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Study of Fuzzy Semi Open Sets in Fuzzy Topological Spaces

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Abstract: ***The present paper we introduce the concept of fuzzy γ-semi super open sets and fuzzy γ-semi T_i spaces and study some of their properties. Finally we introduce fuzzy (γ,β) –semi super continuous maps and explore the basic properties.***
Keywords: fuzzy topology, fuzzy continuity, fuzzy open set, fuzzy closed set fuzzy γ-semi open set, fuzzy γ-semi T_i spaces (i= 0, ½, 1, 2), fuzzy (γ,β) –semi continuous maps.

I. INTRODUCTION

Let X be a non-empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta (y)=\beta$ for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \leq B$ if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

II. PRELIMINIARIES

In this section we recall some of the basic definitions and some theorems.

1) *Definition 2.1:* Let (X, τ) be a fuzzy topological space. An operation γ on the fuzzy topology τ is mapping from τ on to power set P(X) of X such that $V \subseteq V^\gamma$ for each $V\in\tau$, where V^γ denotes the value of τ at V. It is denoted by $\gamma : \tau \to P (X)$.
2) *Definition 2.2:* A subset A of a fuzzy topological space (X,τ) is called fuzzy γ - open set if for each $x \in A$ there exists a fuzzy open set U such that $x \in U$ and $U^\gamma \subseteq A$. τ_γ denotes set of all fuzzy γ - open sets in (X, τ).
3) *Definition 2.3:* The fuzzy point $x \in X$ is in the fuzzy γ - closure of a fuzzy set $A \subseteq X$ if $U^\gamma \cap A \neq \phi$ for each open set U of x. The fuzzy γ - closure of a set A is denoted by $cl_\gamma(A)$.
4) *Definition 2.4:*Let (X,τ) be a fuzzy topological space and A be subset of X then τ_γ-cl (A)= $\cap$ { F: $A \subseteq F$, $X – F \in \tau_\gamma$ }
5) *DEFINITION 2.5 :* A subset A fuzzy topological space (X, τ) is said to be fuzzy γ - generalized closed if $cl_\gamma (A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy γ - open in (X, τ).
6) *Definition 2.6 :* A fuzzy topological space (X,τ) is called
1) fuzzy γ - T_0 space if for each distinct points $x,y \in X$, there exists an fuzzy open set U such that either $x \in U$ and $y \notin U^\gamma$ or $y \in U$ and $x \notin U^\gamma$.
2) fuzzy γ - T_1 space if for each distinct points $x,y \in X$ there exists fuzzy open set U, containing x and y respectively, such that $y \notin U^\gamma$ and $x\notin V^\gamma$.
3) fuzzy γ - T_2 space if for each distinct points x,y of X , there exists fuzzy open set $x\in U$, $y\in V$ and $U^\gamma \cap V^\gamma = \phi$.

7) *DEFINITION 2.7 :* Let (X, τ) be a fuzzy topological space. An operation γ is said to be fuzzy regular if, for every fuzzy open neighborhood U and V of each $x\in X$, there exists an fuzzy open neighborhood W of x such that $W^\gamma \subseteq U^\gamma \cap V^\gamma$

8) *Definition 2.8::* A fuzzy topological space (X, τ) is said to be fuzzy γ-regular ,where γ is an operation on τ , if for each x $\in$ X and for each fuzzy open neighborhood V of x, there exists an fuzzy open neighborhood U of x such that U^{γ} contained in V.
9) *Remark 2.1:* Let (X, τ) be a fuzzy topological space, then for any subset A of X, A $\subseteq$ cl (A) $\subseteq$ cl_{γ} (A) $\subset$ τ_{γ} - cl(A)

III. FUZZY γ - SEMIOPEN SETS

A. *Definition 3.1*: A subset A of fuzzy topological space (X, τ) is said to be a fuzzy γ - semi open set if and only if there exists a fuzzy γ - open set U such that U $\subseteq$ A $\subseteq$ cl_{γ} (U).
1) *Remark 3.1 :* The set of all fuzzy γ -semi open set in (X, τ)fuzzy topological space is denoted as fuzzy τ_{γ}- SO(X).
2) *Definition 3.2:* Let A be any subset of X. The fuzzy τ_{γ}- int(A) is defined as fuzzy τ_{γ}- int(A)= $\cup$ { U : U is a fuzzy γ - open set and U $\subseteq$ A}.
3) *Remark 3.2:* A subset A of fuzzy topological space (X,τ) is fuzzy γ - open then fuzzy τ_{γ}- int(A)= A.
4) *Proof:* Proof is straight forward from the definition.
5) *Theorem 3.1:* If A and B are any two subsets of (X, τ) fuzzy topological space , the
6) *PROOF:* Proof of (i) and (ii) is follows from the definition.

B. *Theorem 3.2:* A subset A of (X, τ) fuzzy topological space is fuzzy γ - semi open if and only if A $\subseteq$ cl_{γ}(τ_{γ}-int(A)).
1) *Proof:* Let A $\subseteq$ cl_{γ}(τ_{γ}-int(A)). Take U =τ_{γ}- int(A)then U is a fuzzy γ - open set. We have U $\subseteq$ A $\subseteq$ cl_{γ}(U) .Hence A is a fuzzy γ - semi open set.

Conversely, Given A is a fuzzy γ - semi open set in X, Then U $\subseteq$ A $\subseteq$ cl_{γ}(U) for some fuzzy γ-open set in (X,τ).Since U $\subseteq$ τ_{γ}-int(A),thus we have cl_{γ}(U)$\subseteq$$cl_{\gamma}$($\tau_{\gamma}$-int(A)).Hence A $\subseteq$ cl_{γ}(τ_{γ}-int (A)).

C. *Theorem 3.3:* If A is a fuzzy γ - open set in (X, τ), then A is a fuzzy γ - semi open set.
1) *Proof:* Given A is a fuzzy γ-open set in (X, τ), therefore by remark 3.5 τ_{γ}-int(A)= A. Since A $\subseteq$ cl_{γ}(A), implies A $\subseteq$ cl_{γ} (τ_{γ}-int(A)). Hence A is a fuzzy γ -semi open set.

D. *Theorem 3.4*: If (X,τ) is γ -regular space, then every γ-semi open set is a semi open set.
1) *Proof:* Easy
E. *Definition 3.3:* A fuzzy subset A of X is said to be fuzzy γ -semi closed if and only if X – A is fuzzy γ-semi open.
F. *Definition 3.4:* Let A be a subset of X. Then τ_{γ}-scl(A) = $\cap${F : F is γ - semi closed and A $\subseteq$ F}
G. *THEOREM 3.6:* For fuzzy point x $\in$ X , x$\in$ τ_{γ}-scl(A) if and only if V$\cap$ A $\neq$ ϕ for any V$\in$$\tau_{\gamma}$- SO(X) such that x $\in$ V.
1) *Proof:* Let F_0 be the set of all y $\in$ X such that V $\cap$ A $\neq$ ϕ for every V $\in$ τ_{γ}- SO(X) and y$\in$ V. Now to prove the theorem it is enough to prove that F_0= τ_{γ}-scl(A).Let x $\in$ τ_{γ}-scl(A). Let us assume x$\notin$ F_0 then there exists a fuzzy γ -semi open set U of x such that U $\cap$ A = ϕ. This implies A $\subseteq$$U^c$. Hence τ_{γ} -scl(A) $\subseteq$ U^c. Therefore x $\notin$ τ_{γ} -scl(A) .This is a contradiction. Hence τ_{γ}- scl(A) $\subseteq$ F_0. Conversely, Let F be a fuzzy set such that A $\subseteq$ F and X – F $\in$$\tau_{\gamma}$ -SO(X). Let x$\notin$ F then we have x $\in$X – F and (X – F) $\cap$ A = ϕ. This implies x $\notin$ F_0. Therefore F_0 $\subseteq$ F. Hence F_0 $\subseteq$ τ_{γ}-scl(A).

IV. FUZZY γ - SEMI τ_i -SPACES

In this section we investigate a general operation approaches on fuzzy T_i spaces .

1) *Definition 4.1:* A fuzzy topological space (X,τ) is called fuzzy γ - semi T_0 space if for each distinct points x,y$\in$ X there exists a fuzzy γ- semi open set U such that x $\in$ U and y$\notin$U or y $\in$U and x$\notin$U.
2) *Definition 4.2:* A fuzzy topological space (X,τ) is called a fuzzy γ- semi T_1 space if for each distinct points x, y $\in$X, there exists a fuzzy γ-semi open set U, V containing x and y respectively such that y$\notin$U and x$\notin$V.
3) *Definition 4.3:* A fuzzy topological space (X,τ) is called a fuzzy γ-semi T_2 space if for each x, y $\in$X there exists a fuzzy γ-semi open sets U, V such that x$\in$U and y$\in$ V and U$\cap$V =ϕ.
4) *Definition 4.4:* A subset A of fuzzy topological space (X, τ) is said to be fuzzy γ-semi g closed if τ_{γ}-scl(A) $\subseteq$ U whenever A$\subseteq$U and U is a fuzzy γ-semi open set in (X,τ).

5) *Definition 4.5:* A fuzzy topological space (X,τ) is said to be fuzzy γ-semi $T_{½}$ space if every fuzzy γ-semi g closed set in fuzzy topological space (X,τ) is fuzzy γ-semi closed.
6) *theorem 4.1:* A subset A of fuzzy topological space (X,τ) is fuzzy γ-semi g closed if and only if τ_γ-scl$(\{x\}) \cap A \neq \phi$ holds for every $x \in \tau_\gamma$-scl(A). Let U be fuzzy γ-semi open set such that $A \subseteq U$. Let $x \in \tau_\gamma$-scl(A). By assumption there exists a $z \in \tau_\gamma$-scl$(\{x\})$ and $z \in A \subseteq U$. then $U \cap \{x\} \neq \phi$.Hence $x \in U$. This implies τ_γ-scl$(A) \subseteq U$. Therefore A is fuzzy γ-semi g closed set in (X,τ).

Conversely, suppose $x \in \tau_\gamma$-scl(A) such that τ_γ-scl$(\{x\}) \cap A = \phi$.Since, τ_γ-scl$(\{x\})$ is fuzzy γ-semi closed set in fuzzy topological space (X,τ). Therefore $(\tau_\gamma\text{-scl}(\{x\}))^c$ is a fuzzy γ-semi open set. Since $A \subseteq (\tau_\gamma\text{-scl}(\{x\}))^c$ and A is fuzzy γ-semi g closed implies that τ_γ-scl$(A) \subseteq (\tau_\gamma\text{-scl}(\{x\}))^c$ holds, and hence $x \notin \tau_\gamma$-scl(A). which is a contradiction. Hence τ_γ-scl$(\{x\}) \cap A \neq \phi$.

7) *Theorem 4.2*: If τ_γ-scl$(\{x\}) \cap A \neq \phi$ holds for every $x \in \tau_\gamma$-scl(A), then τ_γ-scl(A) – A does not contain a non-empty fuzzy γ-semi closed set.

a) *PROOF:* Suppose there exists a non-empty fuzzy γ -semi closed set F such that $F \subseteq \tau_\gamma$-scl(A) – A. ($\subseteq \tau_\gamma$-scl(A)).Let $x \in F$, $x \in \tau_\gamma$-scl(A) holds. It follows from remark 3.23 such that $F \cap A = \tau_\gamma$-scl$(F) \cap A \supseteq \tau_\gamma$-scl$(\{x\}) \cap A \neq \phi$.Hence $F \cap A \neq \phi$.This is a contradiction.

8) *Theorem 4.3:* Let $\gamma : \tau \rightarrow P(X)$ be an mapping .Then for each $x \in X$, $\{x\}$ is fuzzy γ- semi closed or $\{x\}^c$ is fuzzy γ-semi g. closed set in (X, τ).

a) *Proof:* Suppose that $\{x\}$ is not fuzzy γ-semi closed then X-$\{x\}$ is not fuzzy γ-semi open. Let U be any fuzzy γ-semi open set such that $\{x\}^c \subseteq U$. Since U=X, implies τ_γ-scl$(\{x\}^c) \subseteq U$. Therefore $\{x\}^c$ is fuzzy γ-semi g closed.

V. FUZZY (γ, β)- SEMI CONTINUOUS MAPPINGS

Through out this chapter let (X,τ) and $(Y,.\sigma)$ be two fuzzy topological spaces and let $\gamma: \tau \rightarrow P(X)$ and $\beta: \sigma \rightarrow p(Y)$ be mapping on τ and σ respectively.

A. *Definition 5.1:* A fuzzy mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy (γ, β)- semi continuous if for each x of X and each fuzzy β-semi open set V containing f(x) there exists a fuzzy γ-semi open set U such that $x \in U$ and $f(U) \subseteq V$.

B. *Remark 5.1:* If (X,τ) and (Y,σ) are both fuzzy γ - regular spaces then the concept of fuzzy (γ,β) - semi continuity and fuzzy semi continuity are coincide.

C. *Theorem 5.1 :* Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be fuzzy (γ,β)- semi continuous mapping. Then,

f $(\tau_\gamma\text{-scl}(A)) \subseteq \tau_\beta\text{-scl}(f(A))$ holds for every subset A of (X,τ)

Let γ be mapping , then for every fuzzy β-semi closed set B of (y,σ) f^{-1} (B) is fuzzy γ- semi closed in (X, τ).

1) *Proof:* (i) Let $y \in f(\tau_\gamma\text{-scl}(A))$ and V be any fuzzy γ- semi open set containing y. Then there exists a point $x \in X$ and fuzzy γ-semi open set U such that f(x) = y and $x \in U$ and $f(U) \subseteq V$. Since $x \in \tau_\gamma$-scl(A) , we have $U \cap A \neq \phi$ and hence $\phi \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap A$. This implies $x \in \tau_\beta$-scl(f(A)).Therefore we have $f(\tau_\gamma\text{-scl}(A)) \subseteq \tau_\beta\text{-scl}(f(A))$.

Let B be a fuzzy β-semi closed set in (Y,σ) . Therefore τ_β-scl(B)=B. By using (i) we have $f(\tau_\gamma\text{-scl}(f^{-1}(B)) \subseteq \tau_\beta\text{-scl}(B)=B$ Therefore we have $\tau_\gamma\text{-scl}(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is fuzzy γ-semi closed by remark 3.24.

D. *Definition 5.2:* A fuzzy mapping f: $(X.\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy (γ, β) - semi closed if for any fuzzy γ-semi closed set A of (X, τ), f(A) is a fuzzy β-semi closed.

Let id: $\tau \rightarrow P(X)$ be the identity fuzzy mapping . Then, if f is fuzzy (id,β)- semi closed , then f(F) is fuzzy β-semi closed set for any fuzzy semi closed set F of (X, τ).

E. *Theorem 5.3:* Suppose that f is fuzzy (γ, β)- semi continuous mapping and f is fuzzy (γ,β)-semi closed. Then,

for every fuzzy γ-semi g closed set A of (X,τ) the image f(A) is β-semi g. closed.

for every fuzzy β-semi g closed set B of fuzzy space (Y,σ) the inverse set $f^{-1}(B)$ is fuzzy γ-semi g closed.

1) *Proof:*(i) Let V be any fuzzy β-semi open set in (Y, σ) such that $f(A) \subseteq V$. By using theorem5.3(ii) $f^{-1}(V)$ is fuzzy γ -semi open. Since, A is fuzzy γ-semi g closed and $A \subseteq f^{-1}(V)$, we have τ_γ-scl$(A) \subseteq f^{-1}(V)$,and hence $f(\tau_\gamma\text{-scl}(A)) \subseteq V$, then f($\tau_\gamma$-

scl(A)) is a fuzzy β-semi closed set. Therefore τ_β-scl(f(A)) ⊆τ_β-scl(f(τ_γ-scl(A))=f(τ_γ-scl(A))⊆V. This implies f(A) is fuzzy β-semi g closed.

Let U be fuzzy γ-semi open set of (X, τ) such that f^{-1}(B) ⊆ U . Put F = γ-scl(f^{-1} (B))∩ U^c. It follows from remark 3.24 and theorem 3.17, that F is fuzzy γ-semi closed set in (X, τ). Since f is fuzzy (γ,β)-semi closed, f(F) is fuzzy β-semi closed in (Y,σ).f(F)⊆ f(τ_γ-scl(f^{-1}(B)∩U^c)) ⊆ τ_β-scl(f(f^{-1}(B))∩f(U^c)) ⊆ τ_β-scl(B)-B. This implies f(F) =ϕ,and hence F=ϕ.Therefore τ_γ-scl(f^{-1}(B))⊆U. Hence f^{-1}(B) is fuzzy γ-semi g closed in (X,τ).

F. THEOREM5.4: Let f : (X, τ) → (Y, σ) is fuzzy (γ, β)- semi continuous and fuzzy (γ, β) – semi closed. Then,

If f is injective and (y, β) is fuzzy β-semi $T_{½}$, then (x, τ) is fuzzy γ-semi T $_{½}$ space.

If f is surjective and (X, τ) is fuzzy γ-semi $T_{½}$, then (Y, σ) is fuzzy β-semi $T_{½}$.

1) *Proof: (*i) Let A is fuzzy γ -semi g. closed set in (X,τ). Now to show that A is fuzzy γ -semi closed. By theorem 5.5 (ii) and assumption it is obtained that f(A) is fuzzy β -semi g closed and hence f(A) is fuzzy β -semi closed.

Let B be fuzzy β -semi g closed set in (Y, σ) then f^{-1} (B) is fuzzy γ-semi closed. Since,(X, τ) is fuzzy γ -semi $T_{½}$. It follows from the assumption that B is fuzzy β -semi closed in (Y, σ). Therefore (y, σ) is fuzzy β- semi $T_{½}$.

G. Definition5.7: A fuzzy mapping f:(X,τ)→(Y, σ) is said to be fuzzy (γ,β)- semi homeomorphism, if f is bijective, fuzzy (γ,β)-semi continuous and f^{-1} is fuzzy (β, γ) – semi continuous.

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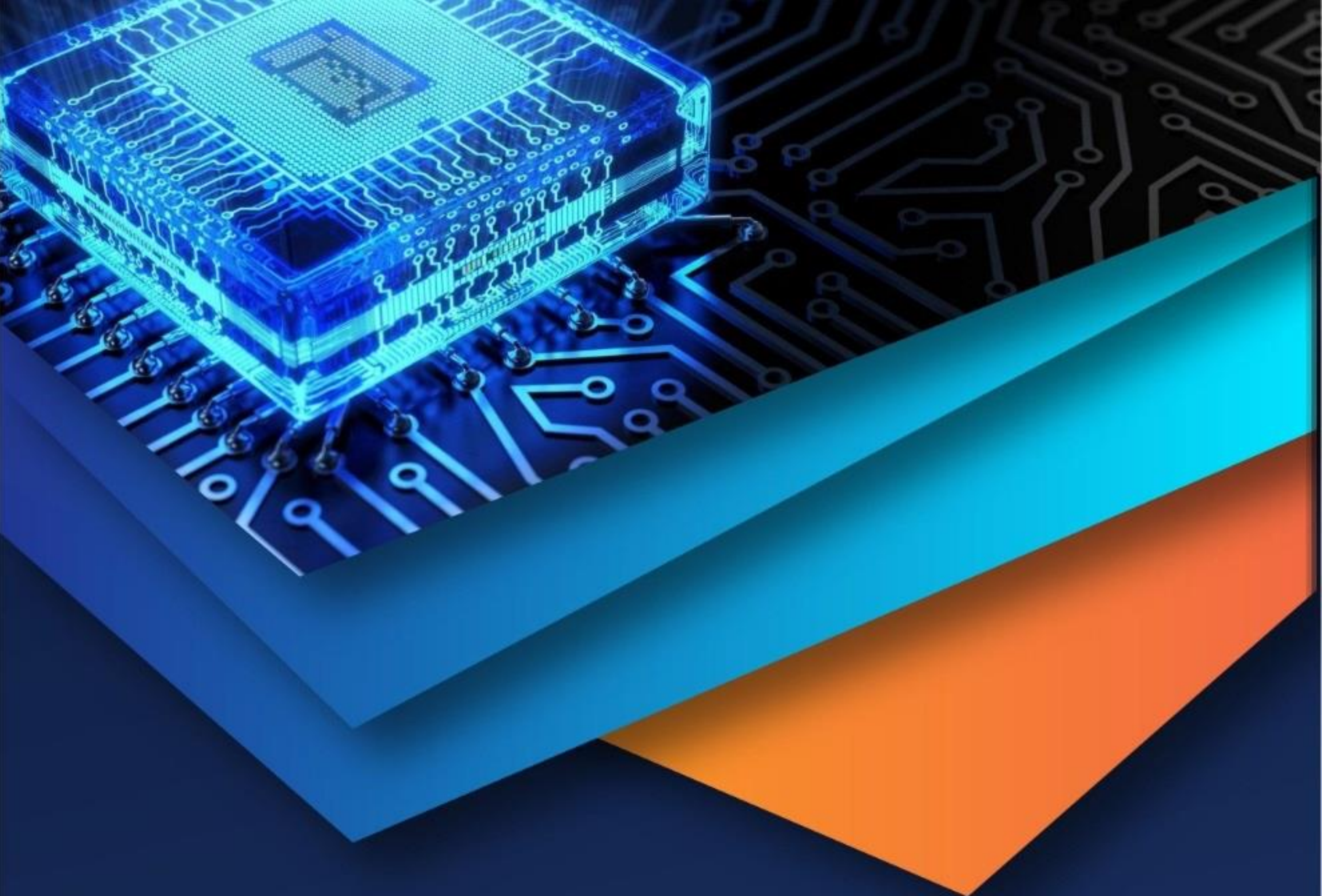

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