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On Almost Cosymplectic Manifolds

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Abstract: In this paper, we have studied almost cosymplectic manifold and it's relation with affinely almost cosymplectic and affinely almost nearly cosymplectic manifold. Affine connexions and geodesics have also been discussed.

Key words: Differentiable manifold, vector field, 1-form, Riemannian metric, curvature tensor.

I. INTRODUCTION

Definition (1.1): Let M be a differentiable manifold of dimension n (odd). Let there exist a tensor F of type (1,1), a 1-form u, a vector field U, a Riemannian metric g, satisfying for arbitrary vector fields X,Y,Z....

(1.1) (a) $\bar{\bar{X}} = -X + u(X)U,$

(b) $\bar{U} = 0$

(c) $u(X) = g(X,U),$

(d) $g(\bar{X},\bar{Y}) = g(X,Y) - u(X)u(Y),$

where $\bar{X} = FX,$ Then M is called an almost contact metric manifold and $\{M,F,U,u,g\}$ is called an almost contact metric structure.

For an almost contact metric manfiold, we can easily prove that,

(e) $u(U) = 1,$

(f) $u(\bar{X}) = 0,$

(g) $g(\bar{X},Y) = -g(X,\bar{Y})$

(h) $\overline{(\nabla_x F)Y} = \overline{\nabla_x \bar{Y}} + \nabla_x Y - u(\nabla_x Y)U,$

(i) $(\nabla_x u)Y = g(\nabla_x U, Y).$ Let us put

(j) $'F(X,Y) = g(\bar{X},Y),$ *then*

(k) $'F(\bar{X},\bar{Y}) = {}'F(X,Y) = -{}'F(Y,X),$

(l) $g((\nabla_x F)Y,Z) = (\nabla'_x F)(Y,Z),$

(m) $(\nabla_x {}'F)(\bar{Y},Z) - (\nabla'_x F)(Y,\bar{Z}) = u(Y)(\nabla_x u)(Z) + u(Z)(\nabla_x u)(Y)$

(n) $(d'F)(X.Y,Z) = (\nabla'_x F)(Y,Z) + (\nabla'_y F)(Z,X) + (\nabla'_z F)(X,Y)$

Also we have,

(o) $(du)(X,Y)=(\nabla_x u)Y-(\nabla_Y u)X.$ Let

(p) $N_0(X,Y)=[\bar{X},\bar{Y}]+\overline{\overline{[X,Y]}}-\overline{[\bar{X},Y]}-\overline{[X,\bar{Y}]}$
$+du(X,Y)U,$ then we get

(q) $N_0(X,Y)=(\nabla_{\bar{X}}F)Y-(\nabla_{\bar{Y}}F)X-\overline{(\nabla_X F)Y}+\overline{(\nabla_Y F)X}$
$+[(\nabla_x u)Y-(\nabla_Y u)X]U.$

Definition (1.2) Let M be an almost contact metric manifold. Let F and u satisfy

(1.2) (a) $(\nabla_x F)Y=0$

(b) $(\nabla_x u)Y=0$

then M is called an affinely almsot cosymplectic manifold.

From (1.1) (i) and (1.2) (b), we get

(c) $\nabla_x U=0$ We also get

(d) $(\nabla'_x F)(Y,Z)=0$

(e) $(d'F)(X,Y,Z)=0$

(f) $(du)(X,Y)=0$

(g) $N_0(X,Y)=0$

Definition (1.3): An almost contact metric manfiold, on which F and u satisfy

(1.3) (a) $(\nabla_x F)X=0,$

(b) $(\nabla_x u)X=0,$

is called affinely almost nearly cosymplectic manifold.

From Definitions (1.2) and (1.3), it is clear that an affinely almost cosymplectic manifold is affinely almost nearly cosymplectic manifold.

II. ALMOST COSYMPLECTIC MANIFOLD

Definition (2.1): An almost contact metric manifold on which F and u satisfy,

(2.1) (a) $d'F=0 \; or \; (\nabla'_X F)(Y,Z)+(\nabla'_Y F)(Z,X)+(\nabla'_Z F)(X,Y)=0$

(b) $du=0 \; or \; (\nabla_X u)Y-(\nabla_Y u)X=0,$ is called an almost cosymplectic manifold, for which

(c) $$N_0\left(X,Y\right)=\left(\nabla_{\bar{X}}F\right)Y-\left(\nabla_{\bar{Y}}F\right)X-\overline{\left(\nabla_X F\right)Y}+\overline{\left(\nabla_Y F\right)X}$$

Theorem (2.1): An affinely almost cosymplectic manifold is an almost cosymplectic manifold.

Proof: from (1.2) (e), (f) the result follows easily.

Theorem (2.2): Let M be an almost cosymplectic manifold. Let

(2.2) $$T\left(X,Y\right)=\left(\nabla_{\bar{X}}F\right)Y-\overline{\left(\nabla_X F\right)Y}+\left(\nabla_X u\right)\left(Y\right)U,$$

(2.3) $$'T\left(X,Y,Z\right)=g\left(T\left(X,Y\right),Z\right),$$ Then

(2.4) $$T\left(X,Y\right)-T\left(Y,X\right)=N_0\left(X,Y\right)$$

(2.5) $$'T\left(X,Y,Z\right)+{}'T\left(X,Z,Y\right)=0$$

(2.6) $$'T\left(X,Y,Z\right)+{}'T\left(Y,Z,X\right)+{}'T\left(Z,X,Y\right)=-\Big[\left(\nabla'_X F\right)\left(\bar{Y},Z\right)+\left(\nabla'_Y F\right)\left(\bar{Z},X\right)+\left(\nabla'_Z F\right)\left(\bar{X},Y\right)\Big]+\left(\nabla_X u\right)\left(Y\right)u\left(Z\right)+\left(\nabla_Y u\right)\left(Z\right)u\left(X\right)+\left(\nabla_Z u\right)\left(X\right)u\left(Y\right)$$

Proof:

$$T\left(X,Y\right)-T\left(Y,X\right)$$
$$=\left(\nabla_{\bar{X}}F\right)Y-\overline{\left(\nabla_X F\right)Y}+\left(\nabla_X u\right)\left(Y\right)U$$
$$\left(\nabla_{\bar{Y}}F\right)X+\overline{\left(\nabla_Y F\right)X}-\left(\nabla_Y u\right)\left(X\right)U$$ Thus,

(2.7) $$T\left(X,Y\right)-T\left(Y,X\right)=\left(\nabla_{\bar{X}}F\right)Y-\left(\nabla_{\bar{Y}}F\right)X-\overline{\left(\nabla_X F\right)Y}+\overline{\left(\nabla_Y F\right)X}+\Big[\left(\nabla_X u\right)Y-\left(\nabla_Y u\right)X\Big]U$$

Using (2.1) (b), (c), we get

$$T\left(X,Y\right)-T\left(Y,X\right)=N_0\left(X,Y\right)$$

Again $'T\left(X,Y,Z\right)=g\left(T\left(X,Y\right),Z\right)$ or

(2.8) $$'T\left(X,Y,Z\right)=g\left(\nabla_{\bar{X}}F\right)Y-\overline{\left(\nabla_X F\right)Y}+\left(\nabla_X u\right)\left(Y\right)U,Z\right)$$

Using (1.1) (c), (g), (l) in (2.8), we get

(2.9) $$'T\left(X,Y,Z\right)=\left(\nabla'_{\bar{X}}F\right)\left(Y,Z\right)+\left(\nabla'_X F\right)\left(Y,Z\right)+\left(\nabla'_X F\right)\left(Y,\bar{Z}\right)+\left(\nabla_X u\right)\left(Y\right)u\left(Z\right)$$

Interchanging Y and Z in (2.9) and adding the resulting equation to (2.9) and making use of (1.1) (m), we get

$$'T\left(X,Y,Z\right)+{}'T\left(X,Z,Y\right)=0$$

From (2.9), writing similar expressions for $'T\left(Y,Z,X\right)$ and $'T\left(Z,X,Y\right)$

and using (2.1) (a), we get

$$'T(X,Y,Z) + 'T(Y,Z,X) + 'T(Z,X,Y) = -\Big[(\nabla'_X F)(\bar{Y},Z) + (\nabla'_Y F)(\bar{Z},X) + (\nabla'_Z F)(\bar{X},Y)\Big] + (\nabla_X u)(Y)u(Z) + (\nabla_Y u)(Z)u(X) + (\nabla_Z u)(X)u(Y)$$

Theorem (2.3): Let *M* be an almost cosymplectic manifold, define

(2.10) $L(X,Y) = (\nabla_X F)Y + (\nabla_Y F)X - u(X)Y - u(Y)X + 2g(X,Y)U,$

(2.11) $'L(X,Y,Z) = g(L(X,Y),Z)$; then

(2.12) $L(X,Y) = L(Y,X)$

(2.13) $'L(X,Y,Z) + 'L(Y,Z,X) + 'L(Z,X,Y) = 0$

Proof: with the symmetry of g, concerned expressions and the use of (2.1) (a), give the required results.

Theorem (2.4): Let *M* be an almost cosymplectic manifold, then

(2.14) $$\underset{0}{N}(X,Y) - L(\bar{X},Y) = -(\nabla_{\bar{Y}} F)X - \overline{(\nabla_X F)Y} + \overline{(\nabla_Y F)X} - (\nabla_Y F)\bar{X} + u(Y)\bar{X} - 2g(\bar{X},Y)U$$

Proof: The expressions for $\underset{0}{N}(X,Y)$ *and* $L(\bar{X},Y)$ give the required result.

III. AFFINE CONNEXION

Definition (3.1): Let ∇ be the Riemannian connexion and B be a connexion, s.t.

(3.1) $B_X Y = \nabla_X Y + H(X,Y),$

then B is an affine connexion.

Theorem (3.1): On an affinely almost cosymplectic manifold M.

(3.2) $B_X U = H(X,U)$

Proof: from (3.1) taking *Y=U*, and using (1.2) (c), we get the required result.

Theorem (3.2): On an affinely almost cosymplectic manifold with (3.1), if

(3.3) $(B'_X F)(Y,Z) = 0 \; 'H(X,Y,Z) = g(H(X,Y),Z)$, then

(3.4) $'H(X,\bar{Y},Z) + 'H(X,Y,\bar{Z}) = 0$

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Proof: from (3.1)

$$B_X\bar{Y} = \nabla_X\bar{Y} + H\left(X,\bar{Y}\right)$$

$$\left(B_X F\right)Y + F\left(B_X Y\right) = \left(\nabla_X F\right)Y + F\left(\nabla_X Y\right) + H\left(X,\bar{Y}\right)$$

(3.5) $$\left(B_X F\right)Y + F\left(H\left(X,Y\right)\right) = \left(\nabla_X F\right)Y + H\left(X,\bar{Y}\right)$$

Using (1.2) (a) in (3.5)

$$\left(B_X F\right)Y + \overline{H\left(X,Y\right)} = H\left(X,\bar{Y}\right)$$

$$\left(B_X F\right)Y = H\left(X,\bar{Y}\right) - \overline{H\left(X,Y\right)}$$

$$g\left(\left(B_X F\right)Y,Z\right) = g\left(H\left(X,\bar{Y}\right),Z\right) - g\left(\overline{H\left(X,Y\right)},Z\right)$$

(3.6) $$\left(B'_X F\right)\left(Y,Z\right) = {}'H\left(X,\bar{Y},Z\right) + {}'H\left(X,Y,\bar{Z}\right)$$

Using (3.3) in (3.6), we get

$${}'H\left(X,\bar{Y},Z\right) + {}'H\left(X,Y,\bar{Z}\right) = 0$$

IV. GEODESIC

Definition (4.1): A curve in a Riemannian manifold with extremum length, is called a geodesic. It's eqn. is

(4.1) $$\nabla_U U = 0$$

Theorem (4.1) On an affinely almost cosymplectic manifold every curve is a geodesic.

Proof: From (1.2) (c), we have

$$\nabla_X U = 0 \;\Rightarrow \nabla_U U = 0$$

Thus every curve is a geodesic

Theorem (4.2): Let *M* be an affinely almost cosymplectic manifold. Define

(4.2) $$K\left(X\right) = K\left(X,U,U,\right),$$ then

(4.3) $$K\left(X\right) = 0$$

Proof: $K\left(X,Y,Z\right) = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$ Thus,

(4.4) $$K\left(X,U,U,\right) = \nabla_X\nabla_U U - \nabla_U\nabla_X U - \nabla_{[X,U]}U$$

Using (1.2) (c) in (4.4), we get -

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$$K(X) = 0$$

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