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Properties of Dominator of an M-Semigroup

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Abstract: In this paper we discuss few properties of a collection of a special type of element of an M-semigroup, namely dominator. In an M-semigroup a dominator may be empty or properly contained in it or equal to the semigroup itself.

Keywords: M- semi group, dominator, idempotent elements, Rectangular Band.

I. INTRODUCTION

In this paper we find a position of a dominator D in an M-semigroup M. We also find a necessary and sufficient condition for the existence of the dominator, a necessary and sufficient condition for a dominator $D = M$. Further, we decompose the dominator D, in which case decomposed part is a semi-inflation of the dominator. We also discuss some properties of dominator of an M-semigroup.

II. PRELIMINARIES

- 1) A subset S' of a semigroup S is said to be a sub semi group of S if S' is a semigroup with the same binary operation of S.
- 2) A semigroup S is said to be right(left) singular if for all x,y in S, $xy=y$ ($xy=x$) such a semigroup is also called a right(left) zero semigroup
- 3) Let X and Y be any two nonempty sets. Then the system $(S=X \times Y; *)$ where $(x,y) * (x',y') = (x,y')$; For all x,x' in X and y,y' in Y is a band. It is called a rectangular band on $X \times Y$ (3)
- 4) A decomposition of a semigroup S is meant a partition of S into union of disjoint subsemigroups S_i of S, where $i \in A$, an index set.

A decomposition as above is sometimes denoted by $\bigcup_{i \in A} S_i$; it is also said that S is decomposed over A

- 5) Let $S = \bigcup_{i \in A} S_i$ be a decomposition of a semigroup S into subsemigroups S_i over an index A. If for each (i,j) in $A \times A$ there exists an element k of A such that $S_i S_j \subseteq S_k$ then A becomes thereby a band. It is then said that S is the union of the band A of semigroups $S_i (i \in A)$; sometimes it is also said that, "S is a band A of semigroups $S_i, i \in A$ "
- 6) If a semigroup S is the union of a band A of semigroups $S_i (i \in A)$ then A is the homomorphic image of S under the homomorphism,
 $f : S \rightarrow A, xf = i$ for x in $S_i (i \in A)$ and the semigroups $S_i (i \in A)$ are the congruence classes of S induced by the homomorphism f.
- 7) If $f : S \rightarrow A$ is a homomorphism of a semigroup S onto a band A then S is the union of the band A of semigroups $S_i = (i)f^{-1}, i \in A$
- 8) If A is band of type ζ , S is a band A of semigroups $S_i (i \in A)$, and each semigroup $S_i (i \in A)$ is a semigroup of type \mathfrak{S} , then S is called as a ζ -band A of \mathfrak{S} -semigroup.

The concept that is being defined now is due to Clifford and Preston (1). Let B be a semigroup. With each i of B, associate a set G_i consisting i (i in B) which are mutually disjoint. Let $G = \bigcup G_i (i \text{ in } B)$ and let the product in B be extended to a product in G by defining $xy = ij$ if x is in G_i and y in $G_j (i,j \text{ in } B)$. Then G is a semigroup which is called an Inflation of B. The following result is also due to the above authors:

- 9) The definition of inflation as given below is due to Tamura (8).

Let B be a given semigroup. Let S be any semigroup. Then S is an inflation of B if and only if,

S contains B as a semigroup,

B contains a homomorphic image of S,

10) An element d of a semigroup S is called a dominator element of S if $dyd=d$ for all y in S . By D , the dominator of S , is meant the set of all dominator elements of S .

III. DOMINATOR OF AN M-SEMIGROUP

A. Definition

An element x of a semigroup S is said to be a dominator of S if and only if $xyx = x$ for all $y \in S$ [2]. The set D of all dominators of S is called the dominator of S denoted by D . The dominator of a semigroup may be empty.

1) Examples: The following are examples of M-semigroups in which the dominator $D = \phi$.

(i)

	e	f	a	b
e	e	f	a	b
f	e	f	a	b
a	a	b	e	f
b	a	b	e	f

(ii)

	e	f	a	b	c	d
e	e	f	a	b	c	d
f	e	f	a	b	c	d
a	a	b	a	b	c	d
b	a	b	a	b	c	d
c	c	d	c	d	a	b
d	c	d	c	d	a	b

The following are two examples of M-semigroups which contains a proper dominator D .

(i)

	e	f	a	b
e	e	f	a	b
f	e	f	a	b
a	a	b	a	b
b	a	b	a	b

Here $D = \{a, b\}$.

(ii)

	e	f	a	b	c	d
e	e	f	a	b	c	d
f	e	f	a	b	c	d
a	a	b	a	b	a	b
b	a	b	a	b	a	b
c	c	d	a	b	a	b
d	c	d	a	b	a	b

Here $D = \{a, b\}$.

Examples of M-semigroups in which the dominator is itself is aright zero semigroup.

B. Lemma: A dominator $D \neq \phi$ of an M-semigroup $M \cong R \times S$ has the following properties:

- (i) $D \cap R = \phi$ or $D = R = M$
- (ii) $D \cap Me \cong D \cap Mf$; $e, f \in R$
- (iii) $D = \bigcup_{e \in R} (D \cap Me)$.

1) Proof: (i) If $D \cap R \neq \phi$. Let e belongs to $D \cap R$. For any x belongs to M , $xe = xe$.

But, $xe = e$ since $e \in D$.

Therefore $xe = e$, for all x belongs to M .

Therefore for any a belongs to M , $ea = xea$.

That is, $a = xa$, for all x belongs to M .

That is, every element of M is a right zero element and hence M is a right zero semigroup [2.2] which is a rectangular band.

Therefore $M = R = D$, i.e. $R \cap D = \phi$ or $R = M = D$.

(ii) Follows from for any ideal I of an M-semigroup $M I \cap Me \cong I \cap Mf$; $e, f \in R$.[4]

A semigroup S contains a dominator if and only if it contains an ideal I which is a rectangular band. Then I is dominator of S [2].

Each $I \cap Me, e \in R$ is a left ideal of I [4].

C. Lemma

Every ideal I of an M-semigroup $M \cong R \times S$ is a disjoint union of subsemigroup $I \cap Me, e \in R$. That is,

$$I = \bigcup_{e \in R} (I \cap Me) . [4]$$

D. Lemma

A semigroup S contains a dominator if and only if it contains an ideal I which is a rectangular band. Then I is the dominator of S [1]. Follows from 3.3

The following lemmas gives the conditions for the existence of the dominator in an M-semigroup.

E. Lemma:

In an M-semigroup $M \cong R \times S$, if the dominator D exists then $D \subseteq E \setminus R$ where E is the set of idempotents of M .

1) Proof: Since D is a rectangular band ideal,

$D \subseteq E$ being a rectangular band and $D \cap R = \phi$ being an ideal.

If $R = E$ and D exists, then $D \cap E = \phi$ and $D \subseteq E$ implies $D = \phi$.

Hence the lemma.

F. Lemma

In a left cancellative M-semigroup $M \cong R \times S$, $D = \phi$ or $D = M$.

1) *Proof:* If M is a left cancellative then every idempotent of M is a left identity. That is, $E = R$.

Therefore $D \subset E = R$.

From 3.2(i), $D = \phi$ or $D = M = R$.

G. Lemma

In an M-semigroup $M \cong R \times S$, if the dominator D of M is equal to M then M is left cancellative.

1) *Proof:* $D = M$ implies $D \cap R \neq \phi$ and $D \neq \phi$

implies $D = R$ by 4.8(i)

implies $D = M = R$.

That is $xyx = x = x^2$ for all x, y belongs to $R = M$.

That is, if $xy = xz$, then $y = z$ since x belongs to R.

Hence M is left cancellative, and hence the lemma. From 3.6 and 3.7 we have:

H. Theorem

If an M-semigroup $M \cong R \times S$ has a nonempty dominator D, then $D = M$ if and only if M is left cancellative.

I. Lemma

In an M-semigroup $M \cong R \times S$ if any one of the left identities e of R is primitive then $D \neq M$ implies $D = \phi$.

1) *Proof:* Let a particular $e \in R$ be primitive. For any idempotent g of Me, $ge = g$ and $eg = g$.

Therefore, $ge = eg = g$.

That is, $e = g$, since e is primitive.

Hence, e is the only idempotent in Me.

Let $D \neq M$, if $D \neq \phi$, D being the kernel of M, D intersects all Me, $e \in R$ and $D \cap R = \phi$. This implies, there are idempotent elements other than e in Me, for all e belongs to R. This contradicts the property that e is primitive.

Hence the lemma.

J. Theorem

An M-semigroup $M \cong R \times S$ contains a dominator D, if and only if S contains a rectangular band ideal.

1) *Proof:* Let S contains a rectangular band ideal De. That is Me, $e \in R$ contains a rectangular band ideal De, $e \in R$. Consider

$$D = \bigcup_{e \in R} De.$$

Since.

For any x belongs to De, a $Me \cong Mf$, $De \cong Df$ ($e, f \in R$) and a belongs to Df,

$$ax = axx = (ax)x$$

$$= (\text{element of } Me) \times \in De.$$

$$xa = xaa \in Df.$$

$$xax = xaxx = x(ax)x = x.$$

Therefore D is a rectangular band. For any xe belongs to De , and af belongs to Mf , $e, f \in R$,

$$xe \cdot af = xaf = xf \cdot af \in Df, \quad \text{since } xf \in Df.$$

$$af \cdot xe = axe \in De.$$

That is D is a rectangular band ideal and hence D is the dominator.

Conversely, let M contain a dominator D. D is a rectangular band ideal of M. That is,

$$D = \bigcup_{e \in R} (D \cap Me), \text{ for a fixed } e \in D \cap Me \subseteq Me.$$

Since D is an ideal of M, $D \cap Me$ is an ideal of Me .

Let xe, ye belong to $D \cap Me$.

Then $xe \cdot xe = xe$, since xe belongs to D.

$$xe \cdot ye \cdot xe = xe, \text{ since } xe, ye \text{ belongs to } D.$$

Therefore $D \cap Me$ is a rectangular band for all $e \in R$.

Since $S \cong Me$, S contains a rectangular band ideal.

Since the dominator of an M-semigroup is an ideal, we have the following:

IV. CONCLUSION

This paper discussed the "Properties of Domination of an M semi group".

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