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On Almost Cosymplectic Manifolds

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Abstract: In this paper, we have studied almost cosymplectic manifold and its relation with affinely almost cosymplectic and affinely almost nearly cosymplectic manifold. Affine connexions and geodesics have also been discussed.

Key words: Differentiable manifold, vector field, 1-form, Riemannian metric, curvature tensor.

I. INTRODUCTION

Definition (1.1): Let M be a differentiable manifold of dimension n (odd). Let there exist a tensor F of type $(1,1)$, a 1-form u , a vector field U , a Riemannian metric g , satisfying for arbitrary vector fields X, Y, Z, \dots

$$(1.1) \quad \begin{aligned} (a) \quad & \bar{X} = -X + u(X)U, \\ (b) \quad & \bar{U} = 0 \\ (c) \quad & u(X) = g(X, U), \\ (d) \quad & g(\bar{X}, \bar{Y}) = g(X, Y) - u(X)u(Y), \end{aligned}$$

where $\bar{X} = FX$, Then M is called an almost contact metric manifold and $\{M, F, U, u, g\}$ is called an almost contact metric structure.

For an almost contact metric manifold, we can easily prove that,

$$\begin{aligned} (e) \quad & u(U) = 1, \\ (f) \quad & u(\bar{X}) = 0, \\ (g) \quad & g(\bar{X}, Y) = -g(X, \bar{Y}) \\ (h) \quad & \overline{(\nabla_x F)Y} = \overline{\nabla_x \bar{Y}} + \nabla_x Y - u(\nabla_x Y)U, \\ (i) \quad & (\nabla_x u)Y = g(\nabla_x U, Y). \quad \text{Let us put} \\ (j) \quad & \mathcal{F}(X, Y) = g(\bar{X}, Y), \text{ then} \\ (k) \quad & \mathcal{F}(\bar{X}, \bar{Y}) = \mathcal{F}(X, Y) = -\mathcal{F}(Y, X), \\ (l) \quad & g((\nabla_x F)Y, Z) = (\nabla_x F)(Y, Z), \\ (m) \quad & (\nabla_x F)(\bar{Y}, Z) - (\nabla_x F)(Y, \bar{Z}) = u(Y)(\nabla_x u)(Z) + u(Z)(\nabla_x u)(Y) \\ (n) \quad & (d'F)(X, Y, Z) = (\nabla_x F)(Y, Z) + (\nabla_y F)(Z, X) + (\nabla_z F)(X, Y) \end{aligned}$$

Also we have,

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- (o) $(du)(X, Y) = (\nabla_x u)Y - (\nabla_y u)X$. Let
- (p) $N_0(X, Y) = [\bar{X}, \bar{Y}] + \overline{[X, Y]} - \overline{[\bar{X}, Y]} - \overline{[X, \bar{Y}]}$
 $+ du(X, Y)U$, then we get
- (q) $N_0(X, Y) = (\nabla_{\bar{X}} F)Y - (\nabla_{\bar{Y}} F)X - \overline{(\nabla_X F)Y} + \overline{(\nabla_Y F)X}$
 $+ [(\nabla_x u)Y - (\nabla_y u)X]U$.

Definition (1.2) Let M be an almost contact metric manifold. Let F and u satisfy

- (1.2) (a) $(\nabla_x F)Y = 0$
 (b) $(\nabla_x u)Y = 0$

then M is called an affinely almost cosymplectic manifold.

From (1.1) (i) and (1.2) (b), we get

- (c) $\nabla_x U = 0$ We also get
 (d) $(\nabla'_x F)(Y, Z) = 0$
 (e) $(d'F)(X, Y, Z) = 0$
 (f) $(du)(X, Y) = 0$
 (g) $N_0(X, Y) = 0$

Definition (1.3): An almost contact metric manifold, on which F and u satisfy

- (1.3) (a) $(\nabla_x F)X = 0$,
 (b) $(\nabla_x u)X = 0$,

is called affinely almost nearly cosymplectic manifold.

From Definitions (1.2) and (1.3), it is clear that an affinely almost cosymplectic manifold is affinely almost nearly cosymplectic manifold.

II. ALMOST COSYMPLECTIC MANIFOLD

Definition (2.1): An almost contact metric manifold on which F and u satisfy,

- (2.1) (a) $d'F = 0$ or $(\nabla'_X F)(Y, Z) + (\nabla'_Y F)(Z, X) + (\nabla'_Z F)(X, Y) = 0$
 (b) $du = 0$ or $(\nabla_x u)Y - (\nabla_y u)X = 0$, is called an almost cosymplectic manifold, for which

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$$(c) \quad N_0(X, Y) = (\nabla_{\bar{X}}F)Y - (\nabla_{\bar{Y}}F)X - \overline{(\nabla_X F)Y} + \overline{(\nabla_Y F)X}$$

Theorem (2.1): An affinely almost cosymplectic manifold is an almost cosymplectic manifold.

Proof: from (1.2) (e), (f) the result follows easily.

Theorem (2.2): Let M be an almost cosymplectic manifold. Let

$$(2.2) \quad T(X, Y) = (\nabla_{\bar{X}}F)Y - \overline{(\nabla_X F)Y} + (\nabla_X u)(Y)U,$$

$$(2.3) \quad T(X, Y, Z) = g(T(X, Y), Z), \text{ Then}$$

$$(2.4) \quad T(X, Y) - T(Y, X) = N_0(X, Y)$$

$$(2.5) \quad T(X, Y, Z) + T(X, Z, Y) = 0$$

$$(2.6) \quad T(X, Y, Z) + T(Y, Z, X) + T(Z, X, Y) = -\left[(\nabla'_X F)(\bar{Y}, Z) \right. \\ \left. + (\nabla'_Y F)(\bar{Z}, X) + (\nabla'_Z F)(\bar{X}, Y) \right] + (\nabla_X u)(Y)u(Z) \\ + (\nabla_Y u)(Z)u(X) + (\nabla_Z u)(X)u(Y)$$

Proof:

$$T(X, Y) - T(Y, X) \\ = (\nabla_{\bar{X}}F)Y - \overline{(\nabla_X F)Y} + (\nabla_X u)(Y)U \\ (\nabla_{\bar{Y}}F)X + \overline{(\nabla_Y F)X} - (\nabla_Y u)(X)U \quad \text{Thus,}$$

$$(2.7) \quad T(X, Y) - T(Y, X) = (\nabla_{\bar{X}}F)Y - (\nabla_{\bar{Y}}F)X - \overline{(\nabla_X F)Y} + \overline{(\nabla_Y F)X} \\ + \left[(\nabla_X u)Y - (\nabla_Y u)X \right] U$$

Using (2.1) (b), (c), we get

$$T(X, Y) - T(Y, X) = N_0(X, Y)$$

Again $T(X, Y, Z) = g(T(X, Y), Z)$ or

$$(2.8) \quad T(X, Y, Z) = g\left((\nabla_{\bar{X}}F)Y - \overline{(\nabla_X F)Y} + (\nabla_X u)(Y)U, Z \right)$$

Using (1.1) (c), (g), (l) in (2.8), we get

$$(2.9) \quad T(X, Y, Z) = (\nabla'_{\bar{X}}F)(Y, Z) + (\nabla'_X F)(Y, Z) + (\nabla'_X F)(Y, \bar{Z}) + (\nabla_X u)(Y)u(Z)$$

Interchanging Y and Z in (2.9) and adding the resulting equation to (2.9) and making use of (1.1) (m), we get

$$T(X, Y, Z) + T(X, Z, Y) = 0$$

From (2.9), writing similar expressions for $T(Y, Z, X)$ and $T(Z, X, Y)$

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and using (2.1) (a), we get

$$\begin{aligned} T(X, Y, Z) + T(Y, Z, X) + T(Z, X, Y) = & - \left[(\nabla'_X F)(\bar{Y}, Z) \right. \\ & \left. + (\nabla'_Y F)(\bar{Z}, X) + (\nabla'_Z F)(\bar{X}, Y) \right] + (\nabla_X u)(Y)u(Z) \\ & + (\nabla_Y u)(Z)u(X) + (\nabla_Z u)(X)u(Y) \end{aligned}$$

Theorem (2.3): Let M be an almost cosymplectic manifold, define

$$(2.10) \quad L(X, Y) = (\nabla_X F)Y + (\nabla_Y F)X - u(X)Y - u(Y)X + 2g(X, Y)U,$$

$$(2.11) \quad \gamma L(X, Y, Z) = g(L(X, Y), Z); \text{ then}$$

$$(2.12) \quad L(X, Y) = L(Y, X)$$

$$(2.13) \quad \gamma L(X, Y, Z) + \gamma L(Y, Z, X) + \gamma L(Z, X, Y) = 0$$

Proof: with the symmetry of g , concerned expressions and the use of (2.1) (a), give the required results.

Theorem (2.4): Let M be an almost cosymplectic manifold, then

$$(2.14) \quad \begin{aligned} N_0(X, Y) - L(\bar{X}, Y) = & - (\nabla_{\bar{Y}} F)X - \overline{(\nabla_X F)Y} + \overline{(\nabla_Y F)X} \\ & - (\nabla_Y F)\bar{X} + u(Y)\bar{X} - 2g(\bar{X}, Y)U \end{aligned}$$

Proof: The expressions for $N_0(X, Y)$ and $L(\bar{X}, Y)$ give the required result.

III. AFFINE CONNEXION

Definition (3.1): Let ∇ be the Riemannian connexion and B be a connexion, s.t.

$$(3.1) \quad B_X Y = \nabla_X Y + H(X, Y),$$

then B is an affine connexion.

Theorem (3.1): On an affinely almost cosymplectic manifold M .

$$(3.2) \quad B_X U = H(X, U)$$

Proof: from (3.1) taking $Y=U$, and using (1.2) (c), we get the required result.

Theorem (3.2): On an affinely almost cosymplectic manifold with (3.1), if

$$(3.3) \quad (B'_X F)(Y, Z) = 0 \quad \gamma H(X, Y, Z) = g(H(X, Y), Z), \text{ then}$$

$$(3.4) \quad \gamma H(X, \bar{Y}, Z) + \gamma H(X, Y, \bar{Z}) = 0$$

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Proof: from (3.1)

$$\begin{aligned}
 B_X \bar{Y} &= \nabla_X \bar{Y} + H(X, \bar{Y}) \\
 (B_X F)Y + F(B_X Y) &= (\nabla_X F)Y + F(\nabla_X Y) + H(X, \bar{Y}) \\
 (3.5) \quad (B_X F)Y + F(H(X, Y)) &= (\nabla_X F)Y + H(X, \bar{Y})
 \end{aligned}$$

Using (1.2) (a) in (3.5)

$$\begin{aligned}
 (B_X F)Y + \overline{H(X, Y)} &= H(X, \bar{Y}) \\
 (B_X F)Y &= H(X, \bar{Y}) - \overline{H(X, Y)} \\
 g((B_X F)Y, Z) &= g(H(X, \bar{Y}), Z) - g(\overline{H(X, Y)}, Z) \\
 (3.6) \quad (B'_X F)(Y, Z) &= H(X, \bar{Y}, Z) + H(X, Y, \bar{Z})
 \end{aligned}$$

Using (3.3) in (3.6), we get

$$H(X, \bar{Y}, Z) + H(X, Y, \bar{Z}) = 0$$

IV. GEODESIC

Definition (4.1): A curve in a Riemannian manifold with extremum length, is called a geodesic. It's eqn. is

$$(4.1) \quad \nabla_U U = 0$$

Theorem (4.1) On an affinely almost cosymplectic manifold every curve is a geodesic.

Proof: From (1.2) (c), we have

$$\nabla_X U = 0 \Rightarrow \nabla_U U = 0$$

Thus every curve is a geodesic

Theorem (4.2): Let M be an affinely almost cosymplectic manifold. Define

$$(4.2) \quad K(X) = K(X, U, U), \text{ then}$$

$$(4.3) \quad K(X) = 0$$

Proof: $K(X, Y, Z) = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z$ Thus,

$$(4.4) \quad K(X, U, U) = \nabla_X \nabla_U U - \nabla_U \nabla_X U - \nabla_{[X, U]} U$$

Using (1.2) (c) in (4.4), we get -

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$$K(X) = 0$$

REFERENCES

- [1] Blair, D.E.: Two remarks on contact metric structures, *Tohoku, Math. J.* 29 (1977), 319-324.
- [2] Chinea, D., Gonzales, C.: An example of an almost cosymplectic homogenous manifold, in : *Lect. Notes Math.* Vol. 1209, Springer-Verlag, Berlin - Heidelberg - New York (1986), 133-142.
- [3] Cordero, L.A., Fernandez, M., DeLeon, M.: Examples of compact almost contact - manifolds admitting neither Sasakian nor cosymplectic structures, *Atti. Sem. Mat. Univ. Modera* (1985-86), 43-54.
- [4] Endo, H.: On Ricci curvatures of almost cosymplectic manifolds, *An. Stunt. Univ. "A.I.I. Cuza" Iasi, Mat.* 40 (1994), 75-83.
- [5] Fuzimoto, A., Muto, H.: On cosymplectic manifolds, *Tensor N.S.* 28 (1974), 43-52.
- [6] Goldberg, S.I. and Yano, K.: Integrability of almost cosymplectic structures, *Pacific J. Matt.* 31 (1969), 373-382.
- [7] Mishra, R.S.: A course in tensors with applications to Riemannian geometry, II edition, Pothishala Pvt. Ltd. Lajpat Road, Allahabad, (1973).
- [8] Olszak, Z.: Almost cosymplectic manifolds with Kahlerian leaves, *Tensor, N.S.* 46 (1987), 1176-124.
- [9] Olszak, Z.: On almost cosymplectic manifolds, *Kodai Mat. J.* 4 (1981), 239-250.



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