					

Where 'a ' indicates the number of main classes, b_i indicates the number of subclasses within the main class and n_{ij} the number of observations.

II. RESULTS AND DISCUSSION

Correlations between V_2 and V_3 in three stage unbalanced nested designs are given in Table 2 :

Table 2

	Design 1					
	Y					
X	0	0.5	1	4	16	24
↓	0	0.0157	0.0309	0.0634	0.0707	0.0819
0	0	0.0087	0.0198	0.0530	0.0756	0.0790
0.5	0	0.0060	0.0146	0.0456	0.0719	0.0763
1	0	0.0021	0.0056	0.0247	0.0556	0.0634
4	0	0.0006	0.0016	0.0087	0.0292	0.0377
16	0	0.0004	0.0011	0.0061	0.0221	0.0297
24						

	Design 2					
	Y					
X	0	0.5	1	4	16	24
↓	0	0.0170	0.0334	0.0685	0.0861	0.0885
0	0	0.0094	0.0214	0.0573	0.0817	0.0854
0.5	0	0.0065	0.0157	0.0492	0.0777	0.0825
1	0	0.0023	0.0061	0.0267	0.0601	0.0685
4	0	0.0006	0.0018	0.0094	0.0315	0.0408
16	0	0.0004	0.0012	0.0064	0.0239	0.0321
24						

	Design 3					
X ↓	Y					
	0	0.5	1	4	16	24
0	0	0.1721	0.2429	0.3401	0.3762	0.3806
0.5	0	0.0863	0.1595	0.2995	0.3631	0.3721
1	0	0.0575	0.1188	0.2675	0.3521	0.3640
4	0	0.0192	0.0469	0.1631	0.2953	0.3218
16	0	0.0052	0.0137	0.0637	0.1795	0.2198
24	0	0.0035	0.0093	0.0453	0.1423	0.1815

	Designs 4 to 18					
X ↓	Y					
	0	0.5	1	4	16	24
0	0	0	0	0	0	0
0.5	0	0	0	0	0	0
1	0	0	0	0	0	0
4	0	0	0	0	0	0
16	0	0	0	0	0	0
24	0	0	0	0	0	0

	Design 19					
X ↓	Y					
	0	0.5	1	4	16	24
0	0	0.0826	0.1028	0.1229	0.1287	0.1294
0.5	0	0.0264	0.0474	0.0934	0.1191	0.1228
1	0	0.0157	0.0308	0.0753	0.1109	0.1168
4	0	0.0046	0.0099	0.0349	0.0783	0.0905
16	0	0.0012	0.0027	0.0111	0.0360	0.0476
24	0	0.0008	0.0018	0.0076	0.0265	0.0361

	Design 20					
X ↓	Y					
	0	0.5	1	4	16	24
0	0	0.0619	0.0784	0.0951	0.1001	0.1006
0.5	0	0.0223	0.0391	0.0745	0.0934	0.0961
1	0	0.0135	0.0258	0.0610	0.0876	0.0919
4	0	0.0040	0.0085	0.0290	0.0634	0.0724
16	0	0.0010	0.0023	0.0093	0.0298	0.0392
24	0	0.0007	0.0015	0.0064	0.0220	0.0299

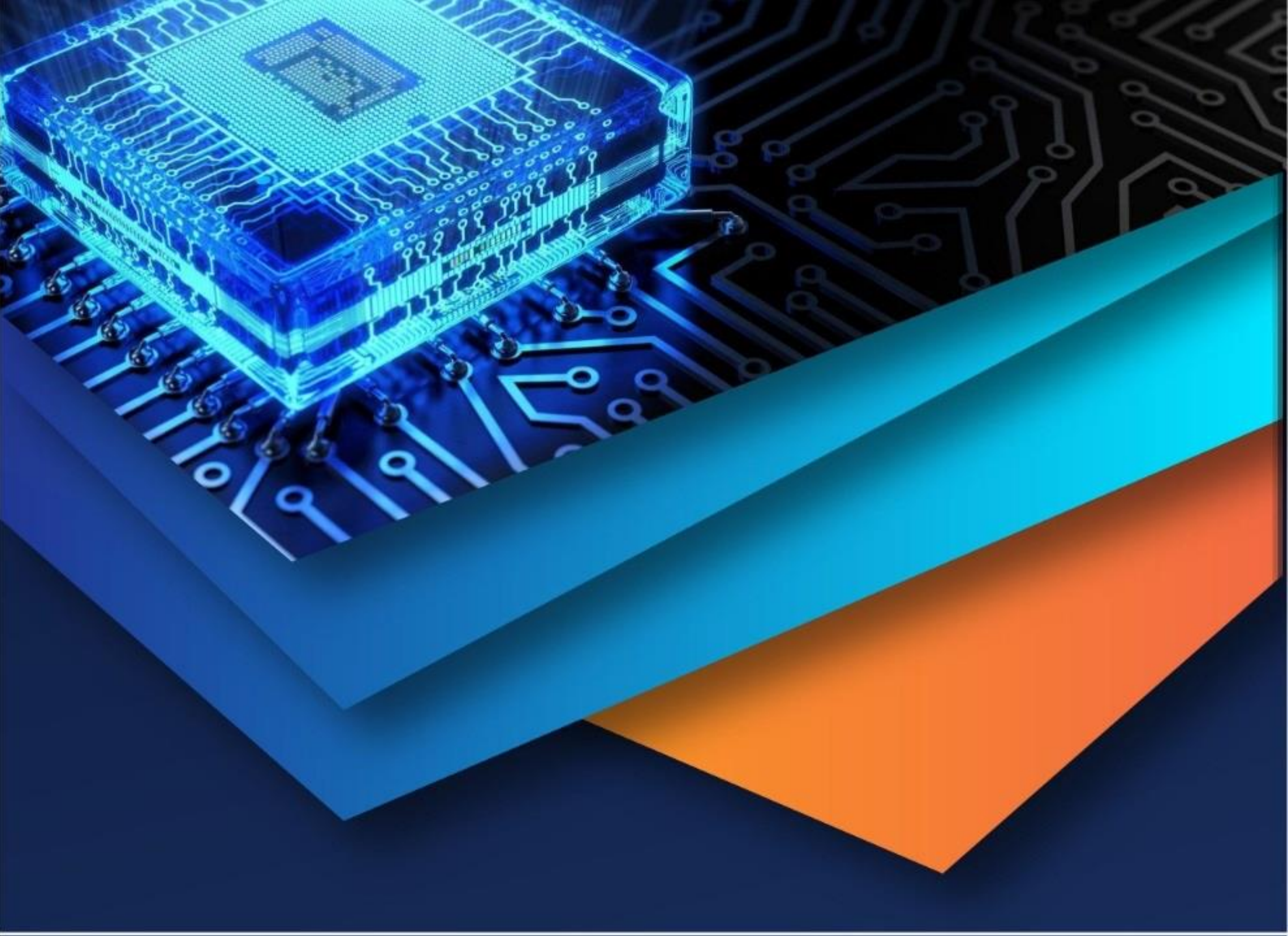
In this study, the ratio of ^{2} / ^{2} and ^{2} / ^{2} are denoted by X and Y respectively. The designs 1 and 2 (Table1) are of special kind in which $b_i = 2$, $n_{i1}= 2$ and $n_{i2}=1$ for all i . In these designs, correlation between mean squares V_2 and V_3 increases for given X and decreases as X increases for given Y. The magnitude of correlation is very small for $X \geq 4$ and $Y \leq 4$ (Table 2). When we increase the number of classes in design2 , the corresponding correlations are slightly increased. When we increase the imbalance at the third stage as in design 3 i.e $n_{i1} = 20$ and $n_{i2} = 1$ with $b_i = 2$ for all i, the correlation increases 4 to 10 times. Further, it is noticed that for each given value of X, the values of correlation decreased as the value of Y increased. A similar trend is also noticed for each given value of Y, the values of correlation decreased as X increased in design 3.When $n_{ij} = k_i$ for all i, that is, balance within the classes, the correlation between mean squares V_2 and V_3 are all zero for all the values of X and Y. This shows that mean squares V_2 and V_3 are independent distributed variables In literature, such designs are called partially balanced nested designs. The designs 4 to 7 are of this kind .When $n_{ij} = k$ for all i and j, that is, balance at the last stage (b_i's are different), the correlation between mean squares V_2 and $V_3$are all zero for all the values of X and Y. Again, this shows that mean squares V_2 and V_3 are independent distributed variables in designs 8 to 18. Such designs are called last stage uniformity designs. The design 19 is balanced at the second stage but highly unbalanced at the third stage. The design 20 is general unbalanced one and the correlations are given in Table 2.

III. CONCLUSION

We have noticed that correlation between V_2 and V_3 are zero when Y = $^{2} / ^{2}$ = 0 for $X \geq 0$ in all the twenty designs. This shows that under the acceptance of the hypothesis that ^{2} is equal to zero, the mean squares V_2 and V_3 are independently distributed. Further, When $n_{ij} = k$ for all i and j, that is, balance at the last stage (b_i's are different) and $n_{ij} = k_i$ for all i, that is, balance within the classes, the mean squares V_2 and V_3 are independently distributed variables.

REFERENCES

[1] Cochran, W.G (1951) : Testing a linear relation among variances. Biometrics, 7, 17 – 32
[2] Anderson,R.L and Bancroft, T.A. (1952) : Statistical theory in Research. McGraw Hill Book Company , NewYork, N.
[3] Eisen, E.J. (1966) : The quasi – F test for an unnested fixed factor in an unbalanced hierarchical design withmixed model. Biometrics, 22, 937 – 942
[4] Cummings , W.B. and Gaylor, D.W. (1974) : Variance components testing in unbalanced nested designs. J.Amer. Stat. Assoc., 69, 675-671
[5] Searle , S.R.(1961): Variance components in the unbalanced 2- way nested classification. Ann. Math. Stat. 32.1161 – 1166
[6] Searle , S.R. (1971) : Linear models. John Wiley & sons, Inc, N.Y.



10.22214/IJRASET

INDEX COPERNICUS 45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY