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M/M/c/N/K Loss and Delay Interdependent Queueing Model with Controllable Arrival Rates and Reverse Balking

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Abstract: In this Paper, an M/M/c/N/K loss and delay interdependent queueing model with controllable arrival rates and reverse balking is considered. For this model, the steady state probabilities are derived and the average waiting times for the two types of customers (elective and emergency) are obtained for varying arrival rates when the arrival and service processes are interdependent. Sensitivity analysis are given for a better understanding.
Keywords: Finite Capacity, Interdependent Controllable Arrival and Service rates, Loss and delay, Reverse balking, Single Server, Bivariate Poisson process.

I. INTRODUCTION

In the loss and decay queueing system, the customers are classified into two classes. They are (i) Elective customers and (ii) Emergency customers. The elective customers have patience to form a queue and wait. While the emergency customers find the server busy on their arrival, leave the system and are lost. The arrival and service processes are taken to be independent in most of these Jain (1998) models. Balking behavior of the customers is also considered due to which the customers may not like to join the queue on seeing it very long. The notion of customer balking appears in queuing theory in the works of Haight [1]. He has analysed M/M/1 queue with balking in which queue length is infinite. Jain and Rakesh Kumar [2] have studied M/M/1/N queueing system with reverse balking (a queueing system that indicates the probability of balking will be low when the queue size is more).

Along with several other assumptions, it is customary to consider that the arrival and service processes are independent. However in many particular situations, it is necessary to consider that the arrival and services processes are inter dependent. A queueing model in which arrivals and services are correlated is known as interdependent queuing Model. Much work has been reported in the literature regarding interdependent standard queuing model with controllable arrival rates. In Jain (1998) [3] analysed the finite population loss and delay queueing system with no passing concept.K. Srinivasa Rao, Shobha and P. Srinivasa Rao [4] have discussed M/M/1/∞ interdependent queuing model with controllable arrival rates. A.Srinivasan and M. Thiagarajan [5,6], have analysed M/M/1/K interdependent queuing model with controllable arrival rates, M/M/C/K/N interdependent queuing Model with controllable arrival rates balking, reneging and spares. Srinivasan and Thiagarajan (2007) have analyzed M/M/c/ /K loss and delay interdependent queueing model with controllable arrival rates and no passing.

In this paper, a finite population, loss and delay, interdependent queueing model with controllable arrival rates and reverse balking has been considered with the assumption that the arrival and service processes of the system are correlated and follows a bivariate Poisson process. Here the arrival rate is considered as, λ_0-a faster rate of arrival and λ_1 –a slower rate of arrival. Whenever the queue size reaches a certain prescribed number R, the arrival rate reduces from λ_o to λ_1 and it continues with that rate as long as the content in the queue is greater than some other prescribed integer r ($r \geq 0$ & $r < R$).When the content reaches r, the arrival rate changes back to λ_o and the same process is repeated. In section 2, the description of the model is given stating the relevant postulates. In section 3, the steady state equations are obtained. In section 4, the characteristics of the model are derived In section5, the analytical results are numerically illustrated .

II. DESCRIPTION OF THE MODEL

Consider a c-server finite capacity loss and delay queueing system with the following assumptions:

(i) The arrival process $\{X_1(t)\}$ and the service process $\{X_2(t)\}$ of the system are correlated and follow a bivariate Poisson process given by

$$P\{X_1(t)=x_1, X_2(t)=x_2\} = e^{-(\lambda_{ij}+\mu_n-\in)t} \sum_{m=0}^{\min(x_1,x_2)} \frac{(\in t)^m[(\lambda_{ij}-\in)tp']^{x_1-m}[(\mu_n-\in)t]^{(x_2-m)}}{m!(x_1-m)!(x_2-m)!}$$

…(2.1)

where $x_1, x_2 = 0,1,2,\ldots; \lambda_{01}, \lambda_{02}, \lambda_{11} > 0$; $\mu_n > 0$

$$0 \le \in < \min(\lambda_{ij}, \mu_n)$$

$$n = 0,1,2,\ldots, c-1, c, c+1, \ldots, r-1, r, r+1, \ldots, R-1, R, R+1, \ldots, K-1, K$$

with parameters $\lambda_{01}, \lambda_{02}, \lambda_{11}, \mu_n, p'$ and $\in$ as mean arrival rate of elective customers when the system is in faster rate of arrival, mean arrival rate of emergency customers when the system is in faster rate of arrival, mean arrival rate of elective customers when the system is in slower rate of arrival, mean service rate of customers of type B, probability of reverse balking and the co-variance between the arrival and service process respectively.

The mean arrival rate and the mean service rate when the system size is n is defined as

$$\lambda_n = \begin{cases} (K-n)\lambda_{0j}; if\ 0 \le n < c,\ j=1,2 \\ (K-n)\lambda_{01}; if c \le n \le R-1 \\ (K-n)\lambda_{11}; if r+1 \le n \le K \end{cases}$$

$$\mu_n = \begin{cases} n\mu; 0 \le \mathrm{n} < \mathrm{c} \\ c\mu; \mathrm{c} \le \mathrm{n} \le K \end{cases}$$

A. *The postulates of the model are*

1) The probability that there is no arrival with loss and delay, reverse balking and no service completion during a small interval of time *h*, when the system is in faster rate of arrivals is $1-[(\lambda_0 - 2\in)Kp' + (\mu_n - \in)]h + o(h)$
2) The probability that there is one arrival with loss and delay, reverse balking and no service completion during a small interval of time *h*, when the system is in faster rate of arrivals is $(\lambda_0 - 2\in)Kp'h + o(h)$
3) The Probability that there is no arrival with loss and delay, reverse balking and no service completion during a small interval of time *h* at state n, when the system is in faster rate of arrival is $1-\left[\left(\frac{n}{N-1}\right)(\lambda_{01}-\in)(K-n) + c(\mu_n - \in)\right]h + o(h)$
4) The Probability that there is no arrival with loss and delay, reverse balking and no service completion during a small interval of time *h* at state *n*, when the system is in slower rate of arrival is $1-\left[\left(\frac{n}{N-1}\right)(\lambda_{11}-\in)(K-n) + c(\mu_n - \in)\right]h + o(h)$
5) The probability that there is no arrival with loss and delay, reverse balking and one service completion during a small interval of time *h* state *n*, when the system is either in faster or slower rate of arrivals is $c(\mu_n - \in)h + o(h)$
6) The Probability that there is one arrival with reverse balking and one service completion during a small interval of time *h*, when the system is either in faster or slower rate of arrivals is $\in h + o(h)$

III. STEADY STATE EQUATIONS

We observe that only $P_n(0)$ exists when *n* = 0,1,2,...,c-1,c,c+1,…, *r -1,r;* both $P_n(0)$ *and* $P_n(1)$ exist when *n = r+1, r+2, …,R-2,R-1* ; only $P_n(1)$ exists when *n=R, R+1,…,K.* Further $P_n(0) = P_n(1) = 0$ when n > K.

The steady state equations are given by

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$$0 = -K(\lambda_0 - 2\in) p' P_0(0) + (\mu - \in) P_1(0) \quad \text{... (3.1)}$$

$$0 = -\left[(K-1)\left(\frac{1}{N-1}\right)(\lambda_0 - 2\in) + (\mu - \in) \right] P_1(0) + 2(\mu - \in) P_2(0) + K(\lambda_0 - 2\in) p' P_0(0)$$

…(3.2)

$$0 = -\left[(K-n)\left(\frac{n}{N-1}\right)(\lambda_0 - 2\in) + n(\mu - \in) \right] P_n(0) + (n+1)(\mu - \in) P_{n+1}(0)$$
$$+ (K - n + 1)\left(\frac{n-1}{N-1}\right)(\lambda_0 - 2\in) P_{n-1}(0) \qquad (2 \le n \le c-1)$$

…(3.3)

$$0 = -\left[(K-c)\left(\frac{c}{N-1}\right)(\lambda_{01} - \in) + c(\mu - \in) \right] P_c(0) + c(\mu - \in) P_{c+1}(0)$$

…(3.4)

$$+ (K - c + 1)\left(\frac{c-1}{N-1}\right)(\lambda_0 - 2\in) P_{c-1}(0)$$

$$0 = -\left[(K-n)\left(\frac{n}{N-1}\right)(\lambda_{01} - \in) + c(\mu - \in) \right] P_n(0) + c(\mu - \in) P_{n+1}(0)$$
$$+ (K - n + 1)\left(\frac{n-1}{N-1}\right)(\lambda_{01} - \in) P_{n-1}(0) \qquad (c+1 \le n \le r-1)$$

…(3.5)

$$0 = -\left[(K-r)\left(\frac{r}{N-1}\right)(\lambda_{01} - \in) + c(\mu - \in) \right] P_r(0) + c(\mu - \in) P_{r+1}(0)$$
$$+ c(\mu - \in) P_{r+1}(1) + (K - r + 1)\left(\frac{r-1}{N-1}\right)(\lambda_{01} - \in) P_{r-1}(0)$$

...(3.6)

$$0 = -\left[(K-n)\left(\frac{n}{N-1}\right)(\lambda_{01} - \in) + c(\mu - \in) \right] P_n(0) + c(\mu - \in) P_{n+1}(0)$$
$$+ (K - n + 1)\left(\frac{n-1}{N-1}\right)(\lambda_{01} - \in) P_{n-1}(0) \qquad (r+1 \le n \le R-2)$$

…(3.7)

$$0 = -\left[(K-R+1)\left(\frac{R-1}{N-1}\right)(\lambda_{01} - \in) + c(\mu - \in) \right] P_{R-1}(0)$$
$$+ (K - R + 2)\left(\frac{R-2}{N-1}\right)(\lambda_{01} - \in) P_{R-2}(0)$$

…(3.8)

$$0 = -\left[(K-r-1)\left(\frac{r+1}{N-1}\right)(\lambda_{11} - \in) + c(\mu - \in) \right] P_{r+1}(1) + c(\mu - \in) P_{r+2}(1)$$

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…(3.9)

$$0=-\left[(K-n)\left(\frac{n}{N-1}\right)(\lambda_{11}-\in)+c(\mu-\in)\right]P_n(1)+c(\mu-\in)\,P_{n+1}(1)$$
$$+(K-n+1)\left(\frac{n-1}{N-1}\right)\,(\lambda_{11}-\in)P_{n-1}(1) \qquad (r+2\le n\le R-1)$$

…(3.10)

$$0=-\left[(K-R)\left(\frac{R}{N-1}\right)(\lambda_{11}-\in)+c(\mu-\in)\right]P_R(1)+c(\mu-\in)\,P_{R+1}(1)$$
$$+(K-R+1)\left(\frac{R-1}{N-1}\right)(\lambda_{01}-\in)P_{R-1}(0)$$
$$+(K-R+1)\left(\frac{R-1}{N-1}\right)\,(\lambda_{11}-\in)\,\mathrm{P}_{R-1}(1)$$

…(3.11)

$$0=-\left[(K-n)\left(\frac{n}{N-1}\right)(\lambda_{11}-\in)+c(\mu-\in)\right]P_n(1)+c(\mu-\in)\mathrm{P}_{n+1}(1)$$
$$+(K-n+1)\left(\frac{n-1}{N-1}\right)\,(\lambda_{11}-\in)P_{n-1}(1) \qquad (\mathrm{R}+1\le \mathrm{n}\le N\text{-}1)$$

…(3.12)

$$0=-[c(\mu-\in)]P_N(1)+(K-N+1)(\lambda_{11}-\in)P_{N-1}(1) \qquad \ldots (3.13)$$

From (3.1) to (3.8) we get

$$P_n(0)=\begin{cases}\left[\frac{1}{(N-1)}\right]^{n-1}\left[\frac{(\lambda_0-2\in)^n}{n(\mu-\in)^n}\right](K)_n\,p'P_0(0),\ ,,n=1,2,\ldots,\ c\\[2ex] \left[\frac{1}{(N-1)}\right]^{n-1}\left[\frac{(\lambda_0-2\in)^c(\lambda_{01}-\in)^{n-c}}{(\mu-\in)^n}\right]\left[\frac{1}{c^{n-c+1}}\right](K)_n\left[\prod_{l=c}^{n-1}l\right]p'P_0(0),\ n=c+1,c+2,\ldots,\ r\\[2ex] \left[\frac{1}{(N-1)}\right]^{r}\left[\frac{(\lambda_0-2\in)^c(\lambda_{01}-\in)^{r-c+1}}{(\mu-\in)^{r+1}}\right]\left[\frac{1}{c^{r-c+2}}\right](K)_{r+1}\left[\prod_{l=c}^{r}l\right]p'P_0(0)-P_{r+1}(1),\ \mathrm{n}=\mathrm{r}+1\\[2ex] \left[\frac{1}{(N-1)}\right]^{n-1}\left[\frac{(\lambda_0-2\in)^c(\lambda_{01}-\in)^{n-c}}{(\mu-\in)^n}\right]\left[\frac{1}{c^{n-c+1}}\right](K)_n\left[\prod_{l=c}^{n-1}l\right]p'P_0(0)\\ -\left[\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)^{n-r-1}(n-1)\,p_{n-r-1}(K-r-1)_{n-r-1}\right.\\ +\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)^{\mathrm{n\text{-}r\text{-}2}}(\mathrm{n}\text{-}1)\ \mathrm{p}_{n-r-2}(K-r-2)_{n-r-2}+\ldots\\ \left.+\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)^{\mathrm{n\text{-}R+1}}(\mathrm{n}\text{-}1)\ \mathrm{P}_{n-R+1}(K-R+1)_{n-R+1}\right]P_{r+1}(1),\ \mathrm{n}=\mathrm{r}+2,\mathrm{r}+3,\ldots,\ \mathrm{R}\text{-}1\end{cases}$$

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… (3.14)

From (3.9) to (3.13) we get

$$P_n(1)=\left[\left(\frac{\lambda_{11}-\in}{c(\mu-\in)(N-1)}\right)^{n-r-1}(n-1)P_{n-r-1}(K-r-1)_{n-r-1}+\left(\frac{\lambda_{11}-\in}{c(\mu-\in)(N-1)}\right)^{n-r-2}(n-1)P_{n-r-2}\right.$$

$$\left.(K-r-2)_{n-r-2}+\ldots+\left(\frac{\lambda_1-\in}{c(\mu-\in)(N-1)}\right)^{n-R}(n-1)P_{n-R}(K-R)_{n-R}\right]P_{r+1}(1) \quad \ldots (3.15)$$

$$n=r+1,r+2,\ldots,R+2,R+3,\ldots,N$$

where

$$P_{r+1}(1)=\frac{\left[\frac{1}{(N-1)}\right]^{R-1}\left[\frac{(\lambda_0-2\in)^c(\lambda_{01}-\in)^{R-c}}{(\mu-\in)^R}\right]\left[\frac{1}{c^{R-c+1}}\right](K)_R\left[\prod_{l=c}^{R-1}l\right]p'P_0(0)}{\left\{\frac{(R-1)!}{r!}\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)^{R-r-1}(K-r-1)_{R-r-1}+\frac{(R-1)!}{(r+1)!}\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)^{R-r-2}(K-r-2)_{R-r-2}+\ldots..+\frac{(R-1)!}{(R-2)!}\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)(K-R+1)_{n-R+1}+1\right\}}$$

The probability $P_0(0)$ that the system is empty can be calculated from the normalizing condition $P(0)+P(1)=1$

$$P(0)+P(1)=P_0(0)+\sum_{n=1}^{c}\left[\frac{1}{(N-1)}\right]^{n-1}\left[\frac{(\lambda_0-2\in)^n}{n(\mu-\in)^n}\right](K)_n\,p'P_0(0)$$

$$+\sum_{n=c+1}^{R-1}\left[\frac{1}{(N-1)}\right]^{n-1}\left[\frac{(\lambda_0-2\in)^c(\lambda_{01}-\in)^{n-c}}{(\mu-\in)^n}\right]\left[\frac{1}{c^{n-c+1}}\right](K)_n\left[\prod_{l=c}^{n-1}l\right]p'P_0(0)$$

$$+\left[-\sum_{n=r+1}^{R-1}\left[\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)^{n-r-1}(n-1)p_{n-r-1}(K-r-1)_{n-r-1}\right.\right.$$

$$+\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)^{n-r-2}(n-1)p_{n-r-2}(K-r-2)_{n-r-2}$$

$$\left.+\ldots+\left(\frac{\lambda_{01}-\in}{(N-1)c(\mu-\in)}\right)^{n-R+1}(n-1)p_{n-R+1}(K-R+1)_{n-R+1}\right]$$

$$+\sum_{n=r+1}^{N}\left[\left(\frac{\lambda_{11}-\in}{(N-1)c(\mu-\in)}\right)^{n-r-1}(n-1)p_{n-r-1}(K-r-1)_{n-r-1}\right.$$

$$+\left(\frac{\lambda_{11}-\in}{(N-1)c(\mu-\in)}\right)^{n-r-2}(n-1)p_{n-r-2}(K-r-2)_{n-r-2}$$

$$\left.\left.+\ldots.+\left(\frac{\lambda_{11}-\in}{(N-1)c(\mu-\in)}\right)^{n-R}(n-1)p_{n-R}(K-R)_{n-R}\right]\right]P_{r+1}(1)P_0(0) \quad \ldots(3.16)$$

IV. CHARACTERSTICS OF THE MODEL

The probability $P(0)$ that the system is in faster rate of arrival is

$$P(0)=\sum_{n=0}^{N} P_n(0)$$

Since $P_n(0)$ exists only when n=0,1,2,...,.r-1,r, r+1, r+2,...,R-2, R-1, we get

$$P(0)=P_0(0)+\sum_{n=1}^{r} P_n(0)+\sum_{n=r+1}^{R-1} P_n(0) \quad \ldots(4.1)$$

The Probability that the system is in slower rate of arrival is

$$P(1)=\sum_{n=0}^{N} P_n(1)=\sum_{n=0}^{r} P_n(1)+\sum_{n=r+1}^{R-1} P_n(1)+\sum_{n=R}^{N} P_n(1) \quad \ldots(4.2)$$

Since $P_n(1)$ exists only when n=r+1, r+2, ...,R-2, R-1,...,N we get

$$P(1)=\sum_{n=r+1}^{N} P_n(1) \quad \ldots(4.3)$$

The expected number of customers in the system is give by

$$L_s=L_{so}+L_{s1} \quad \ldots(4.4)$$

where

$$L_{s_0}=\sum_{n=0}^{c} nP_n(0)+\sum_{n=c+1}^{r} nP_n(0)+\sum_{n=r+1}^{R-1} nP_n(0) \quad \ldots(4.5)$$

and

$$L_{s1}=\sum_{n=r+1}^{R-1} nP_n(1)+\sum_{n=R}^{N} nP_n(1) \quad \ldots(4.6)$$

Therefore

$$L_s=\sum_{n=0}^{c} nP_n(0)+\sum_{n=c+1}^{r} nP_n(0)+\sum_{n=r+1}^{R-1} nP_n(0)+\sum_{n=r+1}^{N} nP_n(1) \quad \ldots(4.7)$$

From (3.14) and (3.16),we get

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$$L_S = \sum_{n=1}^{c} n \left[\frac{1}{(N-1)}\right]^{n-1} \left[\frac{(\lambda_0 - 2\in)^n}{n(\mu - \in)^n}\right] (K)_n \, p' P_0(0)$$

$$+ \sum_{n=c+1}^{R-1} n \left[\frac{1}{(N-1)}\right]^{n-1} \left[\frac{(\lambda_0 - 2\in)^c (\lambda_{01} - \in)^{n-c}}{(\mu - \in)^n}\right] \left[\frac{1}{c^{n-c+1}}\right] (K)_n \left[\prod_{l=c}^{n-1} l\right] p' P_0(0)$$

$$- \left[n \sum_{n=r+1}^{R-1} \left[\left(\frac{\lambda_{01} - \in}{(N-1)c(\mu - \in)}\right)^{n-r-1} (n-1) p_{n-r-1} (K-r-1)_{n-r-1} \right.\right.$$

$$+ \left(\frac{\lambda_{01} - \in}{(N-1)c(\mu - \in)}\right)^{n-r-2} (n-1) p_{n-r-2} (K-r-2)_{n-r-2}$$

$$\left. + \ldots + \left(\frac{\lambda_{01} - \in}{(N-1)c(\mu - \in)}\right)^{n-R+1} (n-1) p_{n-R+1} (K-R+1)_{n-R+1} \right]$$

$$+ n \sum_{n=r+1}^{N} \left[\left(\frac{\lambda_{11} - \in}{(N-1)c(\mu - \in)}\right)^{n-r-1} (n-1) p_{n-r-1} (K-r-1)_{n-r-1} \right.$$

$$+ \left(\frac{\lambda_{11} - \in}{(N-1)c(\mu - \in)}\right)^{n-r-2} (n-1) p_{n-r-2} (K-r-2)_{n-r-2}$$

$$\left.\left. + \ldots. + \left(\frac{\lambda_{11} - \in}{(N-1)c(\mu - \in)}\right)^{n-R} (n-1) p_{n-R} (K-R)_{n-R} \right]\right] P_{r+1}(1) P_0(0)$$

…(4.8)

Using Little's formula, the expected waiting time of the customers in the system is given by

$$W_s = \frac{L_s}{\bar{\lambda}} \qquad \text{Where } \bar{\lambda} = \lambda_0 P(0) + \lambda_1 P(1)$$ …(4.9)

This model includes the particular cases as, when p' =1 this model reduces to M/M/c/N loss and delay interdependent queueing model with controllable arrival rates discussed by Srinivasan and Thiagarajan (2007).

V. SENSITIVITY ANALYSIS

For r=2, R=5, N=6, K=8

Case 1. Probability P(0) that the system is in faster rate of arrivals and Probability P(1) that the system is in slower rate of arrivals by varying λ_0 keeping other parameters fixed

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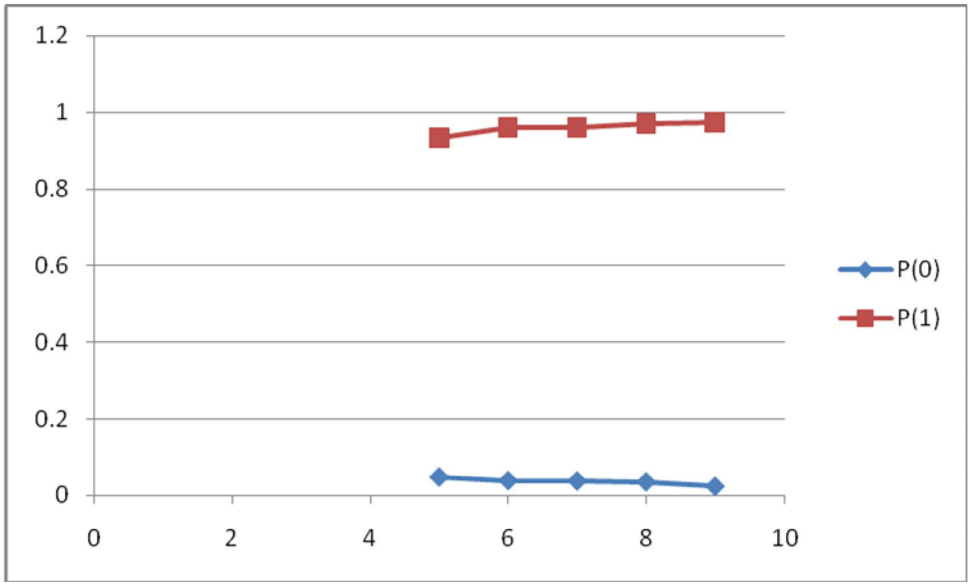
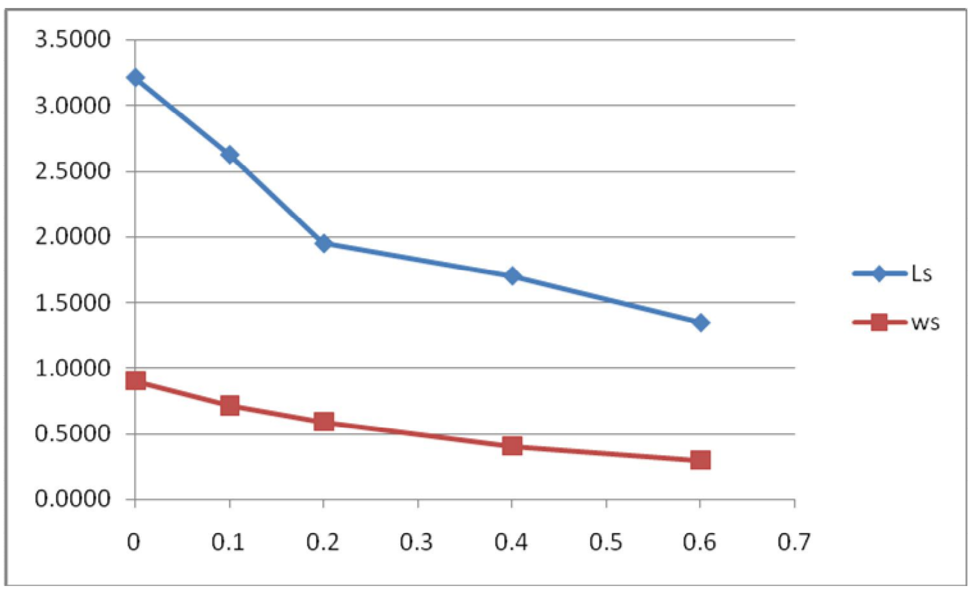


Case 2. Average number of customers in the system (Ls) and expected waiting time of the customers in the system (Ws) by varying β keeping other parameters fixed



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