



IJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 Issue: II Month of publication: February 2018

DOI:

www.ijraset.com

Call:  08813907089

E-mail ID: ijraset@gmail.com

ON s-k –Tripotent Matrices

N. Elumalai¹, R. Anuradha², R. Mathubala³

^{1, 2, 3}PG& Research Department of Mathematics, A.V.C College (Autonomous), Mannampandal, Mayiladuthurai – 609305, Tamilnadu

Abstract: The concept of s-k-tripotent is introduced. Some basic results in s-k-tripotent matrices and their properties are given.

Keywords: Tripotent matrices, k-tripotent matrices.

I. INTRODUCTION

A Matrix ‘A’ that satisfies $A^3=A$ is called a tripotent matrix. In this paper basic concept of s-k- tripotent matrices are introduced. A Theory of k-real and k-Hermitian matrices as a generalization of secondary real and secondary Hermitian Matrices was developed by hill and waters[2]. The secondary symmetric and secondary orthogonal matrices have been analysed by Ann lee [1] in the year 1976. The secondary k-Hermitian matrices are introduced in 2009 by Meenakshi, krishnamoorthy and Ramesh[3]. S. Krishnamoorthy and P.S Meenakshi have studied the basic concepts of k-tripotent matrices as generalization of k-tripotent matrices. The sum and products of k- tripotent matrices are introduced in 2013 by S. Krishnamoorthy and P.S. Meenakshi[4]. Let ‘K’ be the associated permutation matrix of K and let V be the permutation matrix with unit in the secondary diagonal. Clearly K & V satisfies the following properties:

$$K = K^T = \overline{K} = K^*, K^2 = I$$

$$V = V^T = V^{-1} = V^* \text{ and } V^2 = I$$

II. S-K-TRIPOTENT MATRICES

A. Definition

A matrix $A = a_{ij}$ in $C_{n \times n}$ is said to be s-k-tripotent if $\sum_{t=1}^n a_{n-k(i)t} \left[\sum_{m=1}^n a_{tm} a_{m(n-k)t} \right] = a_{ij}$

This equivalent to $KVA^3VK=A$.

It is easy to see that $KVA^3VK=A$

$$KVAVK = A^3$$

$$VKA^3KV = A$$

$$VKAKV = A^3$$

1) Example

$$\text{Let } A = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}, K = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ \& } V = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$KVA^3VK = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

$$KVA^3VK = A$$

Therefore A is s-k-tripotent matrix.

2) *Theorem*

If $A \in C^{n \times n}$ be a s-k-tripotent matrix then

- a) A^T, \bar{A}, A^* and A^{-1} are also s-k-tripotent
- b) A^n is s-k-tripotent for all positive integers 'n'
- c) A is periodic with period 9
- d) A^3 is s-k-tripotent

3) *Proof:*

$$\begin{aligned}
 a) \quad A^T &= (KVA^3VK)^T \\
 &= KV A^T A^T A^T VK \\
 A^T &= KV(A^T)^3VK \\
 \therefore A^T &\text{ is s-k-tripotent}
 \end{aligned}$$

$$\overline{KVA^3VK} = KV(\bar{A}^3)VK$$

$$\begin{aligned}
 &= KV(\bar{A}^3)VK \\
 &= \bar{A}
 \end{aligned}$$

$\therefore \bar{A}$ is s-k-tripotent

$$KVA^3VK = A^*$$

$$\begin{aligned}
 &= KVA^{3*}VK \\
 &= KVA^{*3}VK \\
 &= A^*
 \end{aligned}$$

$\therefore A^*$ is s-k-tripotent.

$$KV A^3 VK^{-1} = KV A^{3^{-1}} VK$$

$$= KV(A^{-1})^3 VK$$

$$= A^{-1}$$

$\therefore A^{-1}$ is s-k-tripotent.

$$b) \quad A^n = (KVA^3VK)^n$$

$$= KVA^3VK \quad KVA^3VK \dots \dots \dots n \text{ times}$$

$$= KVA^{3n}VK$$

$$= KV(A^n)^3VK$$

$\therefore A^n$ is s-k-tripotent.

$$c) \quad A^9 = A^3 A^3 A^3$$

$$= (KVAVK) (KVAVK) (KVAVK)$$

$$= KVAAAVK$$

$$= KVA^3VK$$

$$A^9 = A$$

$\therefore A^9$ is period with 9.

d) *Theorem*

If $A \in C^{n \times n}$ be a s-k-tripotent matrix then $-A$ is s-k-tripotent.

a) *Proof:* Let s-k-tripotent matrix $KVA^3VK = A$

$$-(KVA^3VK)$$

$$\Rightarrow (KV-(A^3)VK)$$

$$\Rightarrow (KV(-A^3)VK)$$

$$-(KVA^3VK) = -A$$

Hence $-A$ is s-k-tripotent.

e) *Theorem*

Let $A, B \in C^{n \times n}$ be two s-k-tripotent matrices, $A+B$ is s-k-tripotent matrices if and only if $A = -B$.

a) *Proof:* Let A and B are two s-k-tripotent matrices

$$\therefore KVA^3VK=A \text{ and } KVB^3VK=B$$

Assume $A = -B$

$$A+B = KVA^3VK+KVB^3VK$$

$$= KV(A^3+B^3)VK$$

$$= KV(A+B)^3VK$$

$$\therefore A+B = KV(A+B)^3VK$$

i.e., $A+B$ is s-k-tripotent.

Conversely,

$A+B$ is s-k-tripotent.

$$A+B = KV(A+B)^3VK$$

$$= KV(A^3+B^3+3AB^2+3A^2B)VK$$

$$= KV(A^3+B^3+3AB[A+B])VK$$

$$= KVA^3VK+KVB^3VK+KV(3AB[A+B])VK$$

$$= A+B+KV(3AB[A+B])VK$$

Hence $KV(3AB[A+B])VK = 0$ if $A = -B$.

f) *Theorem*

If $A^9 = A^*$ then A reduces to orthogonal projection.

$$AB = BA$$

$$AB = KVA^3VK+KVB^3VK$$

$$= KVAAA VK KVB BBVVK$$

$$= KV AAA BBB VK$$

$$= KVAABABBVK$$

$$= KVABABABVK$$

$$\therefore AB = KV(AB)^3VK$$

Hence AB is s-k-tripotent.

B. Generalization

If $A_1 A_2 \dots A_n$ are s-k-tripotent belongs to a commuting family of matrices then $\prod_{i=1}^n A_i$ is s-k-tripotent.

1) *Proof*

Let $A_1 A_2 \dots A_n$ s-k-tripotent.

$$KV \left(\prod_{i=1}^n A_i \right)^3 VK = KV [A_1 A_2 \dots A_n] VK$$

$$= KV [A_1 A_2 \dots A_n] VK$$

$$= KV [A_1^3 A_2^3 A_3^3 \dots A_n^3] VK$$

$$= KVA_1^3 A_2^3 A_3^3 \dots A_n^3] VK$$

$$= A_1 A_2 \dots A_n$$

$$= \prod_{i=1}^n A_i$$

Therefore s-k-tripotent.

C. Theorem

If $A \in C^{n \times n}$ then any two of the following statements implies the other one.

- 1) A is s-k-tripotent
- 2) A is s-k-symmetric
- 3) A is cube symmetric

4) *Proof:*

a) And (b) implies that (c)

$$KVA^3VK = A \text{ and } KVA^T VK = A$$

$$KVA^3VK = KVA^T VK$$

$$A^3 = A^T$$

∴ A is cube symmetric.

b) And (c) implies that (a)

$$KV A^T VK = A \text{ and } A^3 = A^T \text{ substitute } A^3 = A^T \text{ in } KVA^T VK = KVA^3 VK = A.$$

∴ A is s-k-tripotent.

c) And (a) implies that (b)

$$A^3 = A^T \text{ substitute } A^3 = A^T \text{ in } KVA^3 VK = A.$$

$$KVA^T VK = A.$$

∴ A is s-k-symmetric.

D. Theorem

If $A \in C^{n \times n}$ then any two of the following statements implies the other one.

- 1) A is s-k-tripotent
- 2) A is symmetric
- 3) A is s-k cube symmetric

4) *Proof:*

a) And (b) implies that (c)

$$A = A^T \text{ and } KVA^3VK = A \text{ implies that } KVA^3VK = A^T$$

Therefore A is k-cube symmetric.

b) And (c) implies that (a)

$$A = A^T \text{ and } KVA^3VK = A^T \text{ implies that } KVA^3VK = A$$

Therefore A is s-k-tripotent.

c) And (a) implies that (b)

$$KVA^3VK = A^T \text{ and } KVA^3VK = A \Rightarrow A^T = A$$

Therefore A is symmetric.

E. Theorem

If $A \in C^{n \times n}$ then any two of the following statements implies the other one.

- 1) A is s-k-tripotent
- 2) A is s-k-Hermitian
- 3) A is s-k-cube Hermitian

4) *Proof*

a) And (b) implies that (c)

$$KVA^3VK = A \quad \text{and} \quad KVA^*VK = A$$

$$KVA^3VK = KVA^*VK$$

$$A^3 = A$$

A is cube Hermitian.

b) And (c) implies that (a)

$$KVA^*VK = A \quad \text{and} \quad A^3 = A^*$$

$$\text{Substitute } A^3 = A^* \text{ in } KVA^*VK = A$$

$$\Rightarrow KVA^3VK = A$$

Hence A is s-k-tripotent.

c) And (a) implies that (b)

$$A^3 = A \quad \text{and} \quad KVA^3VK = A$$

$$\text{Substitute } A^3 = A^* \text{ in } KVA^3VK = A \Rightarrow KVA^*VK = A.$$

Hence A is s-k-tripotent

REFERENCES

- [1] Ann Lee:secondary symmetric and secondary skew symmetric, secondary orthogonal matrices;Math.hungary7,(1976)
- [2] Hill and Waters S.R: on k-real and k-Hermitian matrices;journalcomput; vol.4,no.2,DEC(2008)
- [3] Meenakshi. A.R, Krishnamoorthy. S and Ramesh .G :on Secondary k-Hermitian matrices; J.Modernsci.,IC(2009).
- [4] S.Krishnamoorthy and PS.Meenakshi: on sums and products of k-tripotent matrices vol.2 issue 7 july 2013



10.22214/IJRASET



45.98



IMPACT FACTOR:
7.129



IMPACT FACTOR:
7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24*7 Support on Whatsapp)