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Self-Weak Complementary Fuzzy Soft Graph

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Abstract: In this paper, we discussed the notion of fuzzy soft graph and concepts of homomorphism, isomorphism, weak isomorphism, self-weak isomorphism of fuzzy soft graphs. Also some properties of isomorphism on fuzzy soft graphs, self-complementary and self-weak complementary fuzzy soft graphs are discussed.

Keywords: Weak isomorphism, self-complementary fuzzy soft graphs, self-weak complementary fuzzy soft graphs.

I. INTRODUCTION

The concept of fuzzy set theory was introduced by zadeh. A [1]. This concept have potential application in various fields such as the smoothness of functions, medical and life sciences, social sciences, Engineering graph theory, artificial intelligence, robotics, computer, networks, decision making and automata theory. The concept of fuzzy soft sets with is a new mathematical tool was firstly introduced by Maji et al [2]. They presented the definition of fuzzy soft sets and investigated some properties at this notion. Therefore many researchers have applied this concept on different branches. In 1975, Rosenfeld [8] introduced the concept of fuzzy graphs. Therefore many researchers have generalized the different notions of graph theory using the notions of fuzzy sets. P. Bhattacharya in [3] showed that a fuzzy graph can be associated with a fuzzy group in a natural way as an automorphism group. K.R. Bhutani in [4] introduced the concept of weak isomorphism and isomorphism between fuzzy graphs. In this paper we define the concept of homomorphism, isomorphism, weak isomorphism, self weak isomorphism of fuzzy soft graph. We also study some of their important properties.

II. PRELIMINARIES

In this section ,We recall some basic notion of graph ,fuzzy graph, soft graph and fuzzy soft graphs

A. Definition:2.1

A *graph* G = (V, E) consists of a non-empty set of objects V called vertices and a set E of two element subset of V called edges.

B. Definition:2.2[5]

Let V be a non-empty finite set μ: V → [0, 1] and v =VxV → [0, 1].If v(x, y) ≤ μ (x) Λ μ(y) for all x, y ∈ V. Then the pair G =(μ,v) is called a *fuzzy graph* over the set V. Here μ and v are called *fuzzy vertex* and *fuzzy edge* of the fuzzy graph (μ, v).

C. Definition:2.3[6]

Let (F, A) be a soft set over V. Then (F, A) is said to be a *soft graph* of G if the subgraph induced by F(x) in G. F(x) is a connected subgraph of G for all x ∈ A.

D. Definition:2.4

A *fuzzy soft graph* $\tilde{G} = (G^*, \tilde{F}, \tilde{K}, A)$ is a 4 tuple such that

(i) G* = (V, E) is a simple graph

(ii) A is a nonempty set of parameters

(iii) $(\tilde{F}, A)$ is a fuzzy soft set over V.

(iv) $(\tilde{K}, A)$ is a fuzzy soft set over E.

(v) $(\tilde{F}(a), \tilde{K}(a))$ is a fuzzy (sub) graph of G*, for all a ∈A.

That is $\widetilde{K}$ (a) (xy) ≤ min $\{\tilde{F}(a)(x), \tilde{F}(a)(y)\}$ for all a ∈ A, x, y ∈ V. The fuzzy graph ($\tilde{F}$(a), $\widetilde{K}$(a)) is denoted by $\widetilde{H}$ (a) for convenience.

On the other hand, a fuzzy soft graph is a parameterized family of fuzzy graphs. The class of all fuzzy soft graphs of G* is denoted by F(G*).

E. Definition:2.5

Let $\widetilde{G_1}$ = (G*,$\widetilde{F_1}$, $\widetilde{K_1}$, A) and $\widetilde{G_2}$ = (G*,$\widetilde{F_2}$,$\widetilde{K_2}$, A) be two fuzzy soft graphs. A *homomorphism* f : $\widetilde{G_1} \rightarrow \widetilde{G_2}$ is a mapping f : $V_1 \rightarrow V_2$ which satisfies the following conditions.

(i) $\widetilde{F_1}$ (a) (x) ≤ $\widetilde{F_2}$ (a)(f(x))
(ii) $\widetilde{K_1}$ (a)(xy) ≤ $\widetilde{K_2}$ (a) (f (x)f(y)) for all a∈A, x, y ∈ V_1, x y ∈ E.

F. Definition:2.6

Let $\widetilde{G_1}$ = (G*, $\widetilde{F_1}$,$\widetilde{K_1}$, A) and $\widetilde{G_2}$ = (G*, $\widetilde{F_2}$, $\widetilde{K_2}$, A) be two fuzzy soft graphs. An *isomorphism*

f :$\widetilde{G_1} \rightarrow \widetilde{G_2}$ is a Bijective mapping f : $V_1 \rightarrow V_2$ which satisfies the following conditions.

(i) F_1 (a) (x) = F_2 (a (f(x))
(ii) K_1(a)(xy) = K_2(a)(f(x)f(y)) for all a ∈A, x, y ∈V_1, xy ∈ E

G. Definition:2.7

Let $\widetilde{G_1}$ = (G*, $\widetilde{F_1}$,$\widetilde{K_1}$, A) and $\widetilde{G_2}$ = (G*, $\widetilde{F_2}$, $\widetilde{K_2}$, A) be two fuzzy soft graphs. Then a *weak isomorphism*

f :$\widetilde{G_1} \rightarrow \widetilde{G_2}$ is a bijective mapping f : $V_1 \rightarrow V_2$ which satisfies the following conditions.

(i) f is homomorphism
(ii) F_1(a) (x) = F_2 (a) (f(x)) for all a ∈A, x ∈V_1

H. Definition:2.8

Let $\widetilde{G_1}$ = (G*, $\widetilde{F_1}$, $\widetilde{K_1}$, A) and $\widetilde{G_2}$ = (G*,$\widetilde{F_2}$, $\widetilde{K_2}$, A) be two fuzzy soft graphs. A *Self Weak Isomorphism*

f :$\widetilde{G_1} \rightarrow \widetilde{G_2}$ is a bijective mapping f : $V_1 \rightarrow V_2$ which satisfies the following conditions.

(i) f is homomorphism
(ii) $\widetilde{K_1}$ (a)(xy) = $\widetilde{K_2}$ (a) (f(x)f(y)) for all a ∈A, x, y ∈ V_1, xy ∈ E.

III. SELF COMPLEMENTARY FUZZY SOFT GRAPHS

A. Definition:3.1

A fuzzy soft graph G is self complementary if G≅ G' where G' is the complement of fuzzy soft graph G.

B. Theorem :3.1

If $\widetilde{G}$ = (G*, $\tilde{F}$, $\widetilde{K}$, A) be a self complementary fuzzy soft graph then

$$\sum_{x=y}\widetilde{K}(a)\,(xy) \le \frac{1}{2}\sum_{x=y} min[\,(\tilde{F}(a)\,(x),\,\tilde{F}(a)\,(y))]$$

Proof:

Let $\tilde{G}$ = (G*, $\tilde{F}$, $\tilde{K}$, A) be a self complementary fuzzy soft graph. Where $\widetilde{G_1}$ is a finite set. Then there exists an isomorphism f :$\widetilde{G_1} \rightarrow \widetilde{G_1}$ is a mapping f : $V_1 \rightarrow V_1$

$\widetilde{F_1}^C$ (a) (x) = $\widetilde{F_1}$ (a) (x) = $\widetilde{F_1}$ (a) (f(x)) --------------------------------- (3.1.1)

$\widetilde{K_1}^C$ (a) (xy) = $\widetilde{K_1}$ (a) (xy) = $\widetilde{K_1}$ (a) (f(x) f(y)) --------------------------------- (3.1.2)

By definition of complement of fuzzy soft graph,

$\widetilde{K_1}^C$ (a) (xy) $=\widetilde{F_1}^C$ (a) (x) $\wedge \widetilde{F_1}^c$ (a) (y) $-\widetilde{K^c{}_1}$ (a) (f(x) f(y))

By using (3.1.1) and (3.1.2) we get,

$\widetilde{K_1}$ (a) (xy) = $\widetilde{F_1}$ (a) (x) $\wedge \widetilde{F_1}$ (a) (y) $-\widetilde{K_1}$ (a) (f(x) f(y))

$\widetilde{K_1}$ (a) (xy) + $\widetilde{K_1}$ (a) (f(x) f(y)) = $\widetilde{F_1}$ (a) (x) $\wedge \widetilde{F_1}$ (a) (y)

again using (3.1.1) and (3.1.2) we get,

$\widetilde{K_1}$ (a) (xy) +$\widetilde{K_1}$ (a)(xy) = $\widetilde{F_1}$ (a) (x) $\wedge \widetilde{F_1}$ (a) (y)

$\Rightarrow 2\widetilde{K_1}$ (a) (xy) = $\widetilde{F_1}$ (a) (x) $\wedge \widetilde{F_1}$ (a) (y)

$\Rightarrow \widetilde{K_1}$ (a) (xy) $=\frac{1}{2}\widetilde{F_1}$ (a) (x) $\wedge \widetilde{F_1}$ (a) (y)

similarly, we have

$\sum \widetilde{K_1}$ (a) (xy) = $\frac{1}{2}\sum_{x=y}$min[$\widetilde{F_1}$(a) (x), $\widetilde{F_1}$ (a) (y)]

In general,

$\sum_{x=y}\tilde{K}$(a) (xy) $\leq \frac{1}{2}\sum_{x=y} min$[($\tilde{F}$(a) (x), $\tilde{F}$(a) (y))] . Hence $\tilde{G}$ be a self complementary fuzzy soft graph.

C. Definition:3.2

A fuzzy soft graph $\tilde{G}$ is self weak complementary if $\tilde{G}$ is weak isomorphic with $\tilde{G}^C$, where $\tilde{G}^C$ is the complement of fuzzy soft graph$\tilde{G}$.

$\widetilde{F_1}$ (a) (x) = $\widetilde{F_1}$ (a) (f(x)) -------------------------------(3.2.1)

$\widetilde{K_1}$ (a) (xy) =$\widetilde{K_1}^C$ (a) (f(x) f(y)) -------------------------------(3.2.2)

D. Theorem 3.2.1

Let $\tilde{G}$ = (G*, $\tilde{F}$, $\tilde{K}$, A) be a self weak complementary fuzzy soft graph then

$\sum_{x=y}\widetilde{K_1}$(a) (xy) $\leq \frac{1}{2}\sum_{x=y}[\widetilde{F_1}$ (a) (x) $\wedge \widetilde{F_1}$(a) (y)]

1) *Proof:* Let $\tilde{G}$ = (G*, $\tilde{F}$, $\tilde{K}$, A) be a self weak complementary fuzzy soft graph. Then there exists a weak isomorphism f :$V_1 \rightarrow V_1$ such that f(x) = x, and $\widetilde{K_1}$(a) (xy) $\leq \widetilde{K_1}^C$ (a) (f(x) f(y))

By definition of complement of fuzzy soft graph

$\widetilde{K_1}^C$ (a) (f(x), f(y)) $=\widetilde{F_1}$ (a) (f(x)) $\wedge \widetilde{F_1}$ (a)(f(y)) - $\widetilde{K_1}$(a)(f(x), f(y)) ----------------------------(3.2.3)

By using (3.2.2) in (3.2.3),we have,

$\widetilde{K_1}$(a) (xy) $\leq \widetilde{F_1}$ (a)(f(x)) $\wedge\widetilde{F_1}$ (a) (f(y))-$\widetilde{K_1}$(a) (f(x) f(y))

By using (3.2.1) in (3.2.3) we have,

$\widetilde{K_1}$(a) (xy)$\leq \widetilde{F_1}$ (a)(x) $\wedge\widetilde{F_1}$ (a) (y) $-\widetilde{K_1}$(a) [f(x) f(y)]

$\widetilde{K_1}$(a) (xy) $+\widetilde{K_1}$(a)(f(x) f(y) $\leq \widetilde{F_1}$ (a) (x)$\wedge\widetilde{F_1}$ (a) (y)

Similarly, we have

$\sum_{x=y}\widetilde{K_1}$(a) (xy) $+\sum_{x=y}\widetilde{K_1}$(a) (f(x) f(y)) $\leq \sum_{x=y}[\widetilde{F_1}(a)\,(x) \wedge \widetilde{F_1}(a\ \ (y)]$ ---------------------------(3.2.4)

By using (3.2.2) in (3.2.4),we have,

$\sum_{x=y}\widetilde{K_1}$(a) (xy) + $\sum_{x=y}\widetilde{K_1}$(a) (xy) $\leq \sum_{x=y}[\widetilde{F_1}(a)(x) \wedge \widetilde{F_1}\ (a)\ (y)]$

2 $\sum_{x=y}\widetilde{K_1}$(a) (xy) $\leq \sum_{x=y}[\widetilde{F_1}\ (a)(x) \wedge \widetilde{F_1}\ (a)(y)]$

$\sum_{x=y}\widetilde{K_1}$(a) (xy) $\leq \frac{1}{2}\sum_{x=y}$ [$\widetilde{F_1}$ (a) (x), $\wedge\widetilde{F_1}\ (a)(y)]$

Hence $\tilde{G}$ is a self weak complementary fuzzy soft graph.

by equation (3.2.5))

$\widetilde{K_1}^c$ (a) (f(x), f(y)) $\geq \frac{1}{2}\Sigma$ (F_1 (a) (x) $\widetilde{F_1}$ (a) (y)]

$\geq \sum_{x=y}\widetilde{K_1}$ (a) (xy) $\forall$ xy ϵ Identity map.

i.e., $\sum_{x=y}\widetilde{K_1}$ (a) (xy) $\leq \widetilde{K_1}$ (a) (f(x), f(y)) --------------------------(3.2.8)

From (3.2.6) and (3.2.8) $\tilde{G} is$ weak isomorphic with $\tilde{G}^c$ therefore G is a self weak complementary fuzzy soft graph.

IV. CONCLUSION

Fuzzy soft graph theory is an extremely useful tool in solving the combinational problems in different areas. In this paper we gave some new concepts such as weak isomorphism, self weak complementary fuzzy soft graph and studied some at their properties.

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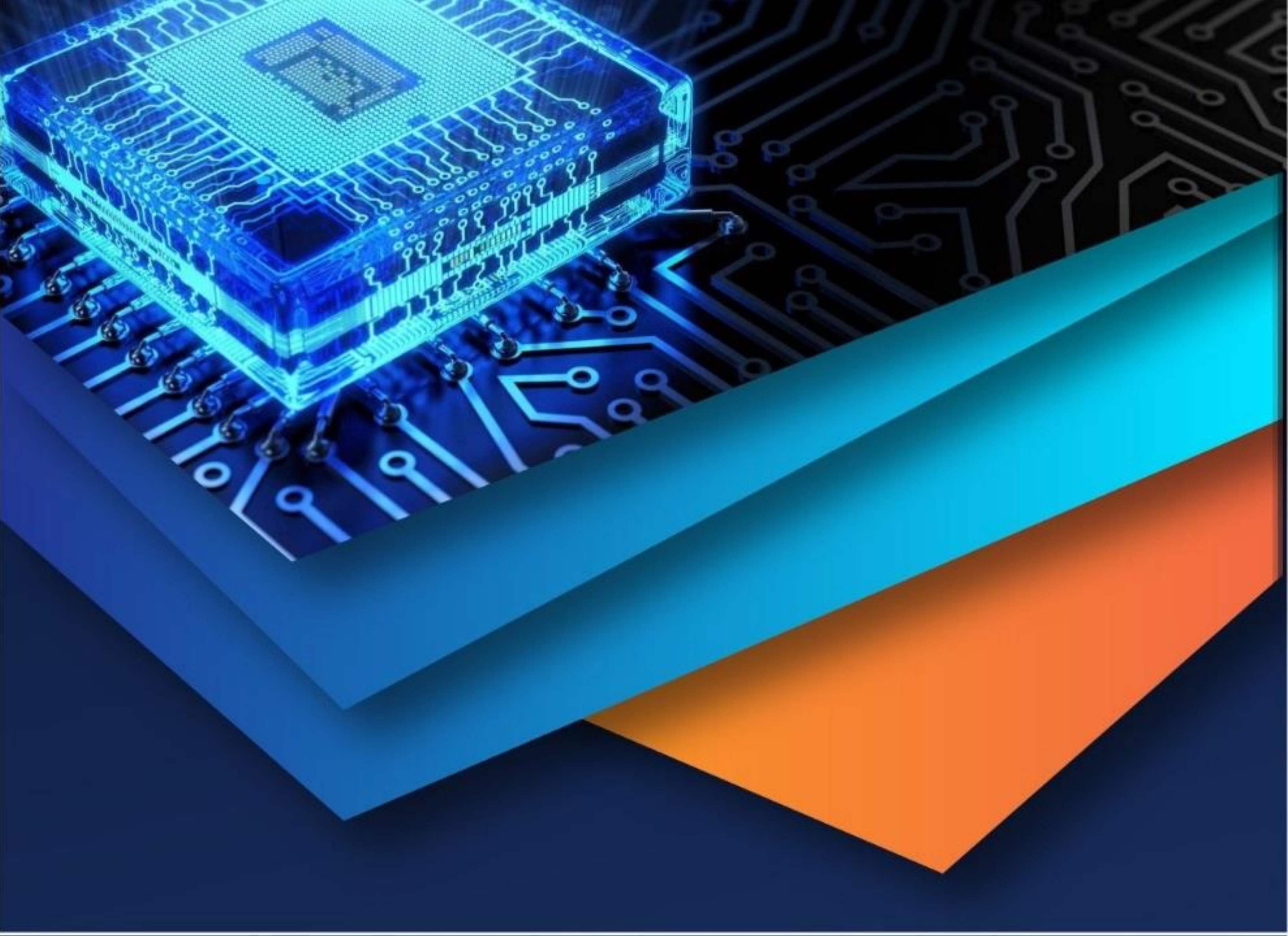

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